Gaussian modification of neutrino energy loss on strongly screening nuclides ^{55}Co and ^{56}Ni by electron capture in stellar interior*

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Abstract: Gaussian modifications of the neutrino energy loss (NEL) by electron capture on the strongly screening nuclides ^{55}Co and ^{56}Ni are investigated. The results show that in strong electron screening (SES), the NEL rates decrease without modifying the Gamow-Teller (G-T) resonance transition. For instance, the NEL rates of ^{55}Co and ^{56}Ni decrease more than two and three orders of magnitude for $\rho_7 = 5.86$, $T_9 \leqslant 5$, $Y_e = 0.47$, $\Delta = 6.3$, respectively. In contrast, due to Gaussian modification, the NEL rates increase about two orders of magnitude in SES. Due to SES, the maximum values of the C-factor (in %) on NEL of ^{55}Co, ^{56}Ni are of the order of 99.80%, 99.56% at $\rho_7 = 5.86$ $Y_e = 0.47$ and 99.60%, 99.65% at $\rho_7 = 106$ $Y_e = 0.43$, respectively.

Key words: neutrino energy loss, strong electron screening, electron capture

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1 Introduction

During the nuclear synthesis processes in the late stage of the evolution of the massive star, the burning of the unstable nuclei and iron core collapse ignites the poignant supernova explosion. Electron capture (EC) plays a key role in this process and can produce a huge number of neutrinos. The neutrinos do not only transport enormous energy outward, but also carry a lot of information about the stellar interiors. Thus, the research on neutrinos and neutrino energy loss (NEL) has been of great interest as a subject on the border line between astrophysics and particle physics. Based on the simple shell model, Fuller G. M., Fowler W. A. and Newman M. J. [1] investigated the NEL rates and accomplished much pioneering work. Based on the Weinberg-Salam theory, Naoki Itoh et al. [2] analyzed the pair, photo-, plasma, bremsstrahlung and recombination NEL rates. But their discussions did not consider the influence of strong electron screening (SES) on the NEL rates.

What role does SES play in stars for NEL and what is the status of measurements for SES on earth? How does SES affect the NEL rates? This issue of SES has already been studied by some authors, such as Gutierrez et al. [3], Bravo & Garcia-Senz [4], Luo & Peng [5, 6]; Liu and Luo [7–10] and Itoh et al. [11]. Their work shows that it is extremely important and necessary to calculate accurately the screening corrections to the NEL rates from the EC processes in dense stars. On the other hand, The Gamow-Teller (G-T) resonance transition contribution has always been a very interesting and challenging issue, especially for beta decay and electron capture. Thus in this paper we pay attention to the influence of the G-T resonance transition contribution, which includes the range of low energy (ground state) to higher energies (excited states) and corrects the G-T resonance transition contribution by a Gaussian function.

Under pre-supernova conditions in the late stages

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of stellar evolution, ^{55}Co and ^{56}Ni are very abundant and in the dynamics of the pre-supernova evolution, the weak interactions on ^{55}Co and ^{56}Ni are believed to contribute effectively and play a key role. Later, Heger et al. [12] regarded ^{55}Co and ^{56}Ni as one of the most important nuclides for the capture processes in the pre-supernova evolution of massive stars. Nabi et al. [13] also discussed the weak interaction in ^{55}Co and ^{56}Ni. Aufderheide et al. [14] placed ^{55}Co and ^{56}Ni in the list of the top ten most important capturing nuclei during the pre-supernova evolution. Due to the importance of ^{55}Co and ^{56}Ni in astrophysical environments, based on the p-f shell model and linear response theory, we investigate in this paper a Gaussian modification of the NEL rates for the strongly screening nuclides ^{55}Co and ^{56}Ni.

2 The NEL rates with and without SES

The NEL rate (in $m_{\rm e}c^2s^{-1}$) due to EC for the kth nucleus (Z, A) in thermal equilibrium at temperature T is given by a sum over the initial parent states i and the final daughter states f [15],

$$\lambda_{if}=\sum_i\frac{(2J_i+1)\,{\rm e}^{-E_i/KT}}{G(Z,A,T)}\sum_f\lambda_{if}^{\nu}, \tag{1}$$

where J_i and E_i are the spin and excitation energy of the parent states and $G(Z,A,T)$ is the nuclear partition function. The NEL rate associated with EC from one of the initial states to all possible final states is λ_{if}^{ν},

$$\lambda_{if}^{\nu}=\frac{\ln 2}{(ft)_{if}}f_{if}$$

with the relation

$$\frac{1}{(ft)_{if}}=\frac{1}{(ft)_{if}^{\rm F}}+\frac{1}{(ft)_{if}^{\rm GT}}.$$

The ft-values and the corresponding Gamow-Teller or Fermi transition matrix elements are related by the equations [15]

$$\frac{1}{(ft)_{if}^{\rm GT}}=\frac{10^{3.596}}{|M_{\rm GT}|_{if}^2},\quad \frac{1}{(ft)_{if}^{\rm F}}=\frac{10^{3.79}}{|M_{\rm F}|_{if}^2}, \tag{2}$$

$$f(Q_{if})=\int_{\varepsilon_0}^{\infty}\varepsilon p(Q_{if}+\varepsilon)^3\frac{F(Z,\varepsilon)}{1+\exp\left(\dfrac{\varepsilon-U_{\rm F}-1}{kT}\right)}{\rm d}\varepsilon. \tag{3}$$

Here, $f(Q_{if})$ is the phase space factor without SES, k is the Boltzmann constant, T is the electron temperature, $U_{\rm F}$ is the electron chemical potential, $Q_{if}=Q_{00}+E_i-E_f$ is the EC threshold energy and $Q_{00}=M_{\rm p}c^2-M_{\rm d}c^2$, with $M_{\rm p}$ and $M_{\rm d}$ being the mass of the parent and daughter nucleus, respectively. E_i, E_f are the excitation energies of the i th and f th nuclear state, respectively, and ε is the total rest mass and kinetic energy. Note that in this paper all of the energies and momenta are given in units of $m_{\rm e}c^2$, $m_{\rm e}c$, where $m_{\rm e}$ is the electron mass and c is the velocity of light. $F(Z,\varepsilon)$ is the coulomb wave correction [1, 14]. Without considering the screening, the threshold energy ε_0 is defined by

$$\varepsilon_0=\begin{cases}Q_{if},\ (Q_{if}<-1)\\ 1,\ (\text{otherwise})\end{cases}. \tag{4}$$

The electron chemical potential is found by inverting the expression for the lepton number density,

$$n_{\rm e}=\frac{1}{\pi^2N_{\rm A}\lambda_{\rm e}^3}\int_0^{\infty}(f_{-{\rm e}}-f_{+{\rm e}}){\rm d}p, \tag{5}$$

where $N_{\rm A}$ is the Avogadro constant, $\lambda_{\rm e}=h/m_{\rm e}c$ is the Compton wavelength and $f_{-{\rm e}}$, $f_{+{\rm e}}$ are the electron and positron distribution functions, respectively.

If the electron is strongly screened and the screening energy is high enough in order not to be neglected in high density plasma, its energy will decrease from ε to $\varepsilon'=\varepsilon-{\rm D}$ in the decay reaction due to electron screening. At the same time, the screening relatively decreases the number of high energy electrons with energies higher than the threshold energy for electron capture. The threshold energy increases from ε_0 to $\varepsilon_{\rm s}=\varepsilon_0+{\rm D}$. Thus the phase space integral with screening becomes

$$f^s(Q_{if})=\int_{\varepsilon_{\rm s}}^{\infty}\varepsilon'(\varepsilon'^2-1)^{1/2}(Q_{if}+\varepsilon')^3F(Z,\varepsilon')f_{\rm e}(\varepsilon){\rm d}\varepsilon. \tag{6}$$

Using the linear response theory, Itoh et al. [11] calculated the screening potential for relativistic degenerate electrons. A more precise screening potential is given by

$$D=7.525\times10^{-3}Z\left(\frac{10Z\rho_7}{A}\right)^{1/3}J(r_{\rm s},R)\,{\rm MeV}, \tag{7}$$

where ρ_7 is the density in units of 10^7g/cm^3, $J(r_{\rm s},R)$, $r_{\rm s}$, R can be found in Ref. [11]. The formula (7) is valid for $10^{-5}\leqslant r_{\rm s}\leqslant10^{-1}$, $0\leqslant R\leqslant50$, conditions, which are usually fulfilled in the pre-supernova environment.

3 A modification of the G-T resonance transition

The G-T resonance transition contribution has always been a very interesting and challenging issue. According to Ref. [16], using a normalized Gaussian function, the strength contribution of the G-T resonance transition is written as [17, 18]

$$B_{\rm GT}^{\rm LP}=\frac{B_{\rm GT}}{\sqrt{\pi}\varDelta}\exp\left[-\left(\frac{\varepsilon''-E_{\rm GT}}{\varDelta^2}\right)^2\right], \tag{8}$$

where $B_{\rm GT}$ is the resonance strength, $B_{\rm GT}^{\rm LP}$ is the modified G-T resonance strength, ε'' is the energy in the excited state and $\varDelta$ is the half-width of the Gaussian function. Thus the NEL rates by EC in the case of SES can be rewritten as

$$\lambda_{if}^{\rm Gauss}=\lambda_{if}^{\nu}=\frac{\ln 2}{(ft)_{if}}f_{if}=\lambda_0+\lambda_{\rm GT}^{\rm Gauss}, \tag{9}$$

$$\lambda_{\rm GT}^{\rm Gauss}=\int_{\varepsilon_{\rm GT}}^{\infty}B_{\rm GT}^{\rm LP}{\rm d}\varepsilon''\int_{Q'}^{\infty}f_{\rm GT}^{\nu}{\rm d}\varepsilon, \tag{10}$$

$$f_{\rm GT}^{\nu}=\varepsilon'\left(\varepsilon'^2-1\right)^{1/2}\left(Q_{00}-\varepsilon''+\varepsilon'\right)^3F\left(Z,\varepsilon''\right)f_{\rm e}\left(\varepsilon\right){\rm d}\varepsilon, \tag{11}$$

where $\varepsilon'=\varepsilon-D$ and λ_0 is the NEL rate of EC in the low energy region, $\lambda_{\rm GT}^{\rm Gauss}$ is the modified G-T resonance transition NEL rate, $f_{\rm GT}^{\nu}$ is the modified phase space factor, $\varepsilon_{\rm GT}$ is the bottom energy of the resonance region, whose exact range depends on the half-width of the Gaussian distribution. Q', the lowest energy in the EC reaction due to SES, is given by

$$Q'=\begin{cases}|Q_{00}-\varepsilon''|+D,(Q_{00}-\varepsilon''<-1)\\ 1+D,(Q_{00}-\varepsilon''\geqslant -1)\end{cases}. \tag{12}$$

Hence, in SES the NEL rate with and without modifying the G-T resonance transition becomes

$$\lambda_{if}^{\rm Gauss(SES)}=\lambda_0^{\rm SES}+\lambda_{\rm GT}^{\rm Gauss(SES)}, \tag{13}$$

$$\lambda_{if}^{0({\rm SES})}=\lambda_0^{\rm SES}+\lambda_{\rm GT}^{0({\rm SES})}. \tag{14}$$

In order to compare the NEL rate with and without modifying the G-T resonance transition in SES, a C-factor is defined as

$$C=\left(\lambda_{if}^{\rm Gauss(SES)}-\lambda_{if}^{0({\rm SES})}\right)/\lambda_{if}^{\rm Gauss(SES)}. \tag{15}$$

4 Numerical analysis and discussion

Figure 1 shows the NEL rates of ^{55}Co, ^{56}Ni as a function of T_9 at the density of $\rho_7=5.86$ and $Y_{\rm e}=0.47$, $\varDelta=6.3$ with and without modifying the G-T resonance transition. One can see that due to SES without modifying the G-T resonance transition at relatively lower temperature (such as $T_9\leqslant 5$) the NEL rates of ^{55}Co decrease more than about two orders of magnitude but about three orders of magnitude for ^{56}Ni. At relatively higher temperature (such as $T_9=50$), the NEL rates of ^{55}Co and ^{56}Ni decrease by about 40%. On the other hand, due to modifying of the G-T resonance transition, the NEL rates will increase by two orders of magnitude.

At the density of $\rho_7=106$ and $Y_{\rm e}=0.43$, $\varDelta=10.3$, Fig. 2 shows the NEL rates of ^{55}Co, ^{56}Ni as a function of T_9 with the modified G-T resonance transition. One can see that in SES without modifying the G-T resonance transition, at relatively lower temperature (such as $T_9\leqslant 5$), the NEL rates of ^{55}Co decrease more than about three orders of magnitude, but about four orders of magnitude for ^{56}Ni under the same condition. However, at relatively higher temperatures (such as $T_9=50$), the NEL rates of ^{55}Co and ^{56}Ni decrease no more than about two orders of magnitude. In contrast, the NEL rate $\lambda_{if}^{\rm Gauss(SES)}$ is about two orders of magnitude larger than $\lambda_{if}^{0({\rm SES})}$.

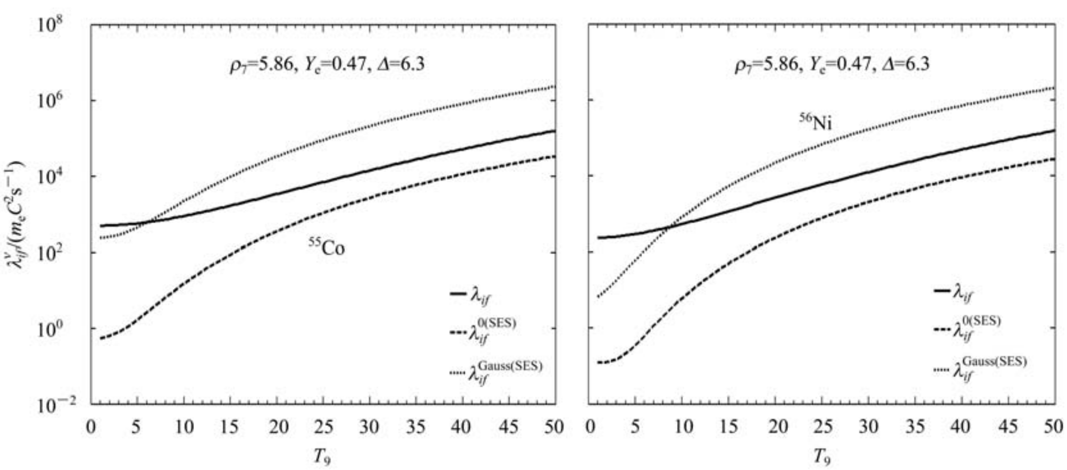

Fig. 1. The NEL rates λ_{if}, $\lambda_{if}^{0({\rm SES})}$, $\lambda_{if}^{\rm Gauss(SES)}$ of ^{55}Co, ^{56}Ni as a function of T_9 for $\rho_7=5.86$, $Y_{\rm e}=0.47$, $\varDelta=6.3$.

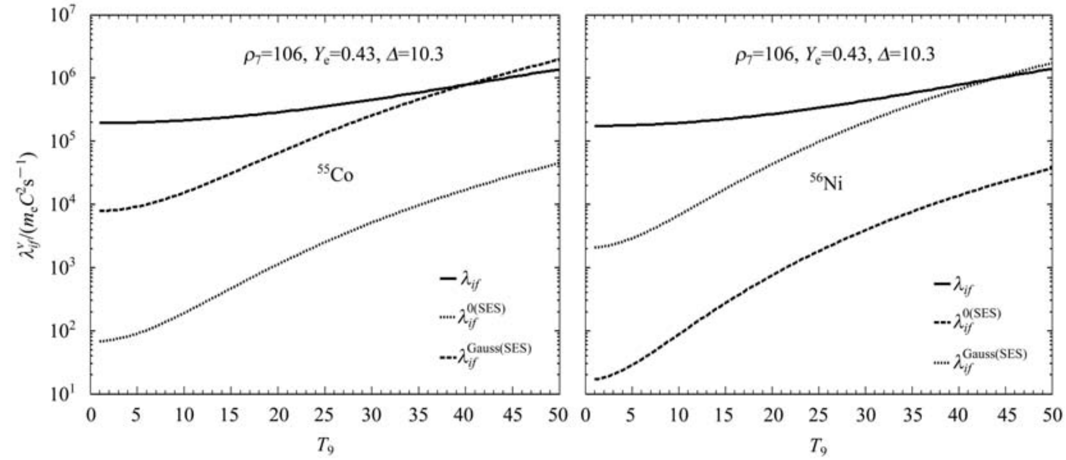

Fig. 2. The NEL rates λ_{if}, $\lambda_{if}^{0(\mathrm{SES})}$, $\lambda_{if}^{\mathrm{Gauss(SES)}}$ of ^{55}Co, ^{56}Ni as a function of T_9 for $\rho_7=106$, $Y_\mathrm{e}=0.43$, $\Delta=10.3$.

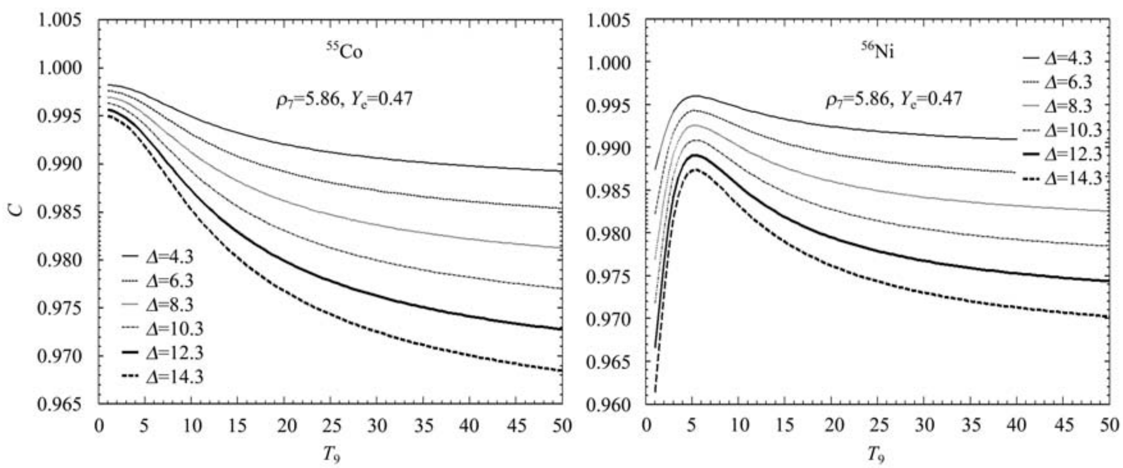

Fig. 3. The C-factors of ^{55}Co, ^{56}Ni as a function of T_9 for $\rho_7=5.86$, $Y_\mathrm{e}=0.47$, $\Delta=4.3$, 6.3, 8.3, 10.3, 12.3, 14.3.

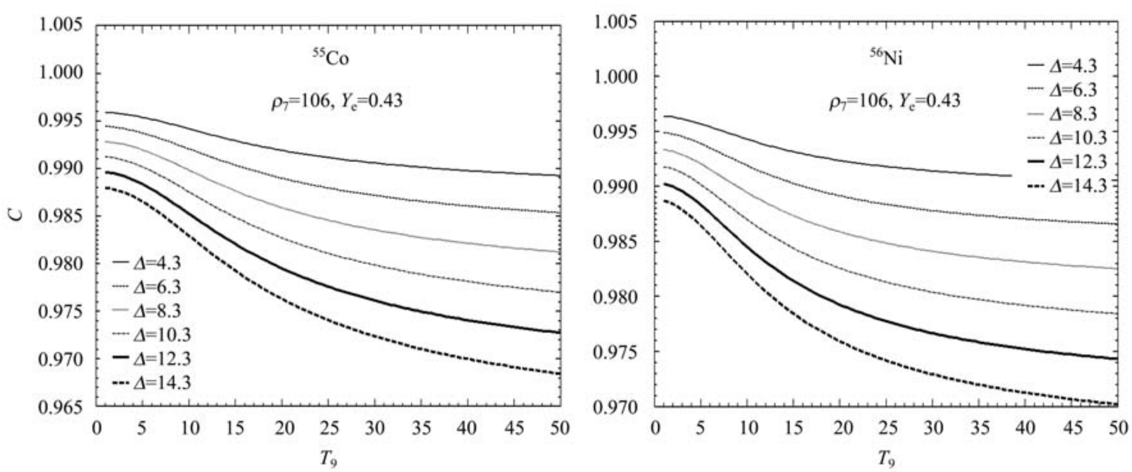

Fig. 4. The C-factors of ^{55}Co, ^{56}Ni as a function of T_9 for $\rho_7=106$, $Y_\mathrm{e}=0.43$, $\Delta=4.3$, 6.3, 8.3, 10.3, 12.3, 14.3.

Figures 3 and 4 show the C-factors of ^{55}Co, ^{56}Ni as a function of T_9 for $\rho_7 = 106$ $Y_e = 0.43$ and $\rho_7 = 5.86$, $Y_e = 0.47$, Δ=4.3, 6.3, 8.3, 10.3, 12.3, 14.3. It can be seen from the two Figures that the maximum values of the C-factor (in %) for the NEL rates of ^{55}Co, ^{56}Ni are of the order of 99.80%, 99.56% at $\rho_7 = 5.86$, $Y_e = 0.47$ and 99.60%, 99.65% at $\rho_7 = 106$ $Y_e = 0.43$, respectively.

From Figs. 1–4, one can see that at the same density, the temperature has a different effect on the NEL rates in and not in SES. The lower the temperature, the larger the influence on NEL is. The reason is the following. Although the electron energies are very low, the number of neutrinos produced by the EC process is very large, so that the influence of the temperature on the NEL becomes very big.

From the above calculation and analysis, comparing the NEL rates of Fig. 1 with those of Fig. 2, one finds that the NEL rates decrease noticeably due to SES. This is due to the fact that the screening changes the Coulomb wave function of the electron and it decreases the energy of the electron in the capture reaction. At the same time, the screening relatively decreases the number of high energy electrons with energy higher than the threshold energy of EC. This may lead to a large decrease in the NEL rates. On the other hand, due to the modification of the G-T resonance transition of an excited level, the NEL rates will increase, especially in the region of higher temperatures. The reason is that the transition strength distribution is Gaussian, which is symmetrical around the resonance location; the electron distribution must obey the Femi-Dirac law, but the energy of the electron in the resonance reaction is not symmetrical and the number of electrons taking part in EC greatly increases. Thus the NEL rate increases by modifying the G-T resonance transition.

As is well known, for massive stars such as white dwarfs and neutron stars, the problem of neutrino production has always been a very challenging subject and it always plays an important role during the late stages of evolution, particularly in terms of the energy loss and cooling in the stellar interior. A precise determination of the neutrino emission rates is therefore a crucial issue in any careful study of the final branches of star evolutionary tracks. In particular, changing the cooling rates at the very last stages of a massive star's evolution may sensibly affect the evolutionary time scale and the iron core configuration at the onset of the supernova explosion, whose triggering mechanism is still lacking a full theoretical understanding. Hence the conclusions we have drawn possibly have a significant influence on further research in nuclear astrophysics, especially on the research on the stellar terminal evolution and on neutrino astrophysics.

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