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$D^+ \to \eta^{(\prime)} l^+ \nu_l$ semileptonic decays in light-cone sum rules*

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Abstract: We calculate the $D \to \eta$ transition form factor in light-cone sum rules by taking improved current correlators to avoid the pollution from the twist-3 wave function. We get consistent results of the $D^+ \to \eta^{(\prime)} l^+ \nu_l$ decays with the experimental data. By comparing the difference between the results of the branching ratios of $D^+ \to \eta^{(\prime)} l^+ \nu_l$ from a two-pole parameterization model and from a BZ parameterization model, we find that the two-pole model and the BZ model are comparably believable. One way is supposed for the determination of the η-η' mixing angle from the dependence of the branching ratios of $D^+ \to \eta^{(\prime)} l^+ \nu_l$ decays on the η-η' mixing angle.

Key words: form factor, chiral current correlator, light-cone sum rules, η-η' mixing angle

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1 Introduction

Semileptonic decay of the charm meson is important for studying strong and weak interactions. It can be used to test the techniques developed to solve perturbative and nonperturbative problems in Quantum Chromodynamics (QCD), and to extract elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

In this work, we study semileptonic decays $D^+ \to \eta^{(\prime)} l^+ \nu_l$. The decays involve $c \to d$ transition, so can be used to determine the CKM matrix element $|V_{cd}|$. Another reason is that the decays can be used to get information about η-η' mixing. The mixing of η and η' and their components are interesting and controversial topics [1–23]. Many attempts have been made to determine the mixing angle and the gluonic component. The η-η' mixing angle θ_p determined by much work is in the range -20° to -10°, which has some uncertainties. To determine the η-η' mixing angle from $D^+ \to \eta^{(\prime)} l^+ \nu_l$, it is necessary to calculate $D^+ \to \eta^{(\prime)} l^+ \nu_l$ accurately, which means that it is essential to calculate the form factor of $D \to \eta$ accurately.

The method of light-cone QCD sum rules has been widely used in hadronic physics since its establishment [24, 25]. In this approach, the non-perturbative dynamics are parameterized as light-cone wave functions classified by their twist. The higher twist the wave function has, the less the contribution to the sum rules of the form factor. Among the different twist wave functions, the twist-2 wave function has a dominant contribution to the form factor, the twist-3 wave function comes second, which has a large contribution amounting to about 50% of what the twist-2 wave function has, and the twist-4 wave function has little contribution, which is about 4%–6% of the form factor [26]. The uncertainties in wave functions will result in uncertainties in the form factor. Among these twist wave functions, the twist-2 wave function has systematically been investigated and the uncertainties from the twist-2 wave function can be controlled well, the twist-3 and twist-4 wave functions are understood poorly, so the twist-3 wave function is the dominant source of the uncertainties in the form factor. To get an accurate calculation of the form factor in light-cone sum rules, it is necessary to decrease the uncertainties from the twist-3 wave function.

In this work, we take an improved correlator as in Ref. [27] to calculate the $D \to \eta$ transition form factor, and then apply it to predict the branching ratio of $D^+ \to \eta^{(\prime)} l^+ \nu_l$ semileptonic decays, which have not been measured correctly. In the calculation of $D \to \eta$ transition form factor, we take a chiral current correlator instead of the usual vector current correlator.

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The advantage with the chiral correlation is that contributions of the twist-3 wave functions vanish completely from the light-cone sum rule, such that the possible pollution by them is effectively avoided. It will be beneficial to decrease the uncertainties in the calculation of the form factor with the light-cone sum rule.

The $D\to\eta$ transition form factor can be reliably calculated in the light-cone QCD sum rules in the region of momentum transfer square q^2, $0\leqslant q^2\leqslant m_c^2-2m_c\Lambda$, where Λ is a typical hadronic scale having the value $\Lambda\approx 0.5$ GeV. To calculate the branching ratios of $D^+\to\eta^{(\prime)}l^+\nu_l$ ($l=e,\ \mu$), the $D\to\eta$ transition form factor in the whole physical region $0\leqslant q^2\leqslant(m_D-m_{\eta^{(\prime)}})^2$ is necessary, so we parametrize the $D\to\eta$ transition form factor in the reliable region and extrapalate it to the whole physical region. The usual parameterization of the heavy to light transition form factor is the two-pole model [28],

$$f^+(q^2)=\frac{f^+(0)}{1+a_1\dfrac{q^2}{m_D^2}+a_2\dfrac{q^4}{m_D^4}},\qquad(1)$$

and the BZ parametrization model [29],

$$f^+(q^2)=\frac{f^+(0)}{1-q^2/m_{D^*}^2}+\frac{f^+(0)rq^2/m_{D^*}^2}{(1-q^2/m_{D^*}^2)(1-\alpha q^2/m_D^2)}.\qquad(2)$$

In order to look for the difference between the two parametrization models, we take the two models to parametrize the form factor in the whole physical region and get the branching ratios of $D^+\to\eta^{(\prime)}l^+\nu_l$. Comparing the results from the two models in Fig. 3, we can find that there is negligible difference between the results of the branching ratio from the two models.

Having gotten an accurate calculation of form factor of $D\to\eta$ from the relation of the branching ratios of $D^+\to\eta^{(\prime)}l^+\nu_l$ with η-η' mixing angle in Eq. (17), we can find one way to determine the η-η' mixing angle with few uncertainties, which is shown in Fig. 4.

This paper is organized as follows. In Section 2, we derive the sum rule for the $D\to\eta$ transition form factor in light-cone sum rules. In Section 3, we present the numerical results. Section 4 is for the summary.

2 Sum rules ror $D\to\eta$ form factor

2.1 η and η' mixing scheme

There are two different mixing schemes in use to describe the η-η' system: the singlet-octet (SO) and the quark-flavour scheme (QF) [17, 30].

In the quark-flavor basis, the η and η' states can be expressed as,

$$\begin{pmatrix}|\eta\rangle\\|\eta'\rangle\end{pmatrix}=\begin{pmatrix}\cos\phi & -\sin\phi\\ \sin\phi & \cos\phi\end{pmatrix}\begin{pmatrix}|\eta_q\rangle\\|\eta_s\rangle\end{pmatrix},\qquad(3)$$

with

$$\begin{aligned}|\eta_q\rangle&=\frac{1}{\sqrt2}|\left(u\bar u+d\bar d\right)\rangle,\\|\eta_s\rangle&=|s\bar s\rangle,\end{aligned}\qquad(4)$$

where ϕ is the η-η' mixing angle in the singlet-octet scheme.

2.2 Correlator and sum rules for $f^+(q^2)$

The $D\to\eta$ transion form factors $f^+(q^2)$ and $\widetilde f(q^2)$ are defined as,

$$\langle\eta(p)|\bar d\gamma_\mu c|D(p+q)\rangle=2f^+(q^2)p_\mu+\widetilde f(q^2)q_\mu,\qquad(5)$$

with q being the momentum transfer.

In this work, in order to eliminate the contribution of twist-3 in the η light-cone wave function, we take a chiral current correlator as Ref. [27],

$$\begin{aligned}\Pi_\mu(p,q)&=i\int d^4x e^{iqx}\langle\eta(p)|T\{\bar d(x)\gamma_\mu(1+\gamma_5)c(x),\\&\qquad\bar c(0)i(1+\gamma_5)d(0)\}|0\rangle\\&=\Pi(q^2,(p+q)^2)p_\mu+\widetilde\Pi(q^2,(p+q)^2)q_\mu,\end{aligned}\qquad(6)$$

which is different from the commonly adopted correlators.

Following the usual steps of the light-cone sum rule, on the one hand, by inserting the complete intermediate states with the same quantum numbers as the current operator $\bar c i(1+\gamma_5)d$ in the correlator, we can get the hadronic representation for the correlator,

$$\begin{aligned}&\Pi_\mu^H(p,q)\\&=\Pi^H(q^2,(p+q)^2)p_\mu+\widetilde\Pi^H(q^2,(p+q)^2)q_\mu\\&=\frac{\langle\eta|\bar d\gamma_\mu c|D\rangle\langle D|\bar c\gamma_5 d|0\rangle}{m_D^2-(p+q)^2}\\&\quad+\sum_H\frac{\langle\eta|\bar d\gamma_\mu(1+\gamma_5)|D^H\rangle\langle D^H|\bar c i(1+\gamma_5)d|0\rangle}{m_{D^H}^2-(p+q)^2}.\end{aligned}\qquad(7)$$

Using Eq. (5) and the definition $\langle D|\bar c i\gamma_5 d|0\rangle=m_D{}^2f_D/m_c$, we can obtain

$$\begin{aligned}\Pi^H(q^2,(p+q)^2)&=\frac{2f^+(q^2)m_D^2f_D}{m_c(m_D^2-(p+q)^2)}\\&\quad+\int_{s_0}^{\infty}\frac{\rho^H(s)}{s-(p+q)^2}ds+\text{subtractions},\end{aligned}\qquad(8)$$

and

$$\widetilde{\Pi}^{\rm H}(q^2,(p+q)^2) = \frac{\widetilde{f}(q^2)m_{\rm D}^2 f_{\rm D}}{m_{\rm c}(m_{\rm D}^2-(p+q)^2)} + \int_{s_0}^{\infty} \frac{\widetilde{\rho}^{\rm H}(s)}{s-(p+q)^2}{\rm d}s + \text{subtractions}, \quad (9)$$

where s_0 is the threshold parameter.

In the calculation of the branching ratios of $D^+ \to \eta^{(\prime)} l^+ \nu_l$ ($l = e,\ \mu$), the contributions of $\widetilde{f}(q^2)$ to the decay amplitudes can be ignored, due to the smallness of the final state lepton masses, we only consider the form factor $f(q^2)$.

On the other hand, using the light-cone operator product expansion (OPE), we can get the QCD representation of the correlators,

$$\Pi^{\rm QCD}(q,(p+q)) = \Pi^{(\bar{q}q)}(q,(p+q)) + \Pi^{(\bar{q}qg)}(q,(p+q)), \quad (10)$$

where

$$\Pi^{(\bar{q}q)}(q^2,(p+q)^2) = 2f^{\rm q}m_{\rm c}\left[\int_0^1 \frac{{\rm d}u}{u}\varphi_\eta(u)\frac{1}{s-(p+q)^2} - 8m_{\rm c}^2\int_0^1 \frac{{\rm d}u}{u^3}g_1(u)\frac{1}{(s-(p+q)^2)^3} + 2\int_0^1 \frac{{\rm d}u}{u^2}G_2(u)\frac{1}{(s-(p+q)^2)^2} + 4\int_0^1 \frac{{\rm d}u}{u^3}G_2(u)\frac{q^2+m_{\rm c}^2}{(s-(p+q)^2)^3}\right], \quad (11)$$

with

$$G_2(u) = \int_0^u g_2(v){\rm d}v,$$

where $\varphi_\eta(u)$ is the twist-2 wave function, while both $g_1(u)$ and $g_2(u)$ have twist-4.

$$\Pi^{(\bar{q}qg)}(q^2,(p+q)^2) = 2m_{\rm c}f^{\rm q}\int_0^1 {\rm d}v\int D\alpha_{\rm i}\left(\frac{2\varphi_\perp(\alpha_{\rm i})+2\tilde{\varphi}_\perp(\alpha_{\rm i})}{[s-(p+q)^2]^2(\alpha_1+v\alpha_3)^2} - \frac{\varphi_\parallel(\alpha_{\rm i})+\tilde{\varphi}_\parallel(\alpha_{\rm i})}{[s-(p+q)^2]^2(\alpha_1+v\alpha_3)^2}\right), \quad (12)$$

with $\varphi_\perp$, $\varphi_\parallel$, $\tilde{\varphi_\perp}$ and $\tilde{\varphi_\parallel}$ being the three-particle wave functions of twist-4.

In the light-cone operator product expansion of the correlator, only the leading nonlocal matrix element $\langle\eta(p)|T\bar{d}(x)\gamma_\mu\gamma_5 d(0)|0\rangle$ contributes to the corrector, while the nonlocal matrix elements $\langle\eta(p)|\bar{d}(x){\rm i}\gamma_5 d(0)|0\rangle$ and $\langle\eta(p)|\bar{d}(x)\sigma_{\mu\nu}\gamma_5 d(0)|0\rangle$, whose leading terms are of twist-3, disappear in our approach, which means that the correlator avoids the uncertainties from the twist-3 wave function.

Matching the two representations of the correlator, using the quark-hadron dulity ansatz,

$$\rho^{\rm H}(s)(\widetilde{\rho}^{\rm H}(s)) = \rho^{\rm QCD}(s)(\widetilde{\rho}^{\rm QCD}(s))\theta(s-s_0), \quad (13)$$

and making Borel transformation, we can get the sum rules for the form factor,

$$f^+(q^2) = \frac{m_{\rm c}^2 f^{\rm q}}{m_{\rm D}^2 f_{\rm D}}{\rm e}^{\frac{m_{\rm D}^2}{M^2}}\left\{\int_\Delta^1 \frac{{\rm d}u}{u}{\rm e}^{-\frac{m_{\rm c}^2-q^2(1-u)}{uM^2}}\left(\varphi_\eta(u) - \frac{4m_{\rm c}^2}{u^2M^4}g_1(u) + \frac{2}{uM^2}\int_0^u g_2(v){\rm d}v\left(1+\frac{m_{\rm c}^2+q^2}{uM^2}\right)\right)\right.$$
$$+\int_0^1 {\rm d}v\int D\alpha_{\rm i}\frac{\theta(\alpha_1+v\alpha_3-\Delta)}{(\alpha_1+v\alpha_3)^2M^2}{\rm e}^{-\frac{m_{\rm c}^2-(1-\alpha_1-v\alpha_3)q^2}{M^2(\alpha_1+v\alpha_3)}}(2\varphi_\perp(\alpha_{\rm i})+2\widetilde{\varphi}_{\rm i}\perp(\alpha_{\rm i})-\varphi_\parallel(\alpha_{\rm i})-\widetilde{\varphi}_\parallel(\alpha_{\rm i}))$$
$$-4m_{\rm c}^2{\rm e}^{\frac{-s_0}{M^2}}\left(\frac{1}{(m_{\rm c}^2-q^2)^2}\left(1+\frac{s_0-q^2}{M^2}\right)g_1(\Delta) - \frac{1}{(s_0-q^2)(m_{\rm c}^2-q^2)}\frac{{\rm d}g_1(\Delta)}{{\rm d}u}\right)$$
$$\left.-2{\rm e}^{\frac{-s_0}{M^2}}\left(\frac{m_{\rm c}^2+q^2}{(s_0-q^2)(m_{\rm c}^2-q^2)}g_2(\Delta) - \frac{1}{(m_{\rm c}^2-q^2)}\left(1+\frac{m_{\rm c}^2+q^2}{m_{\rm c}^2-q^2}\left(1+\frac{s_0-q^2}{M^2}\right)\right)\int_0^\Delta g_2(v){\rm d}v\right)\right\}. \quad (14)$$

where

$$u=\frac{m_c^2-q^2}{s-q^2},\quad \Delta=\frac{m_c^2-q^2}{s_0-q^2},$$

s_0 is the threshold parameter, M is the Borel parameter, and m_c is the mass of c quark.

3 Numerical results

Before calculating the form factor $f^+(q^2)$ with the sum rule in Eq. (14), we need to take the input parameters in the sum rule. The main input parameters of the sum rule are the $\eta^{(\prime)}$ meson wave functions. The definite expressions of the $\eta^{(\prime)}$ meson wave functions are all given in Ref. [31]. The other input parameters are listed below [17, 30, 32],

$$\begin{aligned} M_D &= 1.893\ \text{GeV}, & m_c &= 1.3\pm0.1\ \text{GeV}, \\ f_D &= 170\pm10\ \text{MeV}, & s_0 &= 6\mp1\ \text{GeV}^2, \\ f^q &= (1.07\pm0.02)f_\pi, & f_\pi &= 130\ \text{MeV}. \end{aligned} \tag{15}$$

With these input parameters, we carry out a numerical calculation to the form factor $f^+(q^2)$. The first step is to look for the range of the Borel parameter M^2, where the the form factor $f^+(q^2)$ is stable for a given threshold s_0. The Borel parameter must meet two conditions. It cannot be too small, which ensures that the correlation can be expanded in OPE. At the same time, it cannot be too large, which meets the requirement that the contribution of higher continuum states is not more than 30%. The dependence of the form factor $f^+(q^2)$ on the borel parameter M^2 is shown in Fig. 1. One can see that the form factor $f^+(q^2)$ depends very weakly on the borel parameter M^2 in the interval 10 GeV^2 $\leqslant M^2 \leqslant$ 20 GeV^2 for 0 GeV^2 $\leqslant q^2 \leqslant$ 0.38 GeV^2. For the central value of borel parameter M^2 = 15 GeV^2, Fig. 2 shows the dependence of form factor $f^+(q^2)$ on the momentum in three cases: (1) with $m_c = 1.4$ GeV and $s_0 = 5.0$ GeV^2, (2) $m_c = 1.3$ GeV and $s_0 = 6$ GeV^2, (3) $m_c = 1.2$ GeV and $s_0 = 7.0$ GeV^2, respectively. Taking the central input values, we can get the form factor at 0 momentum trasfer,

$$f^+(0)=0.58. \tag{16}$$

The sources of uncertainties for $f^+(q^2)$ can be estimated in light-cone sum rules. The uncertainties of form factor $f^+(q^2)$ in Eq. (14) are induced by the input parameters M^2, m_c,s_0, f_D and the wave function of η meson, in this work, we avoid the main uncertainties from the uncertainties of twist-3 wave function, so the uncertainties of the form factor in Eq. (14) are decreased to be about 10%, which means the chiral current correlator can lead to the results of form factor with less uncertainties.

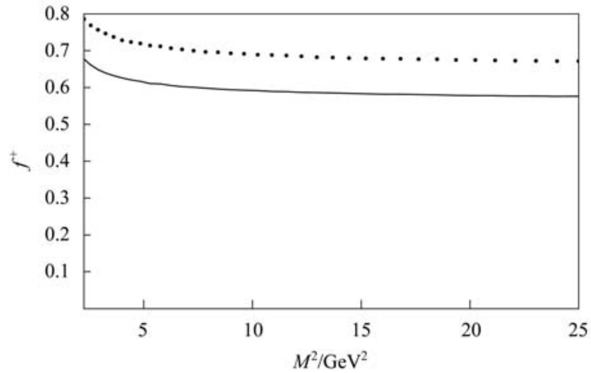

Fig. 1. The form factor f^+ as a function of the Borel parameter M^2. The solid curves: with $q^2 = 0$ GeV^2 and $s_0 = 6$ GeV^2. The dashed curves: with $q^2 = 0.36$ GeV^2 and $s_0 = 6$ GeV^2.

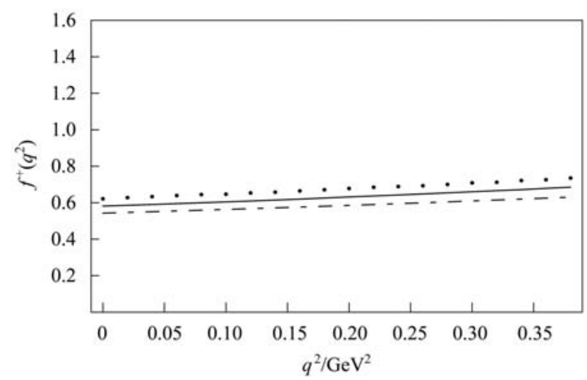

Fig. 2. The form factor $f^+(q^2)$ of $D \to \eta^{(\prime)}$ semileptonic transitions as a function of q^2 calculated in light-cone sum rules in $0 \leqslant q^2 \leqslant m_c^2 - 2m_c\Lambda$ at $M^2 = 16$ GeV^2. The dotted curves: with $m_c = 1.4$ GeV and $s_0 = 5.0$ GeV^2. The solid curves: with $m_c = 1.3$ GeV and $s_0 = 6$ GeV^2. The dotted curves: with $m_c = 1.2$ GeV and $s_0 = 7.0$ GeV^2.

The decay width of the semileptonic decay can be written as

$$\Gamma(D^+\to\eta^{(\prime)}l^+\nu_l) = |F_{d\bar{d}}^{\eta^{(\prime)}}|^2\frac{G_F^2|V_{cd}|^2}{192\pi^3m_D^3}\int_0^{(m_D-m_{\eta^{(\prime)}})^2}\lambda^{3/2}(q^2)|f^+_{\eta^{(\prime)}}(q^2)|^2dq^2, \tag{17}$$

where $\lambda(x) = (m_B^2+m_{\eta^{(\prime)}}^2-x)^2-4m_B^2m_{\eta^{(\prime)}}^2$, and the mixing factor is

$$\begin{aligned} F_{d\bar{d}}^{\eta} &= \frac{1}{\sqrt{6}}\cos\theta_P-\frac{1}{\sqrt{3}}\sin\theta_P, \\ F_{d\bar{d}}^{\eta'} &= \frac{1}{\sqrt{6}}\sin\theta_P+\frac{1}{\sqrt{3}}\cos\theta_P, \end{aligned} \tag{18}$$

where θ_P is the η-η' mixing angle in the singlet-octet scheme.

In order to estimate the width of $D^+\to\eta^{(\prime)}l^+\nu_l$, it is necessary to know the q^2 dependence of the form factors $f^+(q^2)$ in the whole physical region $0\leqslant q^2\leqslant (m_D-m_{\eta^{(\prime)}})^2$. The value of the $D\to\eta$ form factors can be reliably calculated from the QCD light cone sum rules in the reliable range of the region $0\leqslant q^2\leqslant m_c^2-2m_c\Lambda$, which is shown in Fig. 2. To extract the q^2 dependence of the form factors in the whole physical region, we should take parametrization to the form factor in the $0\leqslant q^2\leqslant m_c^2-2m_c\Lambda$ and fit to LCSR results, then extrapolate the fitted results to the whole physical region. The parametrization mostly used to the form factor in the whole physical region is two-pole parametrization [28] and BZ parametrization [29].

The q^2 dependence of the form factors $f^+(q^2)$ can be parametrized with the two-pole form,

$$f^+(q^2)=\frac{f^+(0)}{1-a_1\dfrac{q^2}{m_D^2}-a_2\dfrac{q^4}{m_D^4}}. \tag{19}$$

Fitted to the results of the $D\to\eta$ form factor in light-cone sum rules, the values a_1, a_2 of the parameters $f^+(q^2)$ can be gotten,

$$a_1=1.3162,\qquad a_2=0.2682. \tag{20}$$

The q^2 dependence of the form factors can also be parametrized with the BZ parametrization model,

$$f^+(q^2)=\frac{f^+(0)}{1-q^2/m_{D^*}^2}+\frac{f^+(0)rq^2/m_{D^*}^2}{(1-q^2/m_{D^*})(1-\alpha q^2/m_D^2)}. \tag{21}$$

The BZ parametrization is intuitive and understandable, and it can be obtained from the dispersion relation,

$$f^+(q^2)=\frac{\mathrm{Res}_{q^2=m_{D^*}^2}f^+(q^2)}{q^2-m_{D^*}^2}+\frac{1}{\pi}\int_{(m_D+m_\eta)^2}^{\infty}\mathrm{d}t\,\frac{\mathrm{Im}\,f^+(t)}{t-q^2-\mathrm{i}\epsilon}, \tag{22}$$

by replacing the second term on the right-hand side with an effective pole.

The parameters r, α in Eq. (21) can be determined by fitting to the reliable LCSR values,

$$r=0.5771,\qquad \alpha=2.0636. \tag{23}$$

The q^2 dependence of the $D\to\eta$ form factors $f^+(q^2)$ in the whole physical region from the two-pole parameterization model and BZ parameterization model is shown in Fig. 3, which shows that the two parameterization models to the $D\to\eta$ form factors fit the sum rules prediction quite well. From Fig. 3, we can find that there is a little difference in the large momentum region between the form factors from the two models.

Based on the form factor in the whole physical region extracted from the two models, the branching ratios of $D^+\to\eta^{(\prime)}l^+\nu_l$ can be gotten. Fig. 3 shows that the difference between the results of the decay branching ratio by the two models is negligible when the η-η' mixing angle is taken in the range of $-60°$ to $60°$.

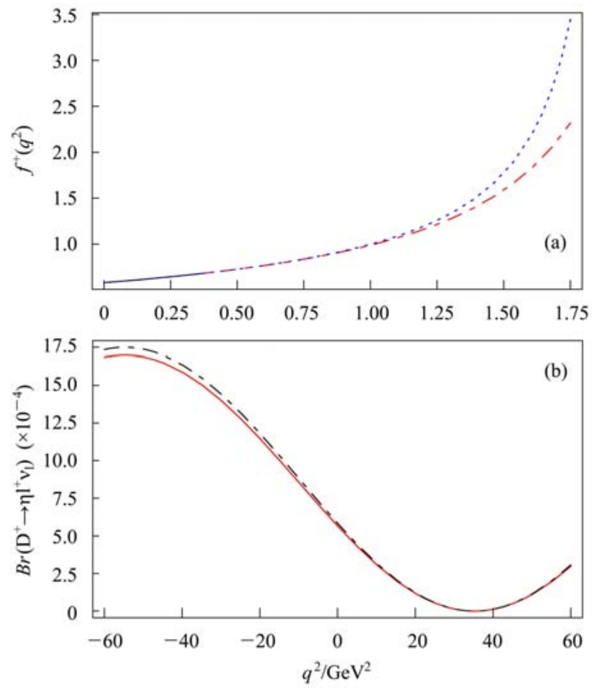

Fig. 3. (a) Difference between the form factors $f^+(q^2)$ of $D\to\eta$ extracted from the two-pole parametrization model (dashed curses) and from the BZ parametrization model (dotted curses). (b) Difference between the branching ratio of $D^+\to\eta l^+\nu_l$ from the two-pole parametrization model (solid curses) and from the BZ parametrization model (dashed curses).

With the form factor in the whole physical region, we can get the branching ratios of $D^+\to\eta^{(\prime)}l^+\nu_l$. Most of the authors obtained the mixing angle θ_p in the range between $-20°$ to $-10°$ by fitting the experimental data [1–23]. If we take the η-η' mixing angle as an input, say, $-20°\leqslant\theta_p\leqslant-10°$, we can predict the branching ratio of $D^+\to\eta^{(\prime)}l^+\nu_l$,

$$7.647\times10^{-4}\leqslant Br(D^+\to\eta l^+\nu_l)\leqslant1.023\times10^{-3}, \tag{24}$$

$$9.597\times10^{-5}\leqslant Br(D^+\to\eta' l^+\nu_l)\leqslant1.464\times10^{-4}. \tag{25}$$

Currently, only the experimental data of the branching ratio of $D^+\to\eta l^+\nu_l$ are available in Ref. [33],

$$Br^{\exp}(D^+\to\eta l^+\nu_l)<7\times10^{-3}. \tag{26}$$

Obviously, our results are consistent with the experimental data.

From Eq. (17), we can find two ways to determine the η-η′ mixing angle. The relation of the branching ratios of $D^+ \to \eta^{(\prime)} l^+ \nu_l$ with the η-η′ mixing angle θ_p

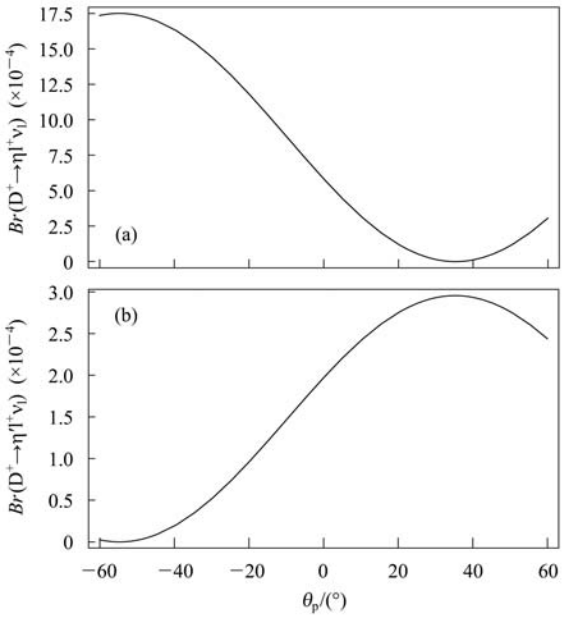

Fig. 4. (a) The variation in the branching ratios of $D^+ \to \eta l^+ \nu_l$ with the η-η′ mixing angle θ_p; (b) The variation in the branching ratios of $D^+ \to \eta' l^+ \nu_l$ with the η-η′ mixing angle θ_p.

can be derived from Eq. (17), which is shown in Fig. 4. From the relation shown in Fig. 4, we can determine the η-η′ mixing angle with the help of the experimental data of the branching ratio of $D^+ \to \eta^{(\prime)} l^+ \nu_l$. We hope the measurement to the branching ratio of $D^+ \to \eta l^+ \nu_l$.

4 Summary

To summarize, we take a chiral current to calculate the form factor, which can decrease the uncertainties arising from the twist-3 operator, and get more reliable values for the form factor in the range of $0 \leqslant q^2 \leqslant m_c^2 - 2m_c\Lambda$. We take the two-pole model and BZ model to parametrize the form factor in the whole physical region and get the branching ratios of $D^+ \to \eta^{(\prime)} l^+ \nu_l$. By comparing the results of the branching ratios of $D^+ \to \eta l^+ \nu_l$ from the two models, we find that the difference between the results extracted from the two parametrization models is negligible. We predict the range of the branching ratio of $D^+ \to \eta^{(\prime)} l^+ \nu_l$, which is consistent with the experimental data. With the accurate calculation of the $D \to \eta$ form factor and the relation of the branching ratio of $D^+ \to \eta^{(\prime)} l^+ \nu_l$ with the η-η′ mixing angle, we suggest one method beneficial to the determination of the η-η′ mixing angle with few uncertainties.

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