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Fine splitting in the charmonium spectrum with a channel coupling effect*

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Abstract: We study the fine splitting in the charmomium spectrum in the quark model with the channel coupling effect, including DD, DD*, D*D* and D_sD_s, $D_sD_s^*$, $D_s^*D_s^*$ channels. The interaction for channel coupling is constructed from the current-current Lagrangian related to the color confinement and the one-gluon exchange potentials. By adopting the massive gluon propagator from the lattice calculation in the nonperturbative region, the coupling interaction is further simplified to four-fermion interaction. The numerical calculation still prefers the assignment 1^{++} of X(3872).

Key words: quark model, four-fermion interaction, coupled-channel, X(3872)

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1 Introduction

A series of hidden charm states, the so-called X, Y, Z, have been discovered and confirmed by experiments since 2003. The nature of these narrow resonances has attracted much attention, because their properties are not consistent with the prediction of the quark model.

The typical X(3872) state, which was discovered in 2003 by the Belle Collaboration [1] and subsequently confirmed by the CDF Collaboration [2] and BABAR Collaboration [3], etc., is now listed with $M_X = 3872.2\pm0.8$ MeV, $\Gamma_X = 3.0^{+1.9}_{-1.4}\pm0.9$ MeV in PDG [4]. Its quantum numbers were inferred $J^{PC}=1^{++}$ or 2^{-+}. The corresponding charmonium candidate in the quark model is 2^3P_1 or 1^1D_2 respectively.

The mass of the 2^3P_1 state in the quark model is ~ 100 MeV above M_X. However, the channel coupling effects by the creation of open charmed meson pairs can produce significant mass shift to the bare charmonium spectrum. In Ref. [5], only the fine splitting in the mass shift induced by open-charm states is considered. In Refs. [6, 7], the whole mass shift is considered to lower the bare mass of the excited charmonium state. The mass shift can also be handily treated by introducing screened potential into the quark model [8].

The proximity of the X(3872) to DD* threshold implies that the cusp scenario may be important [9]. The cusp can be calculated from channel coupling and the result is in qualitative agreement with experiment [10]. The observed but Okubo-Zweig-Iizuka (OZI) forbidden decay channel ρJ/ψ is also considered in Ref. [11].

Recently, a study of the $\pi^+\pi^-\pi^0$ mass distribution from the X(3872) decay by the BABAR Collaboration favors the negative parquantum numbersity assignment 2^{-+} [12]. However, the mass of the corresponding charmonium state 1^1D_2 in the quark model is ~ 100 MeV below M_X. Since the ψ(3770) is assigned to 1^3D_1 in the quark model, the assignment 2^{-+} seems to conflict with the small fine splitting in $c\bar{c}$ $1D$ multiplet from the quark model calculation [13].

The mechanism of channel coupling is the same as strong decay's. The simplest decay model is the so-called $^3P_0$ model based on the flux-tube-breaking model [14, 15]. Another model is the Cornel model which tries to relate the pair-creation interaction to the potential in the quark model [16, 17]. The Cornel

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model assumes the Lorentz vector confinement so the total vector potential is

$$V(r)=-\frac{\kappa}{r}+\frac{r}{a^2}. \tag{1}$$

Thus in the Cornel model the decay amplitude from the one-gluon exchange and that from the confinement add destructively. A similar calculation but using the Lorentz scalar confinement shows that the decay amplitude from the scalar linear confinement is too large [18].

The lattice calculation shows that the gluon propagator is quite different in the nonperturbative region. The gluon may get a mass of about 600–1000 MeV [19–21]. A non-vanishing gluon mass is used in the phenomelogical calculation of the diffractive scattering [22] and radiative decays of the J/ψ and Υ [23].

In this work, we will consider the fine splitting induced by channel coupling with open-charm states, including DD, DD^*, D^*D^* and D_sD_s, $D_sD_s^*$, $D_s^*D_s^*$. Follwing the Cornel model, we will construct the model pair-creation interaction from the potential in the quark model, i.e. the scalar confinement plus the vector one-gluon exchange. With the assumption of the massive gluon propagator in the pair-creation process, we will obtain a simple effective four-fermion interaction which is quite similar to the case of weak interaction. In Sec. 2, we will introduce the channel coupling model. In Sec. 3, the numerical analysis is performed. Finally, we will give a brief summary.

2 The channel coupling model

In the simplest version of the channel coupling model [7], the hadronic state is assumed to be represented by

$$|\Psi_\alpha\rangle=\begin{pmatrix} c_\alpha|\psi_\alpha\rangle \\ \sum_i \chi_{\alpha i}|M_1(i)M_2(i)\rangle \end{pmatrix}, \tag{2}$$

where the bare state $|\psi_\alpha\rangle$ is coupled to several meson-meson channels $|M_1(i)M_2(i)\rangle$. The system Hamiltonian reads

$$\hat{H}=\begin{pmatrix} \hat{H}_c & \hat{V} \\ \hat{V} & \hat{H}_{M_1M_2} \end{pmatrix}, \tag{3}$$

where $\hat{H}_c$ is the meson Hamiltonian of the quark model, with

$$\hat{H}_c|\psi_\alpha\rangle=M_\alpha|\psi_\alpha\rangle. \tag{4}$$

In this work, $\hat{H}_{M_1M_2}$ includes only the free meson Hamiltonian, so

$$\hat{H}_{M_1M_2}=\hat{H}_{M_1}+\hat{H}_{M_2}. \tag{5}$$

The Hamiltonian in the non-relativistic quark potential model can always be written as [7]

$$\hat{H}_c=\hat{H}_0+\hat{H}_{sd}, \tag{6}$$

where $\hat{H}_0$ and $\hat{H}_{sd}$ are the spin-independent and spin-dependent parts respectively. The spin-independent part reads

$$\hat{H}_0=\frac{p^2}{2\mu}+V(r)+C, \tag{7}$$

μ is the reduced mass. The potential $V(r)$ is usually taken to be a sum of the linear confinement plus the one-gluon exchange Coulomb potential:

$$V(r)=\sigma r-\frac{4}{3}\frac{\alpha_s}{r}. \tag{8}$$

$\hat{H}_{sd}$ includes spin-spin, spin-orbit and tensor force:

$$H_{sd}=V_{HF}(r)\boldsymbol{S}_1\cdot\boldsymbol{S}_2+V_{LS}(r)\boldsymbol{L}\cdot\boldsymbol{S}+V_T(r)T, \tag{9}$$

which determines the fine splitting in the spectrum.

The off-diagonal interaction $\hat{V}$ is responsible for channel coupling. It depends on the pair-creation mechanism of the specific hadron decay model. The $^3P_0$ model [14, 15] and the Cornel model [16, 17] are two popular decay models.

To describe the creation of a light-quark pair in the quark model, a plausible approach is to consider the quantum field expression of the quark potential $V(r)$. In the Cornell model, the quark potential is replaced by an instantaneous interaction [16, 17]

$$H_I=\frac{1}{2}\int d^3x d^3y : \rho_a(\boldsymbol{x})\frac{3}{4}V(\boldsymbol{x}-\boldsymbol{y})\rho_a(\boldsymbol{y}):, \tag{10}$$

where

$$\rho_a(\boldsymbol{x})=\sum_{\text{flavors}}\psi^\dagger(\boldsymbol{x})\frac{1}{2}\lambda_a\psi(\boldsymbol{x}), \tag{11}$$

is the quark color-charge-density operator and $\psi(\boldsymbol{x})$ is the quark field operator. As the spin splitting in charmonium spectrum and the lattice gauge calculation indicate that the confinement current should be the Lorentz scalar, in Ref. [18] the instantaneous interaction is replaced by the scalar confinement interaction plus the vector one-gluon exchange.

Following the Cornel model, here we will model the pair-creation from the quark model. We first assume the nonlocal current-current action of the quark interaction [24]:

$$\begin{aligned} A=&-\frac{1}{2}\int d^4x d^4y\bar{\psi}(x)\gamma_\mu\frac{1}{2}\lambda_a\psi(x)G(x-y)\bar{\psi}(y)\gamma^\mu \\ &\times\frac{1}{2}\lambda_a\psi(y)-\frac{1}{2}\int d^4x d^4y\bar{\psi}(x) \\ &\times\frac{1}{2}\lambda_a\psi(x)S(x-y)\bar{\psi}(y)\frac{1}{2}\lambda_a\psi(y). \end{aligned} \tag{12}$$

The vector kernel G is obtained from the one-gluon propagator. In the momentum space

$$G(q^2)=-\frac{4\pi\alpha_s}{q^2}. \tag{13}$$

The scalar kernel $S(x-y)$ is obtained from the linear confinement

$$S(q^2)=-\frac{6\pi b}{q^4}. \tag{14}$$

The lattice calculation shows that the behavior of the gluon propagator is quite different in the nonperturbative region. The gluon may get a mass of about 600–1000 MeV [19–21]. With the gluon getting a mass in the nonperturbative region, we can make the non-relativisitc approximation $q^2\rightarrow q^2-m_g^2\approx -m_g^2$ in the quark-antiquark pair-creation process. Thus

$$D_{\mu\nu}(q^2)\approx\frac{4\pi\alpha_s g_{\mu\nu}}{m_g^2}, \tag{15}$$

$$D(q^2)\approx-\frac{6\pi b}{m_g^4}. \tag{16}$$

Then the channel coupling interaction is simplified to the four-fermion interaction

$$\hat{V}=-\frac{1}{2}\frac{4\pi\alpha_s}{m_g^2}\int d^3x\bar{\psi}(\boldsymbol{x})\gamma_\mu\frac{1}{2}\lambda_a\psi(\boldsymbol{x})\bar{\psi}(\boldsymbol{x})\gamma^\mu\frac{1}{2}\lambda_a\psi(\boldsymbol{x}) +\frac{1}{2}\frac{6\pi b}{m_g^4}\int d^3x\bar{\psi}(\boldsymbol{x})\frac{1}{2}\lambda_a\psi(\boldsymbol{x})\bar{\psi}(\boldsymbol{x})\frac{1}{2}\lambda_a\psi(\boldsymbol{x}). \tag{17}$$

Once we calculate the transition amplitudes

$$f_i(\boldsymbol{p})=\langle\psi_\alpha|\hat{V}|M_1(i)M_2(i)\rangle, \tag{18}$$

where $\boldsymbol{p}$ is the relative momentum between M_1 and M_2, the mass shifts are given by

$$g(M)=\sum_i g_i(M), \tag{19}$$

$$g_i(M)=\int\frac{f_i(\boldsymbol{p})f_i(\boldsymbol{p})}{\left(m_{i1}+m_{i2}+\dfrac{p^2}{2\mu_i}\right)-M}d^3p, \tag{20}$$

where m_{i1} and m_{i2} are the masses of $M_1(i)$ and $M_2(i)$ mesons, μ_i is their reduced mass.

To calculate the coupling matrix element, we will use the simple harmonics oscillator (SHO) wave functions as usual. The partial-wave amplitude f^{ls} can be expressed as

$$f^{ls}(A\rightarrow BC)=\pi^{-\frac{7}{4}}\beta_A^{3/2}e^{-\frac{m_c^2}{2(m_q+m_c)^2(\beta_A^2+\beta_B^2)}p^2}F^{ls}(p), \tag{21}$$

where $\beta_B=\beta_C$, m_c is the mass of charm quark, m_q is the mass of light quarks (u, d, or s). $F^{ls}(p)$ is a polynomial of p which depends on the specific channel (the formulas are collected in Appendix A).

Our calculation is basically non-relativistic. However, the exponential factor in the obtained partial-wave amplitude Eq. (21) is obviously not enough to cut off the high momentum contribution. We will make an additional cutoff to the momentum integration. The mass shift is then replaced by

$$g_i(M)=\int\frac{f_i(\boldsymbol{p})f_i(\boldsymbol{p})}{\left(m_{i1}+m_{i2}+\dfrac{p^2}{2\mu_i}\right)-M}\exp(-p^2/\Lambda^2)d^3p, \tag{22}$$

where Λ is the cutoff parameter.

Since the channel coupling calculation is essentially the virtual charmed meson loop calculation, the quark potential in the quark model should be renormalized [8]. The renormalization process can be outlined as follows. The full Hamiltonian is divided into

$$\hat{H}_{full}=\hat{H}_c+\Delta\hat{H}. \tag{23}$$

$\hat{H}_c$ is the original quark model Hamiltonian. Its spectrum is given by

$$M_{nslj}=M_{nl}+\langle V_{HF}\rangle\langle\boldsymbol{S}_1\cdot\boldsymbol{S}_2\rangle+\langle V_{LS}\rangle\langle\boldsymbol{L}\cdot\boldsymbol{S}\rangle+\langle V_T\rangle\langle T\rangle, \tag{24}$$

where M_{nl} is the centroid of nl multiplet which is obtained from the spin-independent Hamiltonian $\hat{H}_0$ and the remaining terms give the fine splitting. $\langle T\rangle$ is the expectation value of the tensor operator,

$$\langle T\rangle=\begin{cases}-\dfrac{1}{6}\dfrac{l+1}{2l-1} & j=l-1,\\ \dfrac{1}{6} & j=l,\\ -\dfrac{1}{6}\dfrac{l}{2l+3} & j=l+1,\end{cases} \tag{25}$$

where the total spin $s=1$. $\Delta\hat{H}$ is the cancellation term whose contribution should be added to the mass shift from coupled-channels to give the renormalized mass shift. The renormalized mass shift contains both a centroid correction and a fine splitting one. The centroid contribution will modify the quark central potential [8]. It is the fine splitting correction we will consider in this work.

3 Numerical calculation of fine splitting

In our calculation, the quark model is taken from Ref. [7]. The potential parameters are:

$$\alpha_s=0.55,\quad \sigma=0.175\ \text{GeV}^2,\quad m_c=1.7\ \text{GeV},$$
$$C=-0.271\ \text{GeV},\quad m_q=0.33\ \text{GeV},\quad m_s=0.5\ \text{GeV}. \tag{26}$$

The SHO parameter β is determined from the mean square radius of the meson state. The β values of open-charm states are

$$\beta_{\rm D} = 0.385\ \text{GeV}, \qquad \beta_{\rm D_S} = 0.448\ \text{GeV}, \qquad (27)$$

and the β values of charmonium states are listed in Table 1.

Table 1. The β values of charmonium states.

nL	$1S$	$2S$	$1P$	$2P$	$1D$
β/GeV	0.676	0.485	0.514	0.435	0.461

In our calculation we take the gluon mass $m_{\rm g}$ = 640 MeV. This gives

$$\Gamma(\psi(3770) \to {\rm D\bar{D}}) = 28.2\ \text{MeV}, \qquad (28)$$

to fit the expermental value 27.3±1.0 MeV [4].

To calculate the mass shift, we need to know the physical mass M in Eq. (22). For the charmonium $1S$, $1P$ and $2S$ multiplets, we can directly use the experimental masses from PDG [4]. For the $2P$ and $1D$ multiplets, the physical masses are the predicted values calculated from the assignments of ψ(3770) to 1^3D_1 and X(3872) to 2^3P_1.

The mass shifts are listed in Table 2. In our calculation we take the cutoff paramter $\Lambda = 800$ MeV. We also show the mass shifts without the integration cutoff. The cutoff reduces the mass shift by $\sim 15\%$, which means that the contribution from high transfer momentum will be about 85% if we do not make the cutoff in this non-relativistic calculation.

Table 2. The mass shifts of charmonium states in MeV. The last column lists the total mass shifts without the integration cutoff.

$n^{2S+1}L_J$	DD	DD*	D*D*	$\rm D_sD_s$	$\rm D_sD_s^*$	$\rm D_s^*D_s^*$	total	no cutoff
1^3S_1	−9	−36	−64	−6	−26	−49	−190	−1359
1^1S_0	0	−52	−47	0	−39	−36	−175	−1274
1^3P_2	−12	−32	−75	−5	−15	−37	−175	−1035
1^3P_1	0	−53	−52	0	−21	−26	−152	−1021
1^3P_0	−23	0	−67	−7	0	−34	−131	−968
1^1P_1	0	−61	−50	0	−27	−24	−162	−1021
2^3S_1	−6	−18	−31	−1	−4	−8	−68	−872
2^1S_0	0	−28	−21	0	−7	−6	−62	−839
2^3P_2	−1	−9	−16	−1	−3	−7	−37	−691
2^3P_1	0	−17	−10	0	−4	−4	−35	−716
2^3P_0	−5	0	−13	−1	0	−5	−25	−680
2^1P_1	0	−18	−10	0	−5	−4	−36	−701
1^3D_3	−8	−18	−49	−2	−5	−15	−98	−652
1^3D_2	0	−40	−33	0	−9	−11	−93	−665
1^3D_1	−28	−14	−38	−2	−3	−13	−98	−669
1^1D_2	0	−44	−31	0	−11	−9	−95	−657

The fine splittings are listed in Table 3. For $1S$, $1P$, $2S$ states, the physical mass is the experimental mass. Then the fine splitting is calculated for each multiplet and listed as "splitting required". The fine splitting from the quark model is calculated from the bare masses of the quark model which are also taken from Ref. [7]. The fine splitting from coupled-channels are listed in the last column. So the total model fine splitting is the sum of the contributions from the quark model and from the coupled-channels. The results show that the calculated splittings fit the "splitting required" well in $1S$ and $2S$ multiplets. However in the $1P$ multiplet, the model splittings seem too large.

Next, we turn to the $2P$ and $1D$ multiplets. This time, the "required spltting" is the sum of the splitting from the quark model and from the coupled-channels. For the $1D$ multiplet, the ψ(3770) is assig-

Table 3. The physical masses and fine splittings.

$n^{2S+1}L_J$	mass	splitting required	splitting q. m.	splitting c. c.
1^3S_1	3097	+29	+32	−4
1^1S_0	2980	−87	−97	+12
1^3P_2	3556	+31	+36	−13
1^3P_1	3511	−15	−19	+11
1^3P_0	3415	−110	−106	+31
1^1P_1	3525	+0	−5	+0
2^3S_1	3686	+12	+14	−2
2^1S_0	3637	−37	−41	+5
2^3P_2	3918	+30	+32	−2
2^3P_1	3872	−17	−17	+0
2^3P_0	3808	−80	−90	+10
2^1P_1	3881	−7	−6	−1
1^3D_3	3798	+6	+8	−2
1^3D_2	3795	+3	−0	+3
1^3D_1	3773	−19	−17	−2
1^1D_2	3793	+0	−0	+1

ned to the 1^3D_1 state. Then the masses of other states in the multiplet are calculated from the fine splittings as the prediction. The predicted mass of 1^1D_2 is 3793 MeV. So the $c\bar{c}$ 1^1D_2 state is unlikely to be the experimental X(3872) state even when we have considered the fine splitting from coupled-channels. So we assign the X(3872) to the 2^3P_1 state and calculate the masses of the rest states in the $2P$ multiplet.

4 Summary

We have calculated the fine splitting in charmomium spectrum in the quark model with the channel coupling effect. The open charmed meson-meson channels below 4 GeV, including DD, DD^*, D^*D^* and D_sD_s, $D_sD_s^*$, $D_s^*D_s^*$, are considered. The current-current nonlocal interacting action is constructed from the color confinement and the one-gluon exchange interaction in the quark model. Using the massive gluon propagator from the lattice calculation in the nonperturbative region, the coupling interaction is further simplified approximately to the four-fermion interaction. The numerical calculation still prefers the assignment 1^{++} of X(3872) after we consider the fine splitting effect from the coupled-channels. The $2P$ and $1D$ charmonium spectrum are estimated from the assignments of 1^3D_1 to ψ(3770) and 2^3P_1 to X(3872).

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Appendix A

The partial-wave amplitudes

The partial-wave amplitude is the sum of contribution from the confinement and from the coulomb interaction:

$$F^{ls} = \frac{6\pi b}{m_{\mathrm{g}}^4} F_{\mathrm{conf}}^{ls} - \frac{4\pi\alpha_{\mathrm{s}}}{m_{\mathrm{g}}^2} F_{\mathrm{coul}}^{ls}. \tag{A1}$$

In the following,

$$D_k^{ij} = \frac{\beta_{\mathrm{A}}^i \beta_{\mathrm{B}}^j}{(\beta_{\mathrm{A}}^2+\beta_{\mathrm{B}}^2)^{k/2}}, \tag{A2a}$$

$$\xi_{\mathrm{q}} = \frac{m_{\mathrm{q}}}{m_{\mathrm{q}}+m_{\mathrm{c}}}, \tag{A2b}$$

$$\xi_{\mathrm{c}} = \frac{m_{\mathrm{c}}}{m_{\mathrm{q}}+m_{\mathrm{c}}}. \tag{A2c}$$

For the confinement, F_{conf}^{ls} can be represented as

$$F_{\mathrm{conf}}^{ls} = \frac{1}{m_{\mathrm{q}}} F_l(p) C^{ls}, \tag{A3}$$

where C^{ls} is a spin-orbit recoupling coefficient

$$C^{ls} = (-1)^{s_{\mathrm{C}}+s+l_{\mathrm{A}}+j_{\mathrm{A}}} \begin{Bmatrix} s_{\mathrm{A}} & s & 1 \\ l & l_{\mathrm{A}} & j_{\mathrm{A}} \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & s_{\mathrm{B}} \\ \frac{1}{2} & \frac{1}{2} & s_{\mathrm{C}} \\ s_{\mathrm{A}} & 1 & s \end{Bmatrix} \sqrt{6(2s+1)(2l_{\mathrm{A}}+1)(2s_{\mathrm{A}}+1)(2s_{\mathrm{B}}+1)(2s_{\mathrm{C}}+1)}. \tag{A4}$$

The $F_l(p)$ is the polynomial of transfer momentum p:

$$F_p(1S \to 1S+1S) = -\frac{8}{3\sqrt{3}} (\xi_{\mathrm{c}} D_5^{05} + 2\xi_{\mathrm{q}} D_3^{03}) p, \tag{A5}$$

$$F_p(2S \to 1S+1S) = \frac{4\sqrt{2}}{9} \left\{ \left[\xi_{\mathrm{c}}(7D_7^{25} - 3D_7^{07}) + 6\xi_{\mathrm{q}}(D_5^{23} - D_5^{05}) \right] p - 2\xi_{\mathrm{c}}^2 (\xi_{\mathrm{c}} D_9^{25} + 2\xi_{\mathrm{q}} D_7^{23}) p^3 \right\}, \tag{A6}$$

$$F_s(1P \to 1S+1S) = -\frac{8\sqrt{2}}{9\sqrt{3}} \left[3D_5^{15} - \xi_{\mathrm{c}}(\xi_{\mathrm{c}} D_7^{15} + 2\xi_{\mathrm{q}} D_5^{13}) p^2 \right], \tag{A7}$$

$$F_d(1P\to 1S+1S)=-\frac{16}{9\sqrt{3}}\xi_c(\xi_c D_7^{15}+2\xi_q D_5^{13})p^2, \tag{A8}$$

$$F_s(2P\to 1S+1S)=\frac{8}{9\sqrt{15}}\left\{15(D_7^{35}-D_7^{17})-5\xi_c\left[\xi_c(3D_9^{35}-D_9^{17})+2\xi_q(D_7^{33}-D_7^{15})\right]p^2+2\xi_c^3(\xi_c D_{11}^{35}+2\xi_q D_9^{33})p^4\right\}, \tag{A9}$$

$$F_d(2P\to 1S+1S)=\frac{8\sqrt{2}}{9\sqrt{15}}\left\{\xi_c\left[\xi_c(9D_9^{35}-5D_9^{17})+10\xi_q(D_7^{33}-D_7^{15})\right]p^2-2\xi_c^3(\xi_c D_{11}^{35}+2\xi_q D_9^{33})p^4\right\}, \tag{A10}$$

$$F_p(1D\to 1S+1S)=-\frac{16\sqrt{2}}{45}\left[5\xi_c D_7^{25}p-\xi_c^2(\xi_c D_9^{25}+2\xi_q D_7^{23})p^3\right], \tag{A11}$$

$$F_f(1D\to 1S+1S)=-\frac{16}{15\sqrt{3}}\xi_c^2(\xi_c D_9^{25}+2\xi_q D_7^{23})p^3. \tag{A12}$$

For the one-gluon exchange, $F^{ls}_{\rm coul}$ is further decomposed to

$$F^{ls}_{\rm coul}=\frac{1}{m_q}F_{1l}(p)C^{ls}+\frac{1}{m_c}F_{2l}(p)C^{ls}-\frac{1}{m_c}F_{1l}(p)C_2^{ls}, \tag{A13}$$

where C_2^{ls} is another spin-orbit recoupling coefficient.

1) $s_A=s_B=s_C=1$

$$C_2^{ls}=(-1)^{l_A+j_A}\sqrt{2(2s+1)(2l_A+1)}\begin{Bmatrix} l_A & 1 & l\\ s & j_A & 1\end{Bmatrix},$$

2) $s_A=1,\ s_B=s_C=0$

$$C_2^{l=j_A,s=0}=-\sqrt{\frac{2(2l_A+1)}{2j_A+1}},$$

3) $s_A=s_B=1,\ s_C=0$

$$C_2^{l,s=1}=(-1)^{l_A+j_A+1}\frac{\sqrt{3(2l_A+1)}}{2}\begin{Bmatrix} 1 & 1 & 1\\ l & l_A & j_A\end{Bmatrix},$$

4) $s_A=0$

$$C_2^{l,s=1}=0.$$

The polynomials $F_{1l}(p)$ and $F_{2l}(p)$ are:

$$F_{1p}(1S\to 1S+1S)=\frac{8}{3\sqrt{3}}\xi_c(D_3^{03}-D_5^{23})p, \tag{A14}$$

$$F_{2p}(1S\to 1S+1S)=\frac{8}{3\sqrt{3}}\xi_c(D_3^{03}+D_5^{23})p, \tag{A15}$$

$$F_{1p}(2S\to 1S+1S)=-\frac{4\sqrt{2}}{9}\left[\xi_c(7D_7^{25}-3D_7^{43}+3D_5^{23}-3D_5^{05})p+2\xi_c^3(D_9^{43}-D_7^{23})p^3\right], \tag{A16}$$

$$F_{2p}(2S\to 1S+1S)=\frac{4\sqrt{2}}{9}\left[\xi_c(7D_7^{25}-3D_7^{43}-3D_5^{23}+3D_5^{05})p+2\xi_c^3(D_9^{43}+D_7^{23})p^3\right], \tag{A17}$$

$$F_{1s}(1P\to 1S+1S)=\frac{8\sqrt{2}}{9\sqrt{3}}\left[3D_5^{15}+\xi_c^2(D_7^{33}-D_5^{13})p^2\right], \tag{A18}$$

$$F_{2s}(1P\to 1S+1S)=-\frac{8\sqrt{2}}{9\sqrt{3}}\left[3D_5^{15}+\xi_c^2(D_7^{33}+D_5^{13})p^2\right], \tag{A19}$$

$$F_{1d}(1P\to 1S+1S)=-\frac{16}{9\sqrt{3}}\xi_c^2(D_7^{33}-D_5^{13})p^2, \tag{A20}$$

$$F_{2d}(1P\to 1S+1S)=\frac{16}{9\sqrt{3}}\xi_c^2(D_7^{33}+D_5^{13})p^2, \tag{A21}$$

$$F_{1s}(2P\to 1S+1S)=-\frac{8}{9\sqrt{15}}\left[15(D_7^{35}-D_7^{17})-5\xi_c^2(3D_9^{35}+D_9^{53}-D_7^{33}-D_7^{15})p^2-2\xi_c^4(D_{11}^{53}-D_9^{33})p^4\right], \quad (A22)$$

$$F_{2s}(2P\to 1S+1S)=\frac{8}{9\sqrt{15}}\left[15(D_7^{35}-D_7^{17})-5\xi_c^2(3D_9^{35}-D_9^{53}-D_7^{33}+D_7^{15})p^2-2\xi_c^4(D_{11}^{53}+D_9^{33})p^4\right], \quad (A23)$$

$$F_{1d}(2P\to 1S+1S)=-\frac{8\sqrt{2}}{9\sqrt{15}}\left[\xi_c^2(9D_9^{35}-5D_7^{15}+5D_7^{33}-5D_9^{53})p^2+2\xi_c^4(D_{11}^{53}-D_9^{33})p^4\right], \quad (A24)$$

$$F_{2d}(2P\to 1S+1S)=\frac{8\sqrt{2}}{9\sqrt{15}}\left[\xi_c^2(9D_9^{35}+5D_7^{15}-5D_7^{33}-5D_9^{53})p^2+2\xi_c^4(D_{11}^{53}+D_9^{33})p^4\right], \quad (A25)$$

$$F_{1p}(1D\to 1S+1S)=\frac{16\sqrt{2}}{45}\left[5\xi_c D_7^{25}p+\xi_c^3(D_9^{43}-D_7^{23})p^3\right], \quad (A26)$$

$$F_{2p}(1D\to 1S+1S)=-\frac{16\sqrt{2}}{45}\left[5\xi_c D_7^{25}p+\xi_c^3(D_9^{43}+D_7^{23})p^3\right], \quad (A27)$$

$$F_{1f}(1D\to 1S+1S)=-\frac{16}{15\sqrt{3}}\xi_c^3(D_9^{43}-D_7^{23})p^3, \quad (A28)$$

$$F_{2f}(1D\to 1S+1S)=\frac{16}{15\sqrt{3}}\xi_c^3(D_9^{43}+D_7^{23})p^3. \quad (A29)$$

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