System size in relativistic heavy ion collisions*

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Abstract: System size is more than a geometrical quantity in relativistic heavy ion collisions; it is closely related to evolution process, i.e. a different system size corresponds to a different evolution process, and whether QGP is produced depends on the system size. We propose that the system size should be under the same level when comparing the measurements from different colliding nuclei. The equivalence of the peripheral collisions of Au-Au and the central collisions of smaller nuclei is studied using the Monte Carlo method. Comparing the transverse overlapping area of the colliding nuclei, the number of participant nucleons and the number of nucleon-nucleon binary collisions in various colliding nuclei, we give an estimate of the correspondence in system size. This is helpful in the experimental comparison of the measurements from different colliding nuclei.

Key words: relativistic heavy ion collisions, system size

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1 Introduction

High energy nucleus-nucleus collision is proposed as a tool to study the new form of matter — quark gluon plasma (QGP) and its transition since the 1970s [1–3]. Up to now, experiments have collected a large amount of data through the collisions of C, N, O, Al, Si, Ca, Cu, Au, Pb etc. at center-of-mass energies varying from about 2 to 200 A GeV [4–11]. The evidence for QGP has been observed at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory [8–11]. The evidence for the critical point on the phase diagram is still lacking.

Since many nuclei have been used in heavy ion collisions, how to compare the results from different colliding nuclei is an important question. For example, how should one compare the measurements in Cu-Cu collisions and those of Au-Au collisions ? Some papers make a direct comparison between the 0–10% centrality of Cu-Cu and the 0–5% centrality of Au-Au [12, 13]. This seems a bit naive since they are of different system sizes and may undergo different evolution processes. However, this issue didn't seem to gain enough attention.

Generally speaking, the events undergoing the same evolution process can be classed into the same class in order to make comparisons conveniently. To class events, one can estimate the temperature and the baryon chemical potential for each collision and map them on the phase diagram, classing them by their thermodynamical parameters. This means that the collisions with the same thermodynamical parameters create the same thermodynamic condition. But the measurement of these thermodynamical parameters is not so easy and is dependent on certain models. In addition, we argue that one can analyze the initial geometrical characteristic and class events by system size. Here, system size is quantized as the transverse overlapping area of the colliding nuclei $S_\perp$, the number of participant nucleons $N_{\rm part}$ and the number of nucleon-nucleon binary collisions $N_{\rm coll}$.

One may think that system size is a geometrical quantity and comparing measurements from different colliding nuclei under the same geometrical quantity might also be naive.

In fact, system size is more than a geometrical quantity in relativistic heavy ion collisions; it is closely related to evolution processes. Different sys-

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tem size corresponds to different evolution processes. The reasons are as follows:

(1) The RHIC data found that, in the most central (0–5% centrality) Au+Au collisions, the back-side high-$p_{\rm t}$ two-hadron correlations in the azimuthal angle disappear (i.e. the so-called monojet phenomenon [14]), which shows that, in central collisions, QGP is formed and absorbs the back-side jet. As centrality increases, the back-side jet appears gradually. In the most peripheral collisions (60%–80% centrality), the back-side jet is observed in full strength just as in the case of p+p collisions, indicating that there is no QGP formation. Whether QGP is produced depends on centrality, i.e. system size. In other words, whether phase transition takes place depends on system size.

(2) From the theoretical side, system size is related to physical parameters, e.g. energy density. It is multiple scattering between nucleons from two incident nuclei that causes the energy deposition in the collision region. In the Glauber model [15], the probability of n multiscattering in the collision of two nuclei at a given impact parameter is a binomial distribution. For two head-on nuclei with identical nucleon number A, the mean number of multiscatterings is proportional to $A^{4/3}$ [16]. The collision of nuclei with larger nucleon number has more scattering, larger $N_{\rm coll}$, and hence more energy deposition. When the energy density is above about ten times that of normal nuclear matter, phase transition takes place and QGP is created. Therefore the system size can reflect energy density and phase transition as well.

(3) From the experimental side, many observables are investigated as functions of system size, including the colliding nuclear species and centrality. The same system size can be achieved by peripheral collisions of large nuclei as well as central collisions of small nuclei. Most of these experimental measurements show that at a given beam energy the results from different colliding nuclei are nearly the same as long as the number of participant nucleons ($N_{\rm part}$) is the same [17–22], which confirms that events with the same system size experience the same evolution process.

For both theoretical and experimental reasons, therefore, we believe that the evolution process depends on the system size. And when comparing the measurements from different colliding nuclei, the system size should be under the same level.

The importance of correctly comparing the measurements from different colliding nuclei species also originates from the following. For example, people are eager to observe large fluctuation at the critical point or its non-monotonic behavior in a beam energy scan. In that case, the numerical value of these quantities is crucial. However, from SPS to RHIC, the colliding nuclei are different and observables may be both energy dependent and system size dependent. When we draw an observable as a function of energy $\sqrt{s_{\rm NN}}$, the system size of different experiments should be at the same level. However, Ref. [13] does not consider the difference in system size between Cu-Cu and Au-Au and draw the dynamical net charge fluctuation $\nu_{+-,\rm dyn}$ both measured in central collisions (see Fig. 2 in Ref. [13]). This might smooth out the true signal for the critical point.

Based on the above arguments, we propose that when comparing the measurements from different colliding nuclei, the system size should be under the same level. In this paper, we analyze the system size for various colliding nuclei. Calculating the transverse overlapping area of the colliding nuclei $S_\perp$, the number of participant nucleons $N_{\rm part}$ and the number of nucleon-nucleon binary collisions $N_{\rm coll}$ by a simple Monte Carlo method, we give an estimate of the correspondence of their system size, i.e. the correspondence between the centrality of the Au-Au collision and the central collision of small nuclei. The analysis method and the detailed results are given in Section 2 and a summary is given in Section 3.

2 Correspondence between the centrality of Au-Au and the central collisions of a small nucleus

In nucleus-nucleus collision, the impact parameter b is defined as the transverse distance of the colliding nuclei. In head-on collisions, b is small, and in peripheral collisions, b is large. So in theoretical study, b is used to characterize the degree of head-on.

However, b is unmeasurable in experiment. Considering the fact that the smaller the b, the larger the final state multiplicity, people usually draw the multiplicity distribution and select e.g. 5% events with the top multiplicity as the most central collisions, denoted as 0–5% centrality. Similarly, from the theoretical side, to quantify the degree of head-on, the word "centrality" is defined as the percentage of the total cross section, i.e. select e.g. 5% events with the lowest impact parameter $b\in(0,b_0)$, as 0–5% centrality,

$$\text{centrality}(b_0)=\frac{\pi b_0^2}{\pi b_{\max}^2}, \tag{1}$$

where πb_0^2 is the cross section for those events with

$b \in (0, b_0)$ and $\pi b_{\max}^2$ is the total cross section. Ignoring the interactions between the boundaries of nuclei in the very peripheral case, the total cross section is $\sigma_{\text{tot}} = \pi b_{\max}^2 = \pi(2R)^2$.

For different centrality, the initial geometrical characteristic is different, which is usually described by the transverse overlapping area of the colliding nuclei, the number of participant nucleons and the number of nucleon-nucleon binary collisions, which are called system size here. Unfortunately, these quantities cannot be measured directly from experiments. In order to find a correspondence between the centrality of Au-Au collision and the central collisions of small nucleui and provide a guide for experimental comparison, we use a Monte Carlo method and calculate these geometrical quantities as functions of b for Au-Au collisions on the one hand, and, on the other hand, calculate these quantities in central collisions with $b=0$ of various nuclear species (denoted by nucleon number A). According to the same system size, we connect the nucleon number A with b in Au-Au, and hence with centrality in Au-Au.

2.1 The transverse overlapping area $S_\perp$

Because of Lorentz contraction in beam direction, the transverse overlapping area determines the initial volume of the reaction region. Approximating the nucleus as a symmetrical sphere with a certain radius, the transverse overlapping area can be analytically calculated as the overlapping area of two circles in the transverse direction. For example, the Au nucleus has a radius $R = aA^{1/3} \approx 7.7$ fm, with A being the nucleon number 197 and a being a parameter determined by the experiment (here takes 1.33 fm) [23]. For each impact parameter b in Au-Au collision, a transverse overlapping area can be determined (see Fig. 1). In the same way, for the central collisions ($b = 0$) of various nuclear species, a transverse overlapping area can be determined. According to the same $S_\perp$, we connect the nucleon number A with the centrality in Au-Au (see curves in Fig. 4).

2.2 The number of participant nucleons N_{part}

In one nucleus-nucleus collision, only a part of the nucleons participate in the reaction while the others just pass by. Those that participate in the reaction are called participants and the others are called spectators, whose number depends on the impact parameter b. Usually those that are in the overlapping area participate in the reaction, so we just count the numbers in a Monte Carlo simulation.

Nucleons are distributed as Wood-Saxon form in the nucleus, i.e.

$$\rho(r) = \frac{\rho_0}{1+\exp\left[(r-r_0)/a\right]}, \tag{2}$$

with parameters $r_0 = 6.624$ fm and $a = 0.550$ fm [23]. The normalization factor $\rho_0 = 0.1587\ \text{fm}^{-3}$ is fixed by $\int_0^R \rho(r) 4\pi r^2 \mathrm{d}r = 197$. Given the impact parameter b, we count the number of participant s. Fig. 2 shows N_{part} as a function of b for Au-Au. In this plot and in the following, the error of the point is the statistical error, which is small and within the markers. In the same way, for the central collisions ($b=0$) of various nuclear species, N_{part} is also calculated. According to the same N_{part}, we connect the nucleon number A with the centrality in Au-Au (see solid circles in Fig. 4).

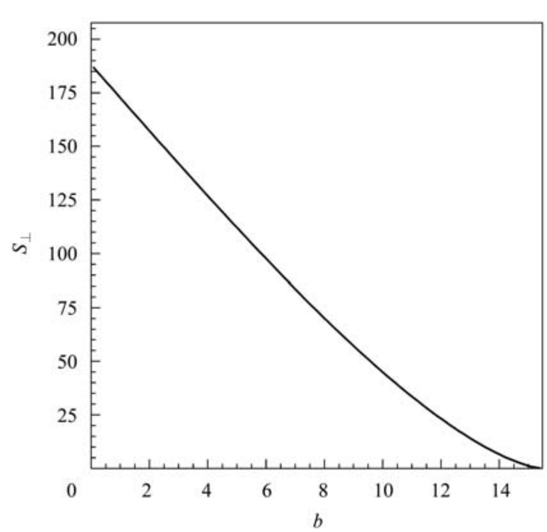

Fig. 1. $S_\perp$ as a function of b in Au-Au collision.

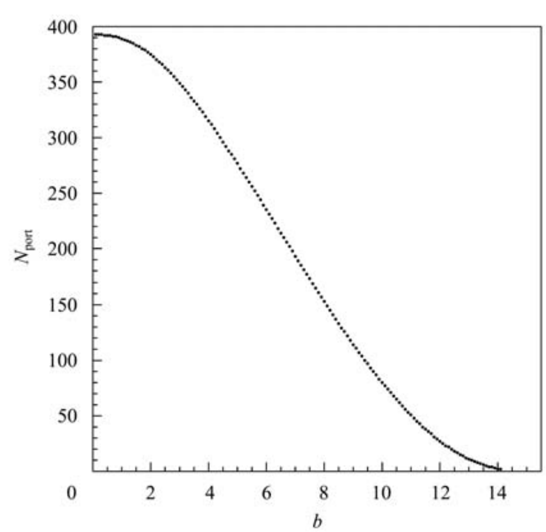

Fig. 2. N_{part} as a function of b in Au-Au.

2.3 The number of nucleon-nucleon binary collisions $N_{\rm coll}$

Sometimes the violence of the reaction can be descri-
bed by the number of nucleon-nucleon binary collisions. For the participating nucleons, the number of collisions it could suffer is dependent on the nucleon-nucleon cross section, $\sigma_{\rm NN}$, which is essentially a probability. The values of $\sigma_{\rm NN}$ usually take 41 mb for 200 GeV [24]. The distance within which collisions would occur is about $\sqrt{\frac{\sigma}{\pi}} \approx 1.14$ fm when $\sigma = 41$ mb. Considering the cross section and the maximum collision distance, for each nucleon we draw a tube along the beam direction with a radius $r = 0.8$ fm. Those nucleons that are in the tube will collide with the tube host. Counting the nucleons falling in the tube and summing them up gives the number of nucleon-nucleon binary collisions $N_{\rm coll}$. Fig. 3 shows $N_{\rm coll}$ as a function of b for Au-Au. In the same way, for the central collisions ($b = 0$) of various nuclear species, $N_{\rm coll}$ is calculated. According to the same $N_{\rm coll}$, we connect the nucleon number A with the centrality in Au-Au (see star points in Fig. 4).

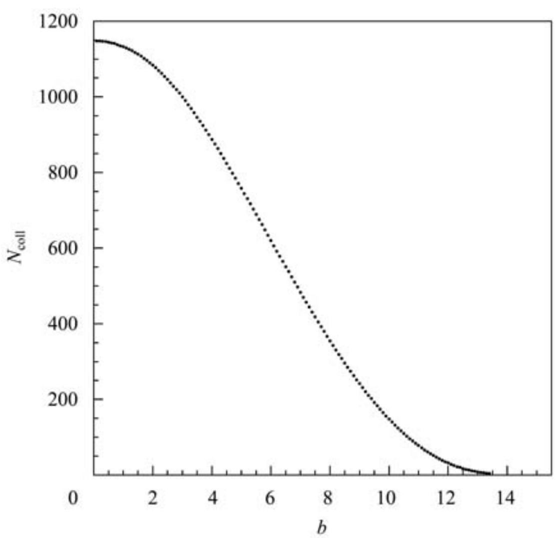

Fig. 3. $N_{\rm coll}$ as a function of b in Au-Au.

From Fig. 4, we find that the correspondence between the centrality of Au-Au and the central collisions of small nuclei has a similar trend for the three methods. The method with $S_\perp$ is just a rough estimation and so the curve in Fig. 4 can be ignored. There exists a minor difference between the other two results. The dashed line marks the nucleon number of Cu. We can equate the central Cu-Cu collision and the Au-Au collision with centrality of 30%–40%. That means that we should not compare the central Cu-Cu result with the central Au-Au result, but with Au-Au the 30%–40% centrality.

3 Summary

We argue that when comparing the measurements from different colliding nuclei, the system size should be under the same level since the system size is closely related to the evolution process. Through a simple analysis of the initial system size, we conclude that we should not compare the central Cu-Cu result with the central Au-Au result, but with Au-Au 30%–40% centrality. Under the same system size, we can draw reasonable conclusions. This just reminds the experimentalists to compare the measurements from different colliding nuclei under the same system size level, especially when we are exploring the signals of the critical point, which are sensitive to system size.

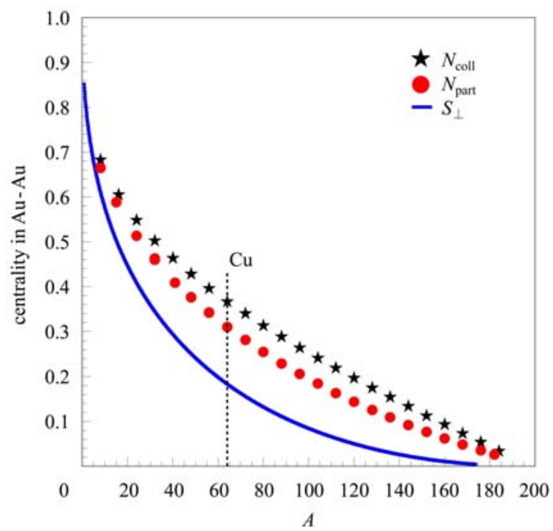

Fig. 4. Correspondence between the central collisions of a small nucleus with nucleon number A and the centrality of Au-Au for the three methods — $S_\perp$ (the curve), $N_{\rm part}$ (the solid circles) and $N_{\rm coll}$ (the stars).

Strictly speaking, the correspondence in Fig. 4 only holds for 200 GeV since the nucleon-nucleon cross section $\sigma_{\rm NN}$ takes the value at 200 GeV. When comparing 200 GeV Au-Au with 62.4 GeV Cu-Cu, the nucleon-nucleon cross section should take 39 mb at 62.4 GeV for Cu [24], which leads to less $N_{\rm coll}$ for Cu. The corrected curve will lie a bit higher than the star points in Fig. 4. The central Cu-Cu collision corresponds to a bit more peripheral Au-Au collision than the previous case. Similar analysis can be done again. The Glauber model can also be used in further studies.

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