Radiation spectrum of rotating Gödel black hole and correction entropy*

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Abstract: We study the Hawking radiation of the scalar field in the rotating Gödel black hole in minimal five-dimensional supergravity. We not only derive radiation spectra that satisfy the unitary principle but also obtain the correction term of Bekenstein-Hawking entropy. The conclusion will help us learn more about the rotating Gödel black hole in minimal five-dimensional supergravity. This provides a greater understanding of the thermal radiation of black holes.

Key words: rotating Gödel black hole, energy conservation, Bekenstein-Hawking entropy correction

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1 Introduction

In 1974, Hawking discovered thermal radiation of black holes [1]. This discovery not only solves the problem in black hole thermodynamics but also illustrates the relationship between quantum mechanics, thermodynamics and gravitation, which sets a milestone in black hole physics. Investigating the physical mechanism of black hole Hawking radiation is an important subject in theoretical physics. So far there have been many methods to calculate Hawking radiation, such as the Hawking method [1], the Damour-Ruffini method [2, 3], the temperature Green function [4], the path integral, the renormalization energy-momentum tensor and a technique developed more recently called the generalized tortoise coordinate transformation to deal with Hawking radiation of an evaporating black hole [5, 6], etc. However, all results show that the black hole radiation is a black body spectrum. The conclusion has posed a difficult problem to theoretical physicists: is the information conservative in the black hole evaporation process?

In 2000, Parikh and Wilczek proposed the tunneling method [7], and in 2005, Robinson and Wilczek developed the covariant abnormal method [8]. In recent years, by using these two methods, thermal radiation of black holes has been investigated in depth [9–42]. All research groups have determined that the outgoing rate of a black hole radiation particle is

$$\Gamma=\exp[\Delta S], \tag{1}$$

where ΔS is the Bekenstein-Hawking(B-H) entropy difference before and after the black hole radiation. This satisfies the unitary principle and supports the principle of information conservation. When the black hole radiation spectra are calculated by the tunneling method, the Wentzel-Kramers-Brillouin (WKB) approach is often adopted. However, because the wave equation is not changed to a differential equation for WKB solving, the Bekenstein-Hawking entropy correction term in the calculation result is lost.

The correction value of the B-H entropy of black holes is a focal point of research. There are many methods for investigating the correction value of B-H entropy [14, 16, 17, 20, 21, 23, 30, 43–60]. Most people believe that the correction expression of B-H entropy of a Schwarzschild black hole is

$$S=\frac{A}{4G}+\chi\ln\frac{A}{4G}, \tag{2}$$

where A is the area of the black hole horizon and χ is a dimensionless constant. At present, the exact value of the logarithmic term coefficient in the correction to black hole B-H entropy is not clear. The result of

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correction to B-H entropy of black holes in complex spacetime has not been reported.

A great deal of effort [61–65] has been spent in recent years to study the Gödel-type solutions in the context of five-dimensional minimal supergravity. These solutions are related by T-duality to *pp*-waves, which make the Gödel-type universes important since they might provide the possibility of quantizing strings in these backgrounds and building relations to the corresponding limits of super-Yang-Mills theories.

The black hole solution must be embedded in the Gödel universe. The solution of the neutral rotating black hole embedded in the rotating Gödel universe was found in the five-dimensional minimal supergravity [61], which is called the Kerr Gödel black hole. The resulting Smarr formula was in full agreement with the first law of black hole thermodynamics like the four-dimensional back hole. To get further insight, therefore, the study of quantum phenomena like Hawking [27, 28, 66, 67] from the Gödel black hole would be highly motivating.

We extend the Damour-Ruffini method [25] and discuss the radiation spectrum and entropy correction in the Kerr Gödel black hole. Under the condition that the total energy and angular momentum are conserved, taking the reaction of the radiation of particles to the spacetime into consideration, we obtain the result that in the general case Hawking radiation spectra satisfy the unitary principle. This radiation is no longer a strict pure thermal spectrum; it is related to the Bekenstein-Hawking entropy change. In our calculation, because the wave equation is reduced to a differential equation for WKB solving, we derive the correction term of B-H entropy considering the radiation reaction to spacetime. This information can improve our knowledge of the rotating Gödel black hole in minimal five-dimensional supergravity.

2 The Kerr Gödel black hole

The Kerr Gödel black hole, which describes a five-dimensional rotating black hole (with the two equal rotation parameters) embedded in the Gödel universe, can be written as [61]

$$\mathrm{d}s^2 = -u(r)\mathrm{d}t^2-2g(r)\sigma_3\mathrm{d}t+h(r)\sigma_3^2+\frac{1}{f(r)}\mathrm{d}r^2 +\frac{1}{4}\mathrm{d}\Omega_3^2, \tag{3}$$

with

$$\mathrm{d}\Omega_3^2 = \mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2+\sigma_3^2,\ \sigma_3=\mathrm{d}\psi+\cos\theta\mathrm{d}\varphi,$$

$$u(r) = 1-\frac{2M}{r^2},\ h(r)=-j^2r^2(r^2+2M)+\frac{Ma^2}{2r^2},$$

$$g(r) = jr^2+\frac{Ma}{r^2}, f(r)=1-\frac{2M}{r^2}+\frac{8jM(a+2jM)}{r^2} +\frac{2Ma^2}{r^4}. \tag{4}$$

Here θ, φ and ψ are Euler angles, and j is the Gödel parameter and is responsible for the rotation of the Gödel universe. M is the mass parameter of the black hole whereas a characterizes its angular momentum. When $j=0$, the solution reduces to the five-dimensional Kerr black hole with the two possible rotation parameters setting equal to a. When $M=a=0$, the metric reduces to that of the five-dimensional Gödel universe [68]. When $a=0$, the solution becomes the Schwarzschild Gödel black hole.

The metric (3) has two spherically symmetric horizons (r_+,r_-) defined by $f(r)=0$. They are expressed as

$$r_\pm^2 = M(1-4aj-8j^2M) \pm\sqrt{M^2(1-4aj-8j^2M)^2-2Ma^2}. \tag{5}$$

The Hawking temperature T_{H} of the black hole and the angular velocity Ω_{H} at the horizon are described as

$$T_{\mathrm{H}}=\frac{r_+^2-r_-^2}{r_+^2\sqrt{4h(r_+)+r_+^2}}, \tag{6}$$

and

$$\Omega_\psi^{\mathrm{H}}=\frac{4g(r_+)}{4h(r_+)+r_+^2},\ \Omega_\varphi^{\mathrm{H}}=0, \tag{7}$$

respectively.

In the following, we will investigate the Hawking radiation of a scalar field in the small j case since the small rotation of the Gödel cosmological background seems to be the most reasonable in phenomenology [69].

3 Klein-Gordon equation and tortoise coordinate transformation

In curved spacetime, the Klein-Gordon equation of particles with rest mass μ_0 is

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi)-\mu_0^2\Phi=0, \tag{8}$$

where the determinant is given by the product of the base metric and higher-dimensional spherical harmonics,

$$\sqrt{-g}=\frac{r^3}{8}\sin\theta. \tag{9}$$

The separation of the wave equation is implemented by making the ansatz

$$\Phi = \mathrm{e}^{\mathrm{i}\omega t + \mathrm{i}m\varphi + \mathrm{i}\lambda\psi}\phi(r)S(\theta), \tag{10}$$

where ω is the energy of radiation particles, and m and λ are the projections of the angular momentum of radiation particle on rotation axis. $S(\theta)$ is the so-called spherical harmonics. We can obtain the equation

$$\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left[\sin\theta\frac{\mathrm{d}S(\theta)}{\mathrm{d}\theta}\right] - \left[\frac{(m-\lambda\cos\theta)^2}{\sin^2\theta} - E_{lm\lambda}\right]S(\theta) = 0 \tag{11}$$

for the angular part, where $E_{lm\lambda} = l(l+1) - \lambda^2$. The radial part reads

$$\frac{1}{r^3}\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{\Delta}{r}\frac{\mathrm{d}\phi(r)}{\mathrm{d}r}\right] + \left[\frac{K^2}{\Delta} - \frac{4E_{lm\lambda}+\Lambda}{r^2} - \mu_0^2\right]\phi(r) = 0, \tag{12}$$

with

$$K = \sqrt{r^4+2Ma^2}\left[\omega - \frac{4\lambda(\mathrm{j}r^4+Ma)}{r^4+2Ma^2}\right], \tag{13}$$

$$\Lambda = \frac{4\lambda^2 r^4}{r^4+2Ma^2}, \Delta = r^4 - 2M(1-4aj)r^2 + 2Ma^2.$$

When Eq. (12) is solved, WKB is often adopted [4, 9, 50]. However, because the wave equation is not changed to a differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + f(x)y = 0$$

suitable for WKB solving, when radiation spectra of black holes are discussed by the tunneling method, the obtained radiation spectra satisfy the unitary principle of quantum mechanics, but the B-H entropy correction term is lost.

In our calculation, Eq. (12) is changed into the second-order differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + f(x)y = 0$$

suitable for WKB solving. Let

$$\phi(r) = \frac{R(r)}{r^{1/2}(r^4+2Ma^2)^{1/4}}, \tag{14}$$

and the tortoise coordinate transformation

$$\mathrm{d}r_* = \frac{r^2(r^4+2Ma^2)^{1/2}}{\Delta}\mathrm{d}r. \tag{15}$$

Thus Eq. (12) can be reduced to

$$\frac{\mathrm{d}^2 R(r)}{\mathrm{d}r_*^2} + Q(r)R(r) = 0, \tag{16}$$

where

$$Q(r) = \left[\left(\omega - \frac{4\lambda(jr^4+Ma)}{r^4+2Ma^2}\right)^2 - \frac{\Delta_{r\omega}(r)}{(r^4+2Ma_\omega^2)}U(r)\right], \tag{17}$$

$$U(r) = \frac{4E_{lm\lambda}+\Lambda}{r^2} + \mu_0^2 - \frac{\Delta}{r^4}\frac{\mathrm{d}^2}{\mathrm{d}r^2}\left(\frac{1}{r^{1/2}(r^4+2Ma^2)^{1/4}}\right) - \frac{1}{r^3}\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{\Delta}{r}\right)\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{1}{r^{1/2}(r^4+2Ma^2)^{1/4}}\right). \tag{18}$$

After the black hole radiated particles with energy ω and angular momentum λ, energy E in spacetime line element (3) will be replaced by $E-\omega$, and J will be replaced by $J-\lambda$. Therefore, after considering the radiation, Δ will be replaced with Δ_ω and $Q(r)$ will be replaced with $Q_\omega(r)$. Considering the reaction of the radiation to spacetimes, (16) is rewritten as

$$\frac{\mathrm{d}^2 R(r)}{\mathrm{d}r_*^2} + Q_\omega(r)R(r) = 0, \tag{19}$$

where

$$E = \frac{3\pi}{4}M - \pi jMa,\ J = \frac{1}{2}\pi Ma - \pi jMa^2$$

[64], and r_ω satisfies $\Delta_\omega(r_\omega) = 0$.

4 Radiation spectrum and B-H entropy correction

Based on $\Delta_\omega(r_\omega) = 0$, near $r = r_\omega$, Eq. (19) is reduced to

$$\frac{\mathrm{d}^2 R(r)}{\mathrm{d}r_*^2} + (\omega-\omega_0)^2 R(r) = 0, \tag{20}$$

where

$$\omega_0 = \lambda\Omega_\omega,\ \Omega_\omega = \frac{4(jr_\omega^2+M_\omega a)}{r_\omega^4+2M_\omega a^2}.$$

The solution of (20) is

$$R(r) = \mathrm{e}^{\pm\mathrm{i}(\omega-\omega_0)r_*}. \tag{21}$$

According to the method proposed by Ref. [21], after the black hole radiates particles with energy ω and angular momentum λ, on surface $r = r_\omega$ the outgoing rate is

$$\Gamma_\omega = \left|\frac{\Phi_{\mathrm{out}}(r>r_\omega)}{\Phi_{\mathrm{out}}(r<r_\omega)}\right|^2 = \mathrm{e}^{-4\pi(\omega-\omega_0)/\kappa_\omega}, \tag{22}$$

where

$$\kappa_\omega = \frac{\Delta'_\omega(r_\omega)}{r_\omega^2(r_\omega^4+2M_\omega a^2)^{1/2}},\ \Delta'_\omega(r_\omega) = \left.\frac{\mathrm{d}\Delta_\omega(r)}{\mathrm{d}r}\right|_{r=r_\omega}.$$

Since the process that the black hole radiates particles with energy ω and angular momentum λ is an integration process [25, 70], that is $\omega=\int_0^\omega \mathrm{d}\omega'$, $\lambda=\int_0^\lambda \mathrm{d}\lambda'$. So the outgoing rate that the black hole radiates particles with energy ω and angular momentum λ is

$$\varGamma(i\rightarrow f)=\prod_i \varGamma_{\omega_i}=\exp\left[-\int_0^\omega \frac{4\pi\mathrm{d}\omega'}{\kappa_{\omega'}}+\int_0^\lambda \frac{4\pi\varOmega_{\omega'}\mathrm{d}\lambda'}{\kappa_{\omega'}}\right]. \tag{23}$$

Thermodynamic quantities corresponding to black holes meet the first law of thermodynamics [64],

$$\mathrm{d}E=\frac{\kappa}{8\pi}\mathrm{d}A+\varOmega\mathrm{d}J. \tag{24}$$

We have

$$\mathrm{d}(\Delta S)=-\frac{2\pi\mathrm{d}\omega'}{\kappa_{\omega'}}+\frac{2\pi\varOmega_{\omega'}}{\kappa_{\omega'}}\mathrm{d}\lambda', \tag{25}$$

the black hole B-H entropy $S=\frac{1}{4}A$, $A=2\pi r_{\mathrm{H}}\sqrt{r_{\mathrm{H}}^4+2Ma^2}$ is the area of the black hole horizon.

The B-H entropy difference before and after the black hole radiation is

$$\Delta S=S_{\omega'}(E-\omega',J-\lambda')-S(E,J). \tag{26}$$

Considering the reaction of the radiation of particles to the spacetime, the Hawking radiation spectrum is

$$\varGamma(i\rightarrow f)=\prod_i \varGamma_{\omega_i}=\exp\int \mathrm{d}(\Delta S)=\mathrm{e}^{\Delta S}. \tag{27}$$

In the radiation rate formula (22), we do not consider the factor $1/[r^{1/2}(r^4+2Ma^2)^{1/4}]$ in (14). After we consider this factor $1/[r^{1/2}(r^4+2Ma^2)^{1/4}]$ and the black hole radiates particles with energy ω and angular momentum λ, the outgoing rate formula (27) should be written as

$$\varGamma(i\rightarrow f)=\frac{r_i^2(r_i^4+2M_ia^2)}{r_f^2(r_f^4+2M_fa^2)}\mathrm{e}^{\Delta S}$$

$$=\exp\left[\left(\frac{A_f}{4}-\ln\frac{A_f}{4}\right)-\left(\frac{A_i}{4}-\ln\frac{A_i}{4}\right)\right]. \tag{28}$$

Thus the first order correction to B-H entropy is

$$S=\frac{A}{4}-\ln\frac{A}{4}. \tag{29}$$

Comparing (29) with (2), we derive $\chi=-1$ in (2). So we obtain the correction to the B-H entropy of a Kerr Gödel black hole.

5 Conclusion

We extend the method that Damour-Ruffini used to discuss Hawking radiation and investigate the radiation spectrum of a rotating Gödel black hole under the condition that the total energy is conserved and self-gravitation exists. Our result is consistent with that of Parikh and Wilczek. The radiation spectrum satisfies the unitary principle.

In our calculation, in order to make the radial equation after separation of variables change into the second-order differential equations

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+f(x)y=0$$

suitable for WKB solving, we introduce coordinate transformation (14). That is, we introduce factor $r^{-1/2}(r^4+2Ma^2)^{-1/4}$ in (10), which produces a black hole B-H entropy correction term. Most research on the black hole radiation spectra is used to obtain solutions by the WKB method. In their calculations, they all neglect the factor $r^{-1/2}(r^4+2Ma^2)^{-1/4}$. So we have given a more comprehensive conclusion and obtained a further understanding of the black hole thermal radiation. This conclusion will help us learn more about the rotating Gödel black hole in minimal five-dimensional supergravity.

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