CPC(HEP & NP), 2011, **35**(6): 543–544 Chinese Physics C Vol. 35, No. 6, Jun., 2011

Branching fraction of the isospin violating process $\phi \to \omega\pi^0$ *

YUAN Chang-Zheng(苑长征)[1)] MO Xiao-Hu(莫晓虎)[2)] WANG Ping(王平)[3)]

Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

Abstract: We have examined the parametrization of the $e^+e^- \to \omega\pi^0$ cross section in the vicinity of the ϕ resonance and the extraction of the branching fraction of the isospin violating process $\phi \to \omega\pi^0$ from experimental data. We found that there are two possible solutions of the branching fraction: one is 4×10^{-5}, and the other is 7×10^{-3}. The latter is two orders of magnitude higher than the former, which is the commonly accepted one.

Key words: multiple solution, ϕ resonance, branching ratio

PACS: 14.40.Be, 13.25.Gv, 13.66.Bc **DOI:** 10.1088/1674-1137/35/6/006

1 Introduction

As has been pointed out in a recent study [1, 2], there are many cases where multiple solutions are found in fitting one dimensional distribution with the coherent sum of several amplitudes and free relative phase between them. The fit to the $e^+e^- \to \omega\pi^0$ cross sections in the vicinity of the ϕ resonance was shown as an example of the existence of the two solutions and how large the difference could be between them.

However, Ref. [1] found these two solutions only through a fit to the experimental data, which raises suspicion that the two solutions may be due to the statistical fluctuation, other reasons associated with the data handling or fitting procedure, or something else. In this brief report, we show mathematically that two solutions exist in the parametrization of the cross section used in the original publication [3]; and the second solution can be obtained analytically from the solution reported in the literature without carrying out a fit to the experimental data.

2 Deriving the second solution

The cross section of $e^+e^- \to \omega\pi^0$ as a function of the center-of-mass energy, $\sqrt{s}$, is parameterized as

$$\sigma(\sqrt{s}) = \sigma_{\rm nr}(\sqrt{s}) \cdot \left|1 - Z\frac{M_\phi \Gamma_\phi}{D_\phi(\sqrt{s})}\right|^2 \qquad (1)$$

in Ref. [3], where $\sigma_{\rm nr}(\sqrt{s}) = \sigma_0 + \sigma'(\sqrt{s} - M_\phi)$ is the bare cross section for the non-resonant process, parameterized as a linear function of $\sqrt{s}$; M_ϕ, Γ_ϕ, and $D_\phi = M_\phi^2 - s - {\rm i}M_\phi\Gamma_\phi$ are the mass, the width, and the inverse propagator of the ϕ meson, respectively. Here, Z is a complex number that depicts the interference effect. Conventionally, the real and imaginary parts of Z are denoted as $\Re(Z)$ and $\Im(Z)$, respectively.

If we write

$$G(s, Z) = 1 - Z\frac{M_\phi \Gamma_\phi}{D_\phi(\sqrt{s})}, \qquad (2)$$

then in the complex-parameter space (denoted by a complex number Z'), we want to figure out all possible parameters that can satisfy the following relation,

$$|G(s, Z)|^2 = |G(s, Z')|^2. \qquad (3)$$

Note that if the above relation is to be true for any s, it should be true for some special values of s. If we firstly take a special value of s that satisfies $M_\phi^2 - s = 0$, then we obtain

$$|1 - {\rm i}Z|^2 = |1 - {\rm i}Z'|^2, \qquad (4)$$

or

$$|Z|^2 + 2\Im(Z) = |Z'|^2 + 2\Im(Z'). \qquad (5)$$

Secondly, we take another special value of s that

Received 25 October 2010, Revised 8 November 2010

* Supported by National Natural Science Foundation of China (10491303, 10775412, 10825524, 10935008), Instrument Developing Project of Chinese Academy of Sciences (YZ200713), Major State Basic Research Development Program (2009CB825206) and Knowledge Innovation Project of Chinese Academy of Sciences (KJCX2-YW-N29)

1) E-mail: yuancz@ihep.ac.cn 2) E-mail: moxh@ihep.ac.cn 3) E-mail: wangp@ihep.ac.cn

satisfies $M_{\phi}^2 - s = M_{\phi}\Gamma_{\phi}$, and we obtain

$$\left|1-\frac{1+\mathrm{i}}{2}Z\right|^2=\left|1-\frac{1+\mathrm{i}}{2}Z'\right|^2, \qquad (6)$$

or

$$|Z|^2-2\Re(Z)+2\Im(Z)=|Z'|^2-2\Re(Z')+2\Im(Z'). \qquad (7)$$

Subtraction of Eq. (5) from Eq. (7) yields

$$\Re(Z')=\Re(Z). \qquad (8)$$

With this equality, Eq. (5) is recast as

$$[1+\Im(Z')]^2=[1+\Im(Z)]^2, \qquad (9)$$

by virtue of which one gets either $\Im(Z')=\Im(Z)$ or $\Im(Z')=-2-\Im(Z)$. As a summary, we have two sets of solutions,

$$\Re(Z')=\Re(Z), \quad \Im(Z')=\Im(Z); \qquad (10)$$

and

$$\Re(Z')=\Re(Z), \quad \Im(Z')=-2-\Im(Z). \qquad (11)$$

We are ready to check that the above two sets of solutions are true for the relation (3) for any value of s 1). Obviously, the first set of solutions is trivial, which is expected intuitively. However, the second set of solutions is fairly interesting, and it is firstly obtained analytically. The more interesting thing is that according to the Eqs. (10) and (11), the second set of solutions can be obtained from the first one. Both solutions describe the experimental data identically well and one cannot distinguish between them purely from the experimental data. Therefore we conclude that if the cross section of $\mathrm{e^+e^-}\to\omega\pi^0$ as a function of the center-of-mass energy is parameterized as Eq. (1), there must be two sets of solutions of the interference parameter Z.

One remark on our mathematical analysis: more generally we write $G(s,Z)$ in the form $G(s,Z)=1+ZF(s)$, with $F(s)$ being a complex function depending on s. Here the first question is under what condition there will be two solutions for the requirement of Eq. (3). According to our further study, only some special forms of function $F(s)$ can guarantee the existence of two distinctive solutions, and in these cases, it can be proved that utilizing the method suggested here, we can get the other solution if one solution is given.

3 Experimental confirmation

From Ref. [3], $\Re(Z)=0.106$ and $\Im(Z)=-0.103$ are acquired from a fit to the experimental data in the $\omega\to\pi^+\pi^-\pi^0$ decay mode. In the light of Eq. (11), the second set of solution can be acquired immediately, i.e. $\Re(Z)=0.106$ and $\Im(Z)=-1.897$.

It is interesting to compare these results with those obtained from a fit to the experimental data [1]. A check of the fitted results given in Ref. [1] indicates that the -1.90 is indeed from a rounding of -1.897. Keeping one more digit, we find that the sum of the imaginary parts is exactly -2. Both the real parts and the imaginary parts agree perfectly between the fitted results and the analytical evaluation.

4 Summary and discussions

We have shown above that there must be two solutions in extracting the branching fraction of $\phi\to\omega\pi^0$ with the parametrization of the $\mathrm{e^+e^-}\to\omega\pi^0$ cross section around the ϕ resonance in Ref. [3]. While the first solution corresponds to $\mathcal{B}(\phi\to\omega\pi^0)=4\times10^{-5}$, as reported in Ref. [3], the second solution would be $\mathcal{B}(\phi\to\omega\pi^0)=7\times10^{-3}$, which is two orders of magnitude higher than the first one.

One may need to check whether the parametrization of the cross section is meaningful or if there are further constraints to the parametrization or the parameters, in order to pick out the physics solution from the two-fold ambiguities.

It is worth pointing out that $\phi\to\omega\pi^0$ is an isospin violating process and thus should be small. The branching fraction reported in Ref. [3] is already large compared with theoretical calculations [4]. However, if the physics is the second solution showed above, we would find the theoretical calculations are too low.

References

1 YUAN C Z, MO X H, WANG P. arXiv:0911.4791 [hep-ph]
2 Bukin A D. arXiv:0710.5627 [hep-ph]
3 Ambrosino F et al. (KLOE collaboration). Phys. Lett. B, 2008, **669**: 223
4 LI G, ZHANG Y J, ZHAO Q. J. Phys. G, 2009, **36**: 085008

1) In fact, there is another equivalent method that can be used to get the same result without resorting to the special values of s (private communication with Maurice Benayoun). By setting $s-M_{\phi}^2=uM_{\phi}\Gamma_{\phi}$ with u being a real variable, and substituting this relation into Eq. (3), one gets a quartic equation of u with the coefficients of function of $\Re(Z)$, $\Im(Z)$, $\Re(Z')$, and $\Im(Z')$. The requirement of coefficient of the same order of u being equal (so that the results do not depend on u) yields five algebraic equations for $\Re(Z)$, $\Im(Z)$, $\Re(Z')$, and $\Im(Z')$. Two independent and non-trivial equations of them are $\Re(Z')=\Re(Z)$ and $[1+\Im(Z')]^2=[1+\Im(Z)]^2$, which are just the results we obtained in Eqs. (8) and (9).