

# Radiative decays $J/\psi \rightarrow \eta^{(\prime)}\gamma$ in perturbative QCD<sup>\*</sup>

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**Abstract:** With the recent investigations of the  $g^*g^*\eta^{(\prime)}$  transition form factor and  $\eta$ - $\eta'$  mixing scheme, we present an updated study of the radiative decays  $J/\psi \rightarrow \eta^{(\prime)}\gamma$  in perturbative QCD. The decays are taken as a test ground for the  $g^*g^*\eta^{(\prime)}$  transition form factors and the  $\eta$ - $\eta'$  mixing scheme. The form factors are found to be working for gluonic  $\eta'$  production and the mixing angle is constrained to be  $\phi = 35.1^\circ \pm 0.8^\circ$ .

**Key words:** transition form factor,  $\eta$ - $\eta'$  mixing scheme, radiative decays

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## 1 Introduction

As is well known, heavy quarkonium decays to light mesons have played a very important role in testing and understanding QCD from the very beginning. The decays  $J/\psi \rightarrow \eta^{(\prime)}\gamma$  are of great interest since they are closely related to the issues of  $\eta$ - $\eta'$  mixing and  $g^*g^*\eta^{(\prime)}$  transition form factors, which are very important ingredients for understanding many interesting hadronic phenomena of  $\eta$  and  $\eta'$  production. For example, it would be very useful for explaining the large branching ratio of strong penguin dominated decay  $B \rightarrow K\eta'$  [1–3].

In the literature, studies of the decays  $J/\psi \rightarrow \eta^{(\prime)}\gamma$  are different from each other either in the treatments of formatting gluons to  $\eta'$ , namely, direct nonperturbative  $g^*g^*\eta^{(\prime)}$  coupling through a strong anomaly [4], or two off-shell gluons coupled to  $\eta'$  through a quark loop [5]. In this letter, we will take the second approach which was pioneered systematically within perturbative QCD by Körner, Kühn, Kramer and Schneider(KKKS) [5] years ago. In Ref. [5], the nonrelativistic quark model and the weak-binding approximation were used for both heavy and light mesons, and systematic helicity projectors were constructed to reduce loop integrations. In this work, we follow their approach. However, two improvements are included:

1)  $g^*g^*\eta^{(\prime)}$  couplings are improved to be relativis-

tic transition form factors, as advocated in Refs. [6–8], instead of non-relativistic modeling.

2) The  $\eta$ - $\eta'$  mixing scheme is also updated to the Feldmann-Kroll-Stech (FKS) mixing scheme [9].

In the perturbative QCD approach, the decays are depicted by the Feynman diagrams in Fig. 1. To calculate the amplitudes for the decays, we need to know how to deal with the dynamics of bound states. Generally, factorization is employed. Soft nonperturbative QCD bound state dynamics are factorized to the decay constants and the wave functions of  $J/\psi$  and  $\eta'$ , which will convolute with the hard kernel induced by the decay. We shall use the non-relativistic approximation for the heavy  $J/\psi$ , but not for the light mesons  $\eta'$  and  $\eta$ . Although a rigorous theory from the first principles for the light bound-states is still missing, some effective approaches are in progress. In recent years, it has been realized that proper treatment of the  $\eta$ - $\eta'$  system requires a sharp distinction between the mixing states and the mixing properties of the decay constants [9]. Taking the strange-nonstrange flavor basis for the  $\eta$ - $\eta'$  system and the mixing of the decay constants following the same pattern of state mixing, FKS have found a dramatic simplification. They have also tested their mixing scheme against experiment and determined corrections to the first order values of the basic parameters from phenomenology.

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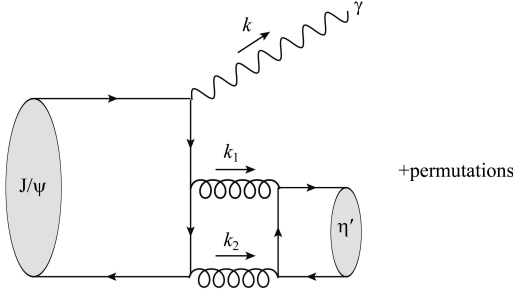


Fig. 1. The lowest order QCD diagrams for  $J/\psi \rightarrow \eta^{(\prime)}\gamma$  decays.

## 2 FKS mixing scheme and $g^*g^*\eta^{(\prime)}$ transition form factor

In the FKS mixing scheme, the parton Fock state decomposition can be expressed as

$$\begin{aligned} |\eta\rangle &= \cos\phi|\eta_q\rangle - \sin\phi|\eta_s\rangle, \\ |\eta'\rangle &= \sin\phi|\eta_q\rangle + \cos\phi|\eta_s\rangle, \end{aligned} \quad (1)$$

where  $\phi$  is the mixing angle,  $|\eta_q\rangle \sim f_q\phi(x,\mu)|u\bar{u}+d\bar{d}\rangle/\sqrt{2}$  and  $|\eta_s\rangle \sim f_s\phi(x,\mu)|s\bar{s}\rangle$ . The decay constants  $f_q$ ,  $f_s$  and the mixing angle  $\phi$  are extracted from the available experimental data with  $f_q = (1.07 \pm 0.02)f_\pi$ ,  $f_s = (1.34 \pm 0.06)f_\pi$ ,  $\phi = 39.3^\circ \pm 1.0^\circ$  [9].

Already in Ref. [10], Baier and Grozin have derived the evolution equations for the distribution functions  $\phi(x,\mu)$  to the first order of  $\alpha_s$ , which eigenfunctions are found to be

$$\phi(x,\mu) = 6x(1-x) \left( 1 + \sum_{n=2,4,\dots} B_n(\mu) C_n^{3/2}(2x-1) \right). \quad (2)$$

In the limit  $\mu \rightarrow \infty$ , the coefficients  $B_n$  evolve to zero and  $\phi(x,\mu)$  turns out to be  $\phi_{AS} = 6x(1-x)$ . When the evolution equations run down to the low energy scale, its quark contents mixed with gluonic states. However, the gluon content enters the  $\eta^{(\prime)}$  wave function from next-to-leading order. This observation encourages the calculations of the  $g^*g^*\eta^{(\prime)}$  transition form factors similarly to the well known  $\gamma^*\pi$  transition form factor at the leading order, which read [6–8]

$$\begin{aligned} \mathcal{M}_{\mu\nu} &= \langle g_a^* g_b^* | \eta^{(\prime)} \rangle \\ &= -4\pi\alpha_s \delta_{ab} i\epsilon_{\mu\nu\alpha\beta} Q_1^\alpha Q_2^\beta F_{g^*g^*\eta^{(\prime)}}(Q_1^2, Q_2^2), \\ &F_{g^*g^*\eta^{(\prime)}}(Q_1^2, Q_2^2) \\ &= \frac{1}{2N_c} f_{\eta^{(\prime)}} \int_0^1 dx \frac{\phi_{\eta^{(\prime)}}(x,\mu)}{\bar{x}Q_1^2 + xQ_2^2 - x\bar{x}m_{\eta^{(\prime)}}^2 + i\epsilon} \\ &+ (x \rightarrow \bar{x}). \end{aligned} \quad (3)$$

Here,  $\bar{x} = 1-x$ ,  $f_{\eta^{(\prime)}} = \sqrt{2}f_q \sin\phi + f_s \cos\phi$  and  $f_{\eta'} = \sqrt{2}f_q \cos\phi - f_s \sin\phi$ . To the accuracy of this paper,  $\phi_{\eta^{(\prime)}}(x,\mu)$  is taken to be the leading twist distribution functions (DAs)  $\phi_{\eta^{(\prime)}}^{AS}(x) = 6x(1-x)$ .

Using  $g^*g^*\eta^{(\prime)}$  in Eq. (3) and following the procedure developed in Refs. [5, 11], it is straightforward to evaluate the amplitudes for the decays as depicted by Feynman diagrams in Fig. 1. We get

$$\begin{aligned} \Gamma(V \rightarrow \eta^{(\prime)}\gamma) &= \frac{1}{6} \left( \frac{2}{3} \right)^2 e_Q^2 \alpha_s^4 (M_V) \alpha_e \\ &\times \frac{f_V^2 f_{\eta^{(\prime)}}^2}{M_V^3} (1-z^2) |H(z)|^2, \end{aligned} \quad (4)$$

where  $z = m_{\eta^{(\prime)}}/M_V$ ,  $e_Q$  is the heavy quark electric charge and  $2/3$  is the color factor. The dimensionless scalar function  $H(z)$  containing loop integrals is given by

$$\begin{aligned} H(z) &= \frac{M_V^2}{2p \cdot k} \frac{1}{16} \frac{1}{i\pi^2} \int_0^1 du \phi_{\eta^{(\prime)}}^{AS}(u) \int d^4q \\ &\times \frac{k_1 \cdot k_2 (p \cdot k q^2 - q \cdot k q \cdot p)}{D_1 D_2 k_1^2 k_2^2 (\bar{u}k_1^2 + uk_2^2 - u\bar{u}m_{\eta^{(\prime)}}^2)}, \end{aligned} \quad (5)$$

where  $D_1 = -k_1 \cdot (k+k_2)$ ,  $D_2 = -k_2 \cdot (k+k_1)$ ,  $q = k_1 - k_2$  and  $p = k_1 + k_2$ .

Obviously in Eq. (5), the  $k_1 \cdot k_2$  numerator would cancel the  $\eta'$  form factor if it is taken to be  $\sim 1/k_1 \cdot k_2$ , and the hard scattering kernel would not convolute with the distribution functions of  $\eta'$ .

With the help of the algebraic identities

$$\begin{aligned} q^2 &= \frac{2}{M_V^2 + m_{\eta^{(\prime)}}^2} [m_{\eta^{(\prime)}}^2 (D_1 + D_2) \\ &+ M_V^2 (k_1^2 + k_2^2)], \end{aligned} \quad (6)$$

$$q \cdot p = k_1^2 + k_2^2,$$

$$\begin{aligned} k_1 \cdot k_2 &= \frac{1}{2} (p^2 - k_1^2 - k_2^2) \\ &= -\frac{1}{2} (p \cdot k + D_1 + D_2), \end{aligned} \quad (7)$$

the integrand in  $H(z)$  can be decomposed into a sum of four, three and two-points functions which is presented in Appendix A. In the calculation of the loop integrals, we have used the dimensional regularization scheme and the methods developed in Ref. [12].

## 3 Numerical results

For numerical results for the decays, we use  $\Gamma_{\text{tot}}(J/\psi) = (87 \pm 5) \text{ keV}$  [13],  $f_{J/\psi} = 400 \text{ MeV}$  and

$\alpha_s(M_{J/\psi}) = 0.2557$  [14]. We get

$$\mathcal{B}^{\text{th}}(J/\psi \rightarrow \eta'\gamma) = 3.9 \times 10^{-3}, \quad (8)$$

$$(\mathcal{B}^{\text{exp}}(J/\psi \rightarrow \eta'\gamma) = (4.3 \pm 0.3) \times 10^{-3}, \text{ PDG [13]}),$$

$$\mathcal{B}^{\text{th}}(J/\psi \rightarrow \eta\gamma) = 3.5 \times 10^{-4}, \quad (9)$$

$$(\mathcal{B}^{\text{exp}}(J/\psi \rightarrow \eta\gamma) = (8.6 \pm 0.8) \times 10^{-4}, \text{ PDG [13]}).$$

While  $\mathcal{B}^{\text{th}}(J/\psi \rightarrow \eta'\gamma)$  agrees with experiment,  $\mathcal{B}^{\text{th}}(J/\psi \rightarrow \eta\gamma)$  turns out to be too small. From the mixing scheme, it is easy to see that  $\mathcal{B}^{\text{th}}(J/\psi \rightarrow \eta'\gamma)$  is insensitive to the mixing angle  $\phi$  when  $\phi$  is about  $35^\circ$ , but  $\mathcal{B}^{\text{th}}(J/\psi \rightarrow \eta\gamma)$  is very sensitive to  $\phi$ . Taking  $\phi = 35.3^\circ$  fitted from  $\eta' \rightarrow \rho\gamma$  and  $\rho \rightarrow \eta\gamma$  [9], we find

$$\mathcal{B}^{\text{th}}(J/\psi \rightarrow \eta'\gamma) = 3.75 \times 10^{-3}, \quad (10)$$

$$\mathcal{B}^{\text{th}}(J/\psi \rightarrow \eta\gamma) = 7.3 \times 10^{-4},$$

which agree with the experimental results quite well. However, if we take  $\alpha_s(\mu) = \alpha_s(m_c)$ , the results turn out to overshoot their experimental data.

The most theoretical uncertainty may arise from the energy scale choice in  $\alpha_s(\mu)$ . Because our calculation is performed at the lowest order in QCD and there is no UV divergence in the loop diagrams which induce the decay, we don't have a strong argument to choose a scale, as in the usual case, to minimize the higher order corrections by setting the logarithm to zero. Naively, the scale could be chosen from  $m_c$  to  $m_{J/\psi}$ . To reduce the scale dependence, we relate  $\mathcal{B}(J/\psi \rightarrow \eta^{(\prime)}\gamma)$  to  $\mathcal{B}(J/\psi \rightarrow ggg)$

$$\mathcal{B}(J/\psi \rightarrow \eta^{(\prime)}\gamma) = \frac{\Gamma(J/\psi \rightarrow \eta^{(\prime)}\gamma)}{\Gamma(J/\psi \rightarrow ggg)} \mathcal{B}(J/\psi \rightarrow ggg). \quad (11)$$

With the help of the known results [15],

$$\frac{\Gamma(V \rightarrow ggg)}{\Gamma(V \rightarrow \mu^+\mu^-)} = \frac{10(\pi^2 - 9)}{81\pi e_Q^2} \frac{\alpha_s^3(M)}{\alpha_e^2} \left\{ 1 + \frac{\alpha_s(M)}{\pi} \left[ -19.4 + \frac{3}{2}\beta_0 \left( 1.16 + \ln \left( \frac{2M}{M_V} \right) \right) \right] \right\}, \quad (12)$$

we can get

$$\mathcal{B}(J/\psi \rightarrow \eta^{(\prime)}\gamma) = \frac{9}{20(\pi^2 - 9)} \frac{e_Q^2}{M_V^2} \frac{\alpha_s(M)\alpha_e f_{\eta^{(\prime)}}^2 (1 - z^2) |H(z)|^2}{1 + \frac{\alpha_s(M)}{\pi} \left[ -19.4 + \frac{3}{2}\beta_0 \left( 1.16 + \ln \left( \frac{2M}{M_V} \right) \right) \right]} \mathcal{B}(J/\psi \rightarrow ggg). \quad (13)$$

We will use the following relation and experimental data [13] for our numerical results,

$$\begin{aligned} \mathcal{B}(J/\psi \rightarrow ggg) &= \mathcal{B}(J/\psi \rightarrow \text{hadrons}) \\ &\quad - \mathcal{B}(J/\psi \rightarrow \text{virtual}\gamma \rightarrow \text{hadrons}) \\ &= (0.877 \pm 0.005) - (0.17 \pm 0.02) \\ &= 0.707 \pm 0.025. \end{aligned} \quad (14)$$

Taking  $\phi = 35.3^\circ$  and  $\alpha_s(M) = \alpha_s(m_c)$ , we obtain

$$\begin{aligned} \mathcal{B}^{\text{th}}(J/\psi \rightarrow \eta'\gamma) &= 4.17 \times 10^{-3}, \\ \mathcal{B}^{\text{th}}(J/\psi \rightarrow \eta\gamma) &= 8.16 \times 10^{-4}, \end{aligned} \quad (15)$$

which agree with the experimental data.

## 4 Conclusions

In Fig. 2, we display the ratio  $\mathcal{R}_{J/\psi} = \mathcal{B}(J/\psi \rightarrow \eta'\gamma)/\mathcal{B}(J/\psi \rightarrow \eta\gamma)$  as a function of  $\phi$ , in which we expect that the relativistic and the higher order QCD corrections may be canceled to a large extent, so the ratio could be predicted much more reliably than the two decay rates respectively. Comparing our results with the experimental measurement  $\mathcal{R}_{J/\psi} = 5.0 \pm 0.6$  [13] as displayed by the horizontal lines in Fig. 2,

we find  $\phi = 35.1^\circ \pm 0.8^\circ$ , which is in good agreement with  $\phi = 35.3^\circ \pm 5.5^\circ$  determined from  $\eta' \rightarrow \rho\gamma$  and  $\rho \rightarrow \eta\gamma$  [9]. However,  $2\sigma$  lower than  $\phi = 39.0^\circ \pm 1.6^\circ$  from  $J/\psi \rightarrow \eta(\eta')\gamma$  [9] by using the QCD anomaly dominance mechanism formula. To make clear the origin of the discrepancy between the two different determinations of mixing angle  $\phi$ , we recapitulate the key formula from the well known work of Novikov et

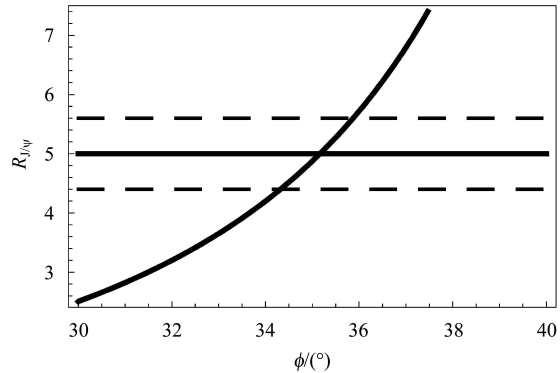


Fig. 2. The ratio  $\mathcal{R}_{J/\psi}$  is shown by a solid curve as a function of  $\phi$  (in degree). The experimental data are shown by the horizontal lines. The thicker solid horizontal line is its center value and the thin horizontal dashed lines are its error bars.

al [4].

$$\mathcal{R}_{J/\psi} = \left| \frac{\langle 0 | G\tilde{G} | \eta' \rangle}{\langle 0 | G\tilde{G} | \eta \rangle} \right|^2 \left( \frac{p_{\eta'}}{p_{\eta}} \right)^3. \quad (16)$$

This formula is frequently employed to determine the  $\eta$ - $\eta'$  mixing angles in the literature. Technologically, the strong anomaly dominance is equivalent to the dominance of the ground state and the neglect of continuum contribution to the dispersion relations, as shown in Refs. [4, 16]. So far, considering the experimental and the theoretical uncertainties, the difference between the predictions for  $\mathcal{R}_{J/\psi}$  by the two mechanisms is still marginal. Although we have improved  $g^*g^*\eta^{(\prime)}$  couplings in Ref. [5] from non-relativistic to relativistic, there is still a large room for theoretical improvements that deserve further stud-

ies. We also note the CLEO/CESR-c project is going, where about one billion  $\psi$  events would be produced. The refined measurements of these decays to be performed at CLEO-c will deepen our understanding of the two  $\eta^{(\prime)}$  production mechanisms.

In this paper, we have studied the radiative decays  $J/\psi \rightarrow \eta^{(\prime)}\gamma$  in perturbative QCD. The relativistic  $g^*g^*\eta^{(\prime)}$  transition form factors have been tested to be working for  $\eta'$  production. The mixing angle in FKS scheme is constrained to be  $\phi = 35.1^\circ \pm 0.8^\circ$ . This study encourages further applications of the form factor for  $\eta^{(\prime)}$  production in hard processes. It is also very helpful for understanding the abnormal large  $\eta^{(\prime)}$  yields in B meson decays, which have attracted much theoretical attention [17, 18] recently.

## Appendix A

In the evolution of the amplitudes for  $J/\psi \rightarrow \eta^{(\prime)}\gamma$ , we encounter the loop integral in Eq. (5), which can be expanded in terms of four, three and two points functions

$$\begin{aligned} H(z) = & \frac{1}{16} \frac{1}{1-z^2} \int du \phi_{AS}(u) \left[ \frac{1-z^2}{2(1+z^2)} 4^4 \left( m^4 D_0^a(u, z) - \frac{1}{2} (1-z^2) m_V^4 D_0^b(u, z) \right) \right. \\ & - \frac{1}{2} (1-uz^2) 4^3 C_0^b(u, z) - \frac{1}{2} (1-2z^2+uz^2) 4^3 C_0^a(u, z) \\ & \left. + \frac{1}{2u} 4^2 \left( B_0^a(u, z) - B_0^b(u, z) - B_0^c(u, z) + B_0^d(u, z) \right) \right], \end{aligned} \quad (A1)$$

with the following functions

$$\begin{aligned} D_0^a(u, z) = & \frac{1}{8m_V^4(1-u)uz^2(1-z^2)} \left[ S_P \left( 1 - \frac{1-\bar{u}z^2}{\bar{u}(1-z^2)} \right) + 2\pi i \ln \left( 1 - \frac{1-\bar{u}z^2}{\bar{u}(1-z^2)} - i\epsilon \right) \right. \\ & + S_P \left( 1 - \frac{1-\bar{u}z^2}{1-(1-2u)z^2} \right) + 2\pi i \ln \left( 1 - \frac{1-\bar{u}z^2}{1-(1-2u)z^2} - i\epsilon \right) - S_P \left( 1 - \frac{\bar{u}-(1-2u)z^2}{\bar{u}(1-z^2)} \right) \\ & \left. + \left( 2\pi i + \ln \left( \frac{\bar{u}-(1-2u)z^2}{1-\bar{u}z^2} \right) \right) \left( \pi i + \ln \left( \frac{(1-z^2)(\bar{u}-(1-3u+2u^2)z^2)}{uz^2} \right) \right) \right], \end{aligned} \quad (A2)$$

$$\begin{aligned} D_0^b(u, z) = & \frac{1}{4m_V^4(1-(1-2u)z^2)u(1-z^2)} \left[ 2S_P \left( -\frac{1-z^2}{uz^2} \right) - 2S_P \left( -\frac{(1-2u)(1-z^2)}{u} \right) \right. \\ & + S_P \left( -\frac{1-z^2}{z^2(\bar{u}-(1-3u+2u^2)z^2)} \right) - S_P \left( -\frac{(1-2u)^2z^2(1-z^2)}{\bar{u}-(1-3u+2u^2)z^2} \right) \\ & \left. + \ln \left( \frac{1-\bar{u}z^2}{z^2(\bar{u}-(1-2u)z^2)} \right) \ln \left( \frac{(1-z^2)(\bar{u}-(1-3u+2u^2)z^2)}{uz^2} + i\pi \right) \right], \end{aligned} \quad (A3)$$

$$C_0^b(u, z) = -\frac{1}{m_V^2} \int_0^1 dy \frac{1}{4u^2 z^2 - y^2(1-2z^2) - 2uy(1-3z^2) - i\epsilon} \\ \times \ln \left( \frac{y(2(1-2z^2) - y(1-2z^2) - 2u(1-3z^2)) - i\epsilon}{2(y(1-z^2) - 2u^2 z^2) - i\epsilon} \right), \quad (\text{A4})$$

$$C_0^a(u, z) = -\frac{1}{m_V^2} \int_0^1 dy \frac{1}{y^2 + 2y\bar{u}(1-z^2) + 4\bar{u}z^2 - i\epsilon} \ln \left( \frac{-y(y + 2\bar{u}(1-2z^2)) + i\epsilon}{4\bar{u}^2 z^2 + i\epsilon} \right), \quad (\text{A5})$$

$$B_0^a(u, z) = \frac{2}{\epsilon} - \gamma_E + \ln 4\pi + \ln \mu^2 + 2 - \ln m_V^2 \\ - \left( 1 - \frac{1}{(1-2u)(1-2uz^2) + i\epsilon} \right) \ln[1 - (1-2u)(1-2uz^2) + i\epsilon], \quad (\text{A6})$$

$$B_0^b(u, z) = \frac{2}{\epsilon} - \gamma_E + \ln 4\pi + \ln \mu^2 + 2 - \ln m_V^2, \quad (\text{A7})$$

$$B_0^c(u, z) = \frac{2}{\epsilon} - \gamma_E + \ln 4\pi + \ln \mu^2 + 2 - \ln m_V^2 \\ - \left( 1 - \frac{1}{(1-2u)(2\bar{u}z^2 - 1) + i\epsilon} \right) \ln[1 - (1-2u)(2\bar{u}z^2 - 1) + i\epsilon], \quad (\text{A8})$$

$$B_0^d(u, z) = \frac{2}{\epsilon} - \gamma_E + \ln 4\pi + \ln \mu^2 + 2 - \ln m_V^2 \\ - \frac{2(1-z^2)}{1-2z^2} \ln[2(1-z^2)] + \ln(1-2z^2), \quad (\text{A9})$$

where  $S_P(x) = \text{Li}_2(x)$  is the Spence function and  $\bar{u} = 1 - u$ .

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