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An approach to dark energy problem through linear invariants*

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Abstract: The time evolution of vacuum energy density is investigated in the coherent states of inflationary universe using a linear invariant approach. The linear invariants we derived are represented in terms of annihilation operators. On account of the fact that the coherent state is an eigenstate of an annihilation operator, the wave function in the coherent state is easily evaluated by solving the eigenvalue equation of the linear invariants. The expectation value of the vacuum energy density is derived using this wave function. Fluctuations of the scalar field and its conjugate momentum are also investigated. Our theory based on the linear invariant shows that the vacuum energy density of the universe in a coherent state is decreased continuously with time due to nonconservative force acting on the coherent oscillations of the scalar field, which is provided by the expansion of the universe. In effect, our analysis reveals that the vacuum energy density decreases in proportion to $t^{-\beta}$ where β is 3/2 for radiation-dominated era and 2 for matter-dominated era. In the case where the duration term of radiation-dominated era is short enough to be negligible, the estimation of the relic vacuum energy density agrees well with the current observational data.

Key words: cosmological constant problem, vacuum energy density, coherent state, linear invariant

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1 Introduction

Needless to say, there are plenty of convincing evidences that the universe is now undergoing an accelerated expansion driven by a dark energy that is yet to be identified. This comes from recent observational data associated with large scale structure surveys [1], type Ia supernova analysis [2–8], and cosmic microwave background radiation mesurement [9–11]. The dark energy is in general responsible for the current phase of accelerated expansion. Though cosmological constant (vacuum energy) seems a natural and the simplest candidate for dark energy, the deep inconsistency between theoretically estimated value of cosmological constant and its observational limit led to a cosmological constant problem. There is a broad consensus that the cosmological constant problem is the greatest challenge in modern cosmology. The search for a complete physical explanation of the origin of an extremely small vacuum expectation value is one of the most puzzling issues in particle physics and astrophysics that we are facing today. An another pending problem in cosmology is why the vacuum energy density is not exactly zero and happens to be of almost the same scale as the matter energy density precisely at the present epoch. It is the so-called cosmic coincidence problem.

The Schrödinger picture of quantum (field) theory is highly attractive in describing the evolution of our universe and has been extensively developed in connection with the problem of modern cosmology [12–25]. In the previous work [12], we studied the evolution of the vacuum energy density in the Fock state using a unitary transformation approach in an inflationary universe and investigated the cosmological constant problem. Inflationary universe models may be the most adequate theories to describe the early epoch of universe since they provide a plausible scientific explanation for the creation of all of the matter and energy in the unvierse [26]. Inflation was

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triggered by enormous vacuum energy density endowed by spontaneous symmetry breaking of the GUT (Grand Unified Theory) gauge symmetry. In general, two types of massive scaler field are often considered in the literature of cosmology, namely, a minimal scalar field and a conformal scalar field.

In this paper, the evolution of vacuum energy density in a coherent state will be investigated on the basis of an inflation scenario by solving quantum mechanical solutions for a time-dependent harmonic oscillator that governs the time behavior of the scalar field. To meet our purpose, we take advantage of the linear invariant theory. Indeed, the construction of invariants (constants of motion) has attracted much attention in the context of quantum mechanics as well as classical mechanics since one of the key points in investigating dynamical systems with time-varying parameters is to find their invariants. It is well known that linear invariants in addition to quadratic invariants are very useful in investigating complete quantum mechanical solutions of dynamical systems, in particular those whose Hamiltonians are explicitly dependent on time [27–32]. The reason why invariant theory is very useful when studying the evolution of quantum scalar field in an inflationary universe is that the Hamiltonian associated with its description in FRW (Friedmann-Robertson-Walker) spacetime is actually a time-dependent form. The behavior of the classical scalar field may be best followed quantum mechanically by constructing an (over)complete set of coherent states since these states are closely related to classical behavior as far as quantum mechanics permits. In quantum mechanics, the coherent states are defined as eigenstates of the annihilation operator. The uncertainty relation in coherent states is the same as the minimum uncertainty relation in the Fock state. Interest in the investigation of cosmology and astroparticle physics in connection with the coherent states has gradually increased in the literature [21–25].

2 Inflationary universe model

A postulation of the inflation scenario is that the universe underwent a period of accelerated expansion based on the early universe dominance of a vacuum energy density characterized by the scalar field. Typically, it is assumed that such an inflation starts by GUT symmetry breaking at the very initial time of the universe and ends at the reheating time for some reason [33]. The inflation paradigm has now become a vital cog in the universe creation theory since it naturally leads to a solution to the horizon problem and provides not only the seeds for the formation of the large scale structure of our universe but also the temperature fluctuations in the CMB (Cosmic Microwave Background). The main predictions of the inflationary universe model are flatness of the universe, flatness of the spectrum of density perturbations, homogeneous and isotopic character of the CMB radiation, etc.

The flat FRW spacetime with the line element is given by

$$\mathrm{d}s^2=-\mathrm{d}t^2+R^2(t)(\mathrm{d}r^2+r^2\mathrm{d}\theta^2+r^2\sin^2\theta\mathrm{d}\varphi^2), \quad (1)$$

where $R(t)$ is a scale factor. We choose $R(t)$ for our expanding universe as a power-law form such that $R(t)=(t/b)^{\beta/3}$, where b and β are positive real constants [34, 35]. Note that b has the dimension of time while β is dimensionless. In an inflationary universe, it is well known that the behavior of the scalar field is governed by a differential equation of the form [36]

$$\ddot{\phi}_{\mathbf{k}}+3H\dot{\phi}_{\mathbf{k}}+V'(\phi_{\mathbf{k}})=0, \quad (2)$$

where $\phi_{\mathbf{k}}$ is a $\mathbf{k}$-th mode scalar field, H is the Hubble parameter, and $V(\phi_{\mathbf{k}})$ is a scalar field potential. Regarding that H takes the form $H=\dot{R}(t)/R(t)=\beta/(3t)$, in this power-law cosmology, the Hubble parameter decreases with time in proportion to $1/t$. On the other hand, if we say for your reference, H is usually assumed to be constant during inflation subjected to de Sitter spacetime. Under the framework of the Hartree approximation, $V(\phi_{\mathbf{k}})$ is given by a time-dependent form [37, 38]: $V(\phi_{\mathbf{k}})=\frac{1}{2}\omega_{\mathbf{k}}^2(t)\phi_{\mathbf{k}}^2$. In this representation, the time-dependent frequency $\omega_{\mathbf{k}}(t)$ can be written as [20]

$$\omega_{\mathbf{k}}(t)=\left(m^2+\xi\mathsf{R}(t)+\frac{\mathbf{k}^2}{R^2(t)}\right)^{1/2}, \quad (3)$$

where m is curvature of the potential and $\mathsf{R}(t)$ is the Ricci scalar defined by

$$\mathsf{R}(t)\,=\,6\left(\frac{\ddot{R}(t)}{R(t)}+\frac{\dot{R}^2(t)}{R^2(t)}\right). \quad (4)$$

The case $\xi=0$ corresponds to the cosmologies with a minimally coupled scalar field (CMCSF) while $\xi=1/6$ yields the cosmologies with a conformally coupled scalar field (CCCSF).

Then, the type of Eq. (2) is a kind of time-dependent harmonic oscillator,

$$\ddot{\phi}_{\mathbf{k}}+\frac{\beta}{t}\dot{\phi}_{\mathbf{k}}+\omega_{\mathbf{k}}^2(t)\phi_{\mathbf{k}}=0. \quad (5)$$

The term including $\dot{\phi}_{\mathbf{k}}$ in Eq. (5) gives nonconservative force to the coherent oscillations of the scalar field. In the Schrödinger picture, let us define a conjugate field momentum such that $\hat{\pi}_{\mathbf{k}} = -(\mathrm{i}/\mathcal{V})(\partial/\partial\phi_{\mathbf{k}})$, where $\mathcal{V}$ is a characteristic volume chosen so that the size of $\mathcal{V}^{-1/3}$ is the same as the energy scale at a boundary time b. To meet this requirement, we assume that the boundary condition at that time is theoretically known or obtainable from rigorous evaluations. The Hamiltonian, then, can be formulated in the form [12]

$$\hat{\mathcal{H}}_{\mathbf{k}}(\hat{\phi}_{\mathbf{k}},\hat{\pi}_{\mathbf{k}},t) = \frac{\hat{\pi}_{\mathbf{k}}^2}{2R^3(t)} + \frac{1}{2}R^3(t)\omega_{\mathbf{k}}^2(t)\hat{\phi}_{\mathbf{k}}^2. \quad (6)$$

Using Hamilton's equations of motion, we can easily check that this Hamiltonian exactly yields the classical equation of motion, Eq. (5). The commutation relation between $\hat{\phi}_{\mathbf{k}}$ and $\hat{\pi}_{\mathbf{k}}$ is given by $[\hat{\phi}_{\mathbf{k}},\hat{\pi}_{\mathbf{k}'}] = (\mathrm{i}/\mathcal{V})\delta_{\mathbf{k}\mathbf{k}'}$. We would like to investigate the evolution of vacuum energy density in a coherent state by solving quantum solutions related to the Hamiltonian of Eq. (6) through linear invariant theory.

3 Coherent state

As a preliminary to full quantum description of the system, let us denote two linearly independent homogeneous real solutions of the classical equation of motion, Eq. (5), as $\varepsilon_{\mathbf{k},1}(t)$ and $\varepsilon_{\mathbf{k},2}(t)$. Then, the general classical solution can be written as

$$\phi_{\mathbf{k}}(t) = c_1\varepsilon_{\mathbf{k},1}(t) + c_2\varepsilon_{\mathbf{k},2}(t). \quad (7)$$

In terms of $\varepsilon_{\mathbf{k},1}(t)$ and $\varepsilon_{\mathbf{k},2}(t)$, it is possible to define a time constant $W_{\mathbf{k}}$ such that [39]

$$W_{\mathbf{k}} = 2R^3(t)[\varepsilon_{\mathbf{k},1}(t)\dot{\varepsilon}_{\mathbf{k},2}(t) - \dot{\varepsilon}_{\mathbf{k},1}(t)\varepsilon_{\mathbf{k},2}(t)]. \quad (8)$$

As you can see at some later time, this is necessary in a quantum description of a time-dependent harmonic oscillator such as ours in this paper. When we find quantum solutions of a dynamical system whose Hamiltonian explicitly depends on time, the introduction of quantum invariant quantities is very useful. In fact, the method of dynamical invariants is widely used in the analysis of the behavior of quantum systems. From the Liouville-von Neumann equation for the linear invariant operator $\hat{I}_{\mathbf{k}}$,

$$\frac{\mathrm{d}\hat{I}_{\mathbf{k}}}{\mathrm{d}t} = \frac{\partial\hat{I}_{\mathbf{k}}}{\partial t} + \frac{\mathcal{V}}{\mathrm{i}}[\hat{I}_{\mathbf{k}},\hat{\mathcal{H}}_{\mathbf{k}}] = 0, \quad (9)$$

we obtain

$$\hat{I}_{\mathbf{k}} = \hat{a}_{\mathbf{k}}\exp[\mathrm{i}\eta_{\mathbf{k}}(t)], \quad (10)$$

where

$$\hat{a}_{\mathbf{k}} = \sqrt{\frac{\mathcal{V}}{W_{\mathbf{k}}}}\left(\frac{W_{\mathbf{k}}}{2\varepsilon_{\mathbf{k}}(t)}[1-\mathrm{i}Y_{\mathbf{k}}(t)]\hat{\phi}_{\mathbf{k}} + \mathrm{i}\varepsilon_{\mathbf{k}}(t)\hat{\pi}_{\mathbf{k}}\right), \quad (11)$$

$$\eta_{\mathbf{k}}(t) = \int_b^t \frac{W_{\mathbf{k}}\mathrm{d}t'}{2\varepsilon_{\mathbf{k}}^2(t')R^3(t')} + \eta_{\mathbf{k}}(b). \quad (12)$$

with $Y_{\mathbf{k}}(t) = 2R^3(t)\varepsilon_{\mathbf{k}}(t)\dot{\varepsilon}_{\mathbf{k}}(t)/W_{\mathbf{k}}$ and $\varepsilon_{\mathbf{k}}(t)$ is a time-dependent classical solution of the following differential equation,

$$\ddot{\varepsilon}_{\mathbf{k}}(t) + \frac{\beta}{t}\dot{\varepsilon}_{\mathbf{k}}(t) + \omega_{\mathbf{k}}^2(t)\varepsilon_{\mathbf{k}}(t) - \frac{W_{\mathbf{k}}^2}{4R^6(t)\varepsilon_{\mathbf{k}}^3(t)} = 0. \quad (13)$$

In fact, $\varepsilon_{\mathbf{k}}(t)$ is related to $\varepsilon_{\mathbf{k},1}(t)$, and $\varepsilon_{\mathbf{k},2}(t)$ by $\varepsilon_{\mathbf{k}}(t) = [\varepsilon_{\mathbf{k},1}^2(t) + \varepsilon_{\mathbf{k},2}^2(t)]^{1/2}$ [39].

We can readily show that $\hat{a}_{\mathbf{k}}$ and its Hermitian adjoint, $\hat{a}_{\mathbf{k}}^\dagger$, satisfy the boson commutation relation of the form $[\hat{a}_{\mathbf{k}},\hat{a}_{\mathbf{k}'}^\dagger] = \delta_{\mathbf{k}\mathbf{k}'}$. This implies that $\hat{a}_{\mathbf{k}}$ and $\hat{a}_{\mathbf{k}}^\dagger$ are ladder operators. Namely, $\hat{a}_{\mathbf{k}}$ is an annihilation operator while $\hat{a}_{\mathbf{k}}^\dagger$ is a creation operator. Indeed, the ladder operators play an important role in formulating quantum mechanics of dynamical systems.

After the publication of Glauber's work [40], which introduced the coherent state of the harmonic oscillator, coherent states and their generalizations became an important concept in modern physics. Coherent states have several outstanding nonclassical properties, such as sub-Poissonian statistics, correlation in the number fluctuations, higher order squeezing, and so on [41, 42]. There are large amounts of physical systems including cosmology for which a description in terms of coherent states is available [21–23].

Let us express the eigenvalue equation for $\hat{I}_{\mathbf{k}}$ in the form $\hat{I}_{\mathbf{k}}|\alpha_{\mathbf{k}}\rangle = \lambda_{\mathbf{k}}|\alpha_{\mathbf{k}}\rangle$, where $\lambda_{\mathbf{k}}$ is the eigenvalue and $|\alpha_{\mathbf{k}}\rangle$ the eigenstate. Since $\hat{I}_{\mathbf{k}}$ is represented in terms of the annihilation operator, $|\alpha_{\mathbf{k}}\rangle$ is a coherent state. If we consider Eq. (10), $\lambda_{\mathbf{k}}$ is given by $\lambda_{\mathbf{k}} = \alpha_{\mathbf{k}}\exp[\mathrm{i}\eta_{\mathbf{k}}(t)]$, where $\alpha_{\mathbf{k}}$ is the eigenvalue of the annihilation operator:

$$\hat{a}_{\mathbf{k}}|\alpha_{\mathbf{k}}\rangle = \alpha_{\mathbf{k}}|\alpha_{\mathbf{k}}\rangle. \quad (14)$$

In addition to the Fock state representation, an (over) complete normalized set of coherent states for the scalar field deserves to be established in order to analyze the vacuum energy density since the coherent state provides a useful basis for expanding the operators of scalar field.

By solving Eq. (14) in configuration space after the substitution of Eq. (11), we have a coherent state in the form

$$\langle\phi_{\mathbf{k}}|\alpha_{\mathbf{k}}\rangle = \left(\frac{\mathcal{V}W_{\mathbf{k}}}{2\varepsilon_{\mathbf{k}}^2(t)\pi}\right)^{1/4}\exp\left\{\frac{\sqrt{\mathcal{V}W_{\mathbf{k}}}}{\varepsilon_{\mathbf{k}}(t)}\alpha_{\mathbf{k}}\phi_{\mathbf{k}} - \frac{\mathcal{V}W_{\mathbf{k}}}{4\varepsilon_{\mathbf{k}}^2(t)}[1-\mathrm{i}Y(t)]\phi_{\mathbf{k}}^2 - \frac{1}{2}|\alpha_{\mathbf{k}}|^2 - \frac{1}{2}\alpha_{\mathbf{k}}^2\right\}. \quad (15)$$

We, however, need to multiply the above equation by an additional time-dependent phase factor in order to obtain a complete coherent state satisfying the Schrödinger equation, such that

$$\langle\phi_{\mathbf{k}}|\psi_{\mathbf{k}}\rangle=\langle\phi_{\mathbf{k}}|\alpha_{\mathbf{k}}\rangle e^{i\vartheta_{\mathbf{k}}(t)}, \tag{16}$$

where $\vartheta_{\mathbf{k}}(t)$ is a time-dependent phase. From the substitution of Eqs. (6) and (16) into the Schrödinger equation, we get

$$\vartheta_{\mathbf{k}}(t)=-\frac{1}{2}\int_b^t\frac{W_{\mathbf{k}}dt'}{2\varepsilon_{\mathbf{k}}^2(t')R^3(t')}+\vartheta_{\mathbf{k}}(b). \tag{17}$$

The direct differentiation of Eq. (11) with respect to time leads to $d\hat{a}_{\mathbf{k}}(t)/dt=-iW_{\mathbf{k}}\hat{a}_{\mathbf{k}}(t)/[2\varepsilon_{\mathbf{k}}^2(t)R^3(t)]$, so that

$$\hat{a}_{\mathbf{k}}(t)=\hat{a}_{\mathbf{k}}(b)\exp\left[-i\int_b^t\frac{W_{\mathbf{k}}dt'}{2\varepsilon_{\mathbf{k}}^2(t')R^3(t')}\right]. \tag{18}$$

By taking into account Eq. (11), we can easily see that the eigenvalue of Eq. (14) can be written as

$$\alpha_{\mathbf{k}}(t)=\sqrt{\frac{\mathcal{V}}{W_{\mathbf{k}}}}\left(\frac{W_{\mathbf{k}}}{2\varepsilon_{\mathbf{k}}(t)}[1-iY_{\mathbf{k}}(t)]\phi_{\mathbf{k}}(t)+i\varepsilon_{\mathbf{k}}(t)\pi_{\mathbf{k}}(t)\right), \tag{19}$$

where $\phi_{\mathbf{k}}(t)$ is the classical scalar field represented in Eq. (7) and $\pi_{\mathbf{k}}(t)$ is the conjugate classical momentum that is given by $\pi_{\mathbf{k}}(t)=R^3(t)\dot{\phi}_{\mathbf{k}}(t)$. If we consider Eq. (7), Eq. (19) can be divided into real and imaginary parts such that $\alpha_{\mathbf{k}}=\alpha_{\mathbf{k},R}+i\alpha_{\mathbf{k},I}$, where

$$\alpha_{\mathbf{k},R} = \frac{\sqrt{\mathcal{V}W_{\mathbf{k}}}}{2\varepsilon_{\mathbf{k}}}(c_1\varepsilon_{\mathbf{k},1}+c_2\varepsilon_{\mathbf{k},2}), \tag{20}$$

$$\alpha_{\mathbf{k},I} = \sqrt{\frac{\mathcal{V}}{W_{\mathbf{k}}}}R^3(t)[\varepsilon_{\mathbf{k}}(c_1\dot{\varepsilon}_{\mathbf{k},1}+c_2\dot{\varepsilon}_{\mathbf{k},2}) -\dot{\varepsilon}_{\mathbf{k}}(c_1\varepsilon_{\mathbf{k},1}+c_2\varepsilon_{\mathbf{k},2})]. \tag{21}$$

Now, it is possible to represent $\alpha_{\mathbf{k}}(t)$ in terms of its amplitude and phase:

$$\alpha_{\mathbf{k}}(t)=\alpha_{\mathbf{k}0}e^{i\sigma_{\mathbf{k}}(t)}, \tag{22}$$

where

$$\alpha_{\mathbf{k}0}=\sqrt{\alpha_{\mathbf{k},R}^2+\alpha_{\mathbf{k},I}^2}$$

and $\sigma_{\mathbf{k}}(t) = \tan^{-1}(\alpha_{\mathbf{k},I}/\alpha_{\mathbf{k},R})$. The substitution of Eqs. (20) and (21) into $\alpha_{\mathbf{k}0}$ gives

$$\alpha_{\mathbf{k}0}=\frac{1}{2}\sqrt{\mathcal{V}W_{\mathbf{k}}(c_1^2+c_2^2)}. \tag{23}$$

This is constant with time as expected. The differentiation of $\sigma_{\mathbf{k}}(t)$ with respect to time after further inserting Eqs. (20) and (21) yields $d\sigma_{\mathbf{k}}(t)/dt=-W_{\mathbf{k}}/[2\varepsilon_{\mathbf{k}}^2(t)R^3(t)]$. Therefore $\sigma_{\mathbf{k}}(t)$ becomes

$$\sigma_{\mathbf{k}}(t)=-\int_b^t\frac{W_{\mathbf{k}}dt'}{2\varepsilon_{\mathbf{k}}^2(t')R^3(t')}+\sigma_{\mathbf{k}}(b)\ [=-\eta_{\mathbf{k}}(t)]. \tag{24}$$

4 Fluctuations and vacuum energy density

Now, taking into account the effect of expansion of the universe, we study the quantum fluctuations. It is emphasized by Sakharov that the cosmological evolution is affected by the quantum fluctuations [43]. Accordingly, the quantum primordial fluctuations should have expanded towards the present time leading not only to classical energy density perturbations but also the decoupling from the cosmological background to the observable galaxies, clusters of galaxies, and superclusters [43]. The fluctuation of a certain quantum variable $\hat{x}_{\mathbf{k}}$ in the coherent state is defined by $(\Delta x_{\mathbf{k}})_{coh}=[\langle\psi_{\mathbf{k}}|\hat{x}_{\mathbf{k}}^2|\psi_{\mathbf{k}}\rangle-(\langle\psi_{\mathbf{k}}|\hat{x}_{\mathbf{k}}|\psi_{\mathbf{k}}\rangle)^2]^{1/2}$. According to this, we can easily evaluate the fluctuations of $\hat{\phi}_{\mathbf{k}}$ and $\hat{\pi}_{\mathbf{k}}$ by taking advantage of Eq. (16):

$$(\Delta\phi_{\mathbf{k}})_{coh} = \varepsilon_{\mathbf{k}}(t)\left(\frac{1}{\mathcal{V}W_{\mathbf{k}}}\right)^{1/2}, \tag{25}$$

$$(\Delta\pi_{\mathbf{k}})_{coh} = \frac{1}{2\varepsilon_{\mathbf{k}}(t)}\left(\frac{W_{\mathbf{k}}}{\mathcal{V}}[1+Y_{\mathbf{k}}^2(t)]\right)^{1/2}. \tag{26}$$

Note that these fluctuations explicitly depend on time. Due to nonconservative force acting on the coherent oscillations of the scalar field, that is provided by the expansion of the universe, it is expected that the amplitude of classical solution Eq. (7) decreases with time. This may also drive $\varepsilon_{\mathbf{k}}(t)$ to be smaller as time goes by. The above two equations therefore imply that $(\Delta\phi_{\mathbf{k}})_{coh}$ decreases with time whereas $(\Delta\pi_{\mathbf{k}})_{coh}$ increases.

The limitation in quantum mechanics, unlike in classical mechanics, on how accurately one can measure any two non-commuting observables is well documented by the famous Heisenberg uncertainty relation. From Eqs. (25) and (26), the uncertainty product is nothing but

$$(\Delta\phi_{\mathbf{k}})_{coh}(\Delta\pi_{\mathbf{k}})_{coh}=\frac{1}{2\mathcal{V}}[1+Y_{\mathbf{k}}^2(t)]^{1/2}\geqslant\frac{1}{2\mathcal{V}}. \tag{27}$$

Thus, we can confirm in this case that the uncertainty principle always holds. The uncertainty product in the coherent state for the time-dependent harmonic oscillator is not 1/2 (or, in the case here, not $1/(2\mathcal{V})$) but more or less larger than that. In fact, the uncertainty product increases depending on $Y_{\mathbf{k}}(t)$ which is closely related to the variation of $\varepsilon_{\mathbf{k}}$ with respect to time. The time variation of $\varepsilon_{\mathbf{k}}$ may give rise to a decreasing feature of scalar field due to the expansion of the universe. If $\xi\to0$ and $\beta\to0$, the Hamiltonian Eq. (6) no longer depends on time and $\varepsilon_{\mathbf{k}}$ becomes a constant. Then, Eq. (27) reduces to $1/(2\mathcal{V})$ which is familiar to us in the coherent states of a simple harmonic oscillator.

From the absolute square of Eq. (16), we see that the probability density can be represented in terms of $(\Delta\phi_{\mathbf{k}})_{\mathrm{coh}}$ such that

$$|\langle\phi_{\mathbf{k}}|\psi_{\mathbf{k}}\rangle|^2 = \frac{1}{\sqrt{2\pi}(\Delta\phi_{\mathbf{k}})_{\mathrm{coh}}}\exp\left[-\frac{(\phi_{\mathbf{k}}-\langle\phi_{\mathbf{k}}\rangle_{\mathrm{coh}})^2}{2(\Delta\phi_{\mathbf{k}})^2_{\mathrm{coh}}}\right], \tag{28}$$

where $\langle\phi_{\mathbf{k}}\rangle_{\mathrm{coh}}$ is the expectation value of $\hat{\phi}_{\mathbf{k}}$ in the coherent state and is given by $\langle\phi_{\mathbf{k}}\rangle_{\mathrm{coh}} = (\Delta\phi_{\mathbf{k}})_{\mathrm{coh}}(\alpha_{\mathbf{k}}+\alpha^*_{\mathbf{k}})$. Clearly, Eq. (28) is the Gaussian form which is very common. Since $(\Delta\phi_{\mathbf{k}})_{\mathrm{coh}}$ explicitly depends on time, it is expected that the shape of the evolution for Eq. (28) may somewhat change as time goes by. However, when there are no extra perturbations, the wave packet preserves Gaussian form once its initial shape is Gaussian.

The very earlier epoch when the universe was dominated not by matter or relativistic particles but by the action of a scalar field was characterized by vacuum energy density which dictated inflation of the universe. Now we are going to investigate the time evolution of the vacuum energy density. The classical vacuum energy density is given by $\rho_{\mathbf{k}} = \dot{\phi}^2_{\mathbf{k}}/2 + \omega^2_{\mathbf{k}}(t)\phi^2_{\mathbf{k}}/2$. If we consider the relation $\dot{\phi}_{\mathbf{k}} = \pi_{\mathbf{k}}/R^3(t)$ and convert the classical variables $\phi_{\mathbf{k}}$ and $\pi_{\mathbf{k}}$ into quantum variables $\hat{\phi}_{\mathbf{k}}$ and $\hat{\pi}_{\mathbf{k}}$, the vacuum energy density operator is constructed in the form $\hat{\rho}_{\mathbf{k}} = \hat{\pi}^2_{\mathbf{k}}/[2R^6(t)] + \omega^2_{\mathbf{k}}(t)\hat{\phi}^2_{\mathbf{k}}/2$. This is somewhat different from the Hamiltonian represented in Eq. (6). In fact, the role of a Hamiltonian is restricted to only the generator of the classical equation of motion when the given Hamiltonian explicitly depends on time [44, 45]. The expectation value of $\hat{\rho}_{\mathbf{k}}$ in the coherent states can be evaluated from

$$\rho_{\mathbf{k}}(t) = \frac{1}{2R^6(t)}\langle\psi_{\mathbf{k}}|\hat{\pi}^2_{\mathbf{k}}|\psi_{\mathbf{k}}\rangle + \frac{1}{2}\omega^2_{\mathbf{k}}(t)\langle\psi_{\mathbf{k}}|\hat{\phi}^2_{\mathbf{k}}|\psi_{\mathbf{k}}\rangle. \tag{29}$$

After a little algebra with the help of Eq. (16) and considering Eq. (22), we get

$$\rho_{\mathbf{k}}(t) = \frac{\varPi_{\mathbf{k},\alpha}(t)}{2\mathcal{V}}\left(\frac{b}{t}\right)^{\beta}, \tag{30}$$

where

$$\begin{aligned}\varPi_{\mathbf{k},\alpha}(t) \;=\; & \frac{1}{4R^3(t)\varepsilon^2_{\mathbf{k}}(t)W_{\mathbf{k}}}\{2\alpha^2_{\mathbf{k}0}\{2W^2_{\mathbf{k}}Y_{\mathbf{k}}(t)\sin[2\sigma_{\mathbf{k}}(t)] \\ & +[4\omega^2_{\mathbf{k}}(t)\varepsilon^4_{\mathbf{k}}(t)R^6(t) \\ & -W^2_{\mathbf{k}}(1-Y^2_{\mathbf{k}}(t))]\cos[2\sigma_{\mathbf{k}}(t)]\} \\ & +(1+2\alpha^2_{\mathbf{k}0})[4\omega^2_{\mathbf{k}}(t)\varepsilon^4_{\mathbf{k}}(t)R^6(t) \\ & +W^2_{\mathbf{k}}(1+Y^2_{\mathbf{k}}(t))]\}.\end{aligned} \tag{31}$$

To understand the evolution of the vacuum energy density, it is necessary to know the time behavior of $\varPi_{\mathbf{k},\alpha}(t)$. In the next section, this quantity will be analyzed for single scalar field cosmology.

5 Single scalar field cosmology

To investigate the wave function derived in the coherent state, we should know the explicit form of the classical solutions of Eq. (5), i.e. $\varepsilon_{\mathbf{k},1}(t)$ and $\varepsilon_{\mathbf{k},2}(t)$. However, the relevant mathematical procedures are somewhat complicated. We therefore consider only a single scalar field cosmology in this section. Then, the frequency Eq. (3) reduces to $\omega(t) = (m^2+\gamma^2/t^2)^{1/2}$, where $\gamma^2 = \frac{2}{3}\xi\beta(2\beta-3)$. By regarding this, the two linearly independent classical solutions of Eq. (5) are given by

$$\varepsilon_1(t) = \varepsilon_0(mt)^{(1-\beta)/2}J_\nu(mt), \tag{32}$$

$$\varepsilon_2(t) = \varepsilon_0(mt)^{(1-\beta)/2}N_\nu(mt), \tag{33}$$

where ε_0 is a constant which has a dimension of energy, J_ν and N_ν are the first and the second kind Bessel functions, and under subscript is $\nu = \sqrt{|(1-\beta)^2-4\gamma^2|}/2$.

According to Ref. [36], β can be expanded about $\varOmega_0 = 1$ where $\varOmega_0$ is the ratio of the present energy density to the critical energy density such that

$$\beta \simeq (3/2)[1-(\varOmega_0-1)/4+\cdots] \quad \text{radiation-dominated era,} \tag{34}$$

$$\beta \simeq 2[1-(\varOmega_0-1)/5+\cdots] \quad \text{matter-dominated era.} \tag{35}$$

Since it turns out from recent observations that $\varOmega_0 \simeq 1$ with fairly reliable precision, we can say that $\beta \simeq 3/2$ for radiation-dominated era and $\beta \simeq 2$ for matter-dominated era. Therefore. for the sake of simplicity, we take $\beta = 3/2$ and $\beta = 2$ for each epoch, respectively.

Based on the above arguments, the probability densities in radiation- and in matter-dominated era are plotted in Fig. 1. From this figure, we see that the probability densities not only oscillate but also converge to the origin ($\phi = 0$) with time. The converging feature of the probability density may reflect the decreasing property of the scalar field along its time evolution. As you can see, the probability density which belongs to the matter-dominated era converges more drastically to the origin than that which belongs to the radiation-dominated era. This means that the ratio of decreasing for the scalar field (and, consequently, vacuum energy density characterized by the scalar field) grows as the evolution period of the universe crosses from the radiation-dominated era to the matter-dominated era.

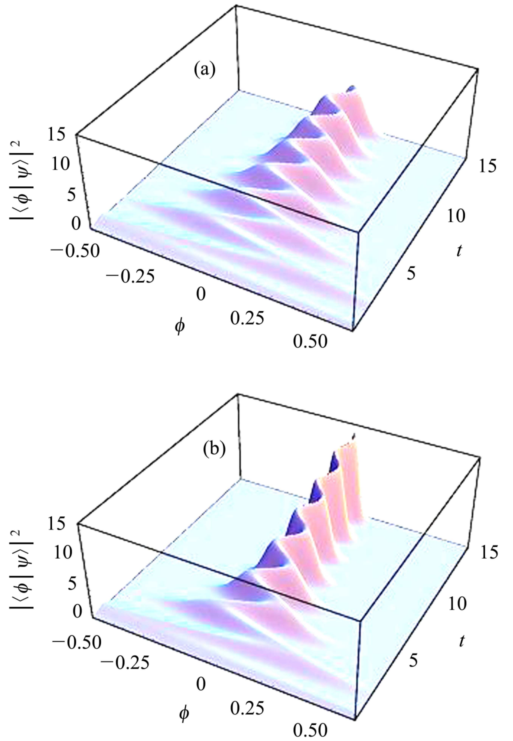

Fig. 1. The time evolution of the probability density $|\langle\phi|\psi\rangle|^2$ for the CMCSF ($\xi=0$) with $\beta=3/2$ (a) and $\beta=2$ (b) in the coherent state as a function of ϕ and t. Note that (a) belongs to the radiation-dominated era whilst (b) to the matter-dominated era. We used $c_1=c_2=1$, $\varepsilon_0=1$, $m=3$, $b=1$, $\delta=0$, and $\mathcal{V}=1$. For convenience and simplicity, all parameters are taken to be dimensionless.

We have also plotted fluctuations $(\Delta\phi)_{\rm coh}$ and $(\Delta\pi)_{\rm coh}$ and uncertainty relation $(\Delta\phi)_{\rm coh}(\Delta\pi)_{\rm coh}$ in Fig. 2. The fluctuation of the scalar field diminishes with time owing to the decrease in its amplitude during the evolution of the universe. On the other hand, the fluctuation of its conjugate momentum increases. But, the corresponding uncertainty product eventually does not significantly deviate from $1/(2\mathcal{V})$ which is the minimum value that quantum mechanics permits, except for a very small region that starts from initial time.

Now, to analyze the behavior of fluctuations and vacuum energy density, we assume

$$mt \gg 1. \tag{36}$$

It is shown in Appendix A that this assumption is in general valid for $t \geqslant t_{\rm r}$ where $t_{\rm r}$ is the time at the beginning of reheating. Then, the asymptotic behavior

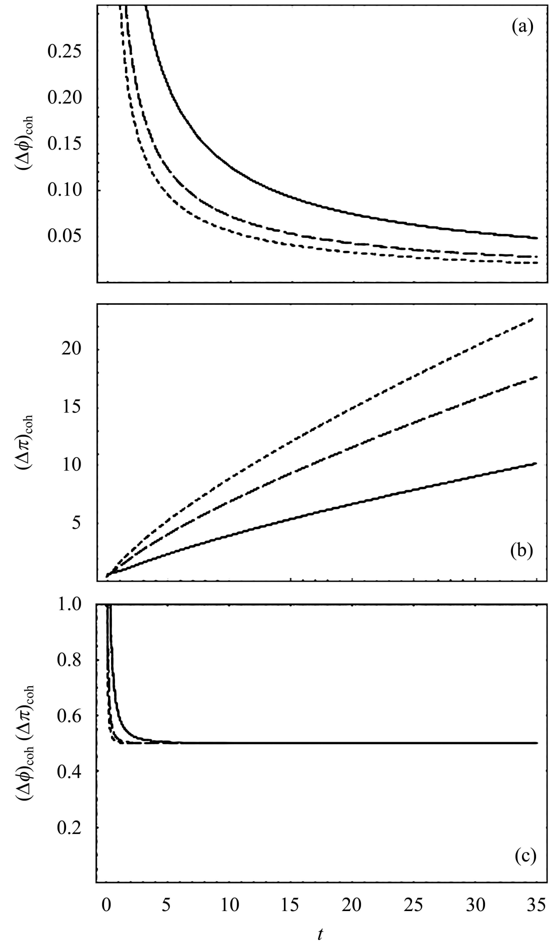

Fig. 2. Fluctuations, $(\Delta\phi)_{\rm coh}$ (a) and $(\Delta\pi)_{\rm coh}$ (b), and uncertainty relations, $(\Delta\phi)_{\rm coh}\times(\Delta\pi)_{\rm coh}$ (c), for various values of m as a function of t. The value of m is 1 for the solid line, 3 for the long dashed line, and 5 for the short dashed line. We used $\xi=0$, $\beta=3/2$, $b=1$, $\mathcal{V}=1$, and $\varepsilon_0=1$. Like the previous figure, all parameters are taken to be dimensionless.

of Bessel functions in Eqs. (32) and (33) becomes [46]

$$J_\nu(x) \simeq \sqrt{\frac{2}{\pi x}}\cos\left(x-\nu\frac{\pi}{2}-\frac{\pi}{4}\right), \tag{37}$$

$$N_\nu(x) \simeq \sqrt{\frac{2}{\pi x}}\sin\left(x-\nu\frac{\pi}{2}-\frac{\pi}{4}\right). \tag{38}$$

In particular, for the case of $\beta=2$ and $\xi=0$ which corresponds to the matter-dominated era in CMCSF, the approximation signs in the above two equations are converted to equal signs. From now on, these approximations for $J_\nu(mt)$ and $N_\nu(mt)$ that appeared in Eqs. (32) and (33) will be used for all of the relevant evaluations. Then, the amplitude and phase given in Eqs. (23) and (24) reduce to $\alpha_0=\varepsilon_0[\mathcal{V}(c_1^2+c_2^2)/(\pi b^\beta m^{\beta-1})]^{1/2}$ and $\sigma(t)=$

$-m(t-b)+\sigma(b)$. And Eq. (22) becomes $\alpha(t) = \alpha_0 \exp[-\mathrm{i}(mt+\delta)]$, where δ is a readjusted phase: $\delta = -mb-\sigma(b)$.

By using these, the coherent state wave function given in Eq. (16) yields

$$\langle\phi|\psi\rangle = \left[\frac{m\mathcal{V}}{\pi}\left(\frac{t}{b}\right)^{\beta}\right]^{1/4} \exp\left\{\alpha_0 \mathrm{e}^{-\mathrm{i}(mt+\delta)} \times\left(\sqrt{2m\mathcal{V}\left(\frac{t}{b}\right)^{\beta}}\phi-\alpha_0\cos(mt+\delta)\right) -\frac{\mathcal{V}}{4t}\left(\frac{t}{b}\right)^{\beta}(2mt+\mathrm{i}\beta)\phi^2-\frac{\mathrm{i}}{2}[m(t-b) -2\vartheta(b)]\right\}. \quad (39)$$

The wave packet associated with this wave function oscillates back and forth about $\phi=0$ like a classical one. For $\beta=0$ and $\mathcal{V}=1$, the above equation exactly recovers to that of the well known simple harmonic oscillator [47].

By the way, the fluctuations and uncertainty relations, Eqs. (25)–(27), reduce to

$$(\Delta\phi)_{\rm coh} = \left[\frac{1}{2\mathcal{V}m}\left(\frac{b}{t}\right)^{\beta}\right]^{1/2}, \quad (40)$$

$$(\Delta\pi)_{\rm coh} = \left\{\frac{m}{2\mathcal{V}}\left[1+\left(\frac{\beta}{2mt}\right)^2\right]\left(\frac{t}{b}\right)^{\beta}\right\}^{1/2}, \quad (41)$$

$$(\Delta\phi)_{\rm coh}(\Delta\pi)_{\rm coh} = \frac{1}{2\mathcal{V}}\left[1+\left(\frac{\beta}{2mt}\right)^2\right]^{1/2}. \quad (42)$$

Now, it is much clearer that $(\Delta\phi)_{\rm coh}$ decreases with time meanwhile $(\Delta\pi)_{\rm coh}$ increases. Therefore, a natural squeezing for the scalar field is brought out as time goes by. If we regard the condition given in Eq. (36) and the situation that the largest value of β does not exceed 2 according to Eqs. (34) and (35), the added term $[\beta/(2mt)]^2$ inside the square bracket of Eqs. (41) and (42) is quite small enough that we can ignore it. Consequently, the uncertainty product is nearly the same as the standard value of the simple harmonic oscillator.

Now let us see the evolution of the vacuum energy density. With the use of Eqs. (37) and (38), Eq. (31) reduces to

$$\mathit{\Pi}_\alpha(t) = \frac{1}{8mt^2}\{2\alpha_0^2\{(\beta^2+4\gamma^2)\cos[2(mt+\delta)] +4mt\beta\sin[2(mt+\delta)]\}+(1+2\alpha_0^2)(8m^2t^2 +\beta^2+4\gamma^2)\}. \quad (43)$$

By further consideration of Eq. (36), the leading term under the assumption that α_0 is not so large yields $\mathit{\Pi}_\alpha \simeq (1+2\alpha_0^2)m$ (constant). Thus, from the substitution of this into Eq. (30), we have

$$\rho(t) \simeq \rho(b)\left(\frac{b}{t}\right)^{\beta}, \quad (44)$$

where $\rho(b)$ is total energy density at a boundary time b, which is given by

$$\rho(b) = (1+2\alpha_0^2)\,m/(2\mathcal{V}). \quad (45)$$

The result Eq. (44) shows that the vacuum energy density decreases in proportion to $t^{-\beta}$. Therefore, we can readily confirm that

$$\rho(t) \propto 1/t^{3/2} \quad \text{radiation-dominated era}, \quad (46)$$

$$\rho(t) \propto 1/t^{2} \quad \text{matter-dominated era}. \quad (47)$$

Thus, from these equations, it should be noted that the decrease in vacuum energy density for the matter-dominated era is more rapid than that of the radiation-dominated era. This consequence thoroughly corresponds to the previous analysis carried out from the time behavior of Fig. 1 associated with the probability densities, which reveals faster convergence of the scalar field for the matter-dominated era than that for the radiation-dominated era. Moreover, these time behaviors for $\rho(t)$ actually agree with the recent result of Fock state analysis [12]. Though we have considered single scalar field cosmology in this section, the time evolution of $\rho(t)$ in multi scalar field cosmology may make no significant difference from the one obtained here since it is expected that the decrease in the vacuum energy density is actually related to the expansion term $(\beta/t)\dot{\phi}_{\mathbf{k}}$ in Eq. (5) rather than $\omega_{\mathbf{k}}^2(t)\phi_{\mathbf{k}}$ term.

From observations [11], it is well known that the order of the vacuum energy density is the same as that of the matter energy density. This may strongly imply that there is every possibility that the origins of the two energy densities are not so different and also their behaviors of time evolution are the same or at least very similar to each other.

6 Summary and conclusion

Using linear invariant theory, we have studied the inflationary universe in a coherent state with emphasis on investigating the time evolution of the vacuum energy density. The Hamiltonian associated with the classical equation of motion Eq. (5) governing the behavior of the scalar field is given by Eq. (6). Since this is evidently a time-dependent form, the evolution of the scalar field in the inflation scenario is described by a time-dependent harmonic oscillator. Indeed, linear invariant theory is one of the useful methods in analyzing quantum properties of the time-dependent Hamiltonian system. By means of the fact that the time derivative of an invariant quantity should vanish, quantum linear invariants $\hat{I}_{\mathbf{k}}$ are constructed. From Eq. (10), we can see that $\hat{I}_{\mathbf{k}}$ is expressed in terms of annihilation operator. Using the fact that the eigenstate of $\hat{a}_{\mathbf{k}}$ is a coherent state, we have established the coherent state of the system through linear invariant formulation. The configuration space representation of the coherent state including full phase factor is given by Eq. (16) with Eqs. (15) and (17). We also showed in Eq. (22) that the eigenvalue of the annihilation operator can be expressed in terms of its time-constant amplitude and time-dependent phase. Apparently, this simple formula makes it easy to describe the quantum feature of the system in the domain of the coherent state. The fluctuations and the corresponding uncertainty relation are given in Eqs. (25)–(27). If we regard that $\varepsilon_{\mathbf{k}}(t)$ decreases with time, the fluctuation $(\Delta\phi_{\mathbf{k}})_{\rm coh}$ also decreases as time goes by. On the other hand, $(\Delta\pi_{\mathbf{k}})_{\rm coh}$ increases. The uncertainty product, however, hardly varies with time except for a very little initial time interval which can be ignored. As you can see from Eq. (28), the probability density in the coherent state follows Gaussian form peaked at $\langle\phi_{\mathbf{k}}\rangle_{\rm coh}$, where the standard deviation is $(\Delta\phi_{\mathbf{k}})_{\rm coh}$. Though the shape of the Gaussian wave packet can be changed by external driving forces, the fluctuations of the Gaussian wave packet do not depend on external forces [48].

The expectation value of the vacuum energy density in the coherent state is given by Eq. (30) with Eq. (31). To analyze the time evolution of the vacuum energy density, it is necessary to make out the time behavior of $\Pi_{\mathbf{k},\alpha}(t)$. For clearer analysis of the time evolution of the vacuum energy density and accompanying quantum behaviors associated with the scalar field, we have taken in Sec. 5 the single scalar field cosmology whose mathematical manipulation seems easier to manage. Our analysis shows that the vacuum energy density decreases in proportion to $t^{-3/2}$ for the radiation-dominated era and in proportion to t^{-2} for the matter-dominated era. In particular, according to this result, the time evolution of the vacuum energy density within the matter-dominated era is in fact exactly the same as that of dust matter or radiation density [49]. In the case where the duration term of the radiation-dominated era is very short enough to be negligible, the estimation of the relic vacuum energy density agrees well with the current observational data. The nonconservative force acting on the coherent oscillations of the scalar field, which is provided by the expansion of the universe, is responsible for the decrease in vacuum energy density.

If we consider that the age of the universe is at least more than 10 billion years, the observational consequence that the order of the vacuum energy density is precisely the same as that of dust matter is remarkable. In the light of this situation, it may be reasonable to think that the two energy densities would have undergone the same or very similar types of time evolution. Recently, the possibility of interaction between dark energy and dark matter has been proposed and it has become a common issue in cosmology [50–54]. The effects of these interactions on the evolution of the universe may offer a mechanism for alleviating the coincidence problem. Though the very early epoch is dominated by the vacuum energy density, we can think that not only the vacuum energy density decreases with time according to the expansion of the universe but also some fraction of the remaining vacuum energy density might have been converted into matter energy density due to their possible interaction. According to our theory, the present energy density would be roughly estimated from Eq. (44) by taking a starting boundary time as $b = t_{\rm r}$ since the boundary condition at that time is well known theoretically. Based on the recent observational data [11, 55] which show that the vacuum energy density is 73% and almost all remnant is matter energy density since radiation density is negligible, we can simply represent matter and vacuum energy densities at the present epoch in terms of the total energy density obtained from such estimation as $\rho_{\rm M}(t) = 0.27\rho(t)$ and $\rho_{\Lambda}(t) = 0.73\rho(t)$, respectively, where the time behavior of $\rho(t)$ follows Eqs. (46) and (47). So long as the present predominance of vacuum energy density over matter energy density continues in the future, the universe would maintain acceleratory expansion owing to the positive acceleration, as it has until now. In other words, the speed of the expansion increases as time goes by.

In conclusion, our theory based on the linear invariant approach shows that the (vacuum) energy density of the universe in the coherent state decreases continuously with time according to the expansion of the universe. This agrees well with our previous investigation [12] performed in the Fock state using another method which was the unitary transformation approach. So far, a large class of alternative dark energy models have emerged due to the difficulty [56] in explaining the current states of dark energy relying on the previous conventional theory of vacuum energy. Most of the alternative theories suggest different candidates for the dark energy instead of the cosmological constant. On the other hand, the procedure in this work does not demand exclusion of the possibility that the identity of the dark energy is the cosmological constant, which is the early primary presumption in the history of the inflationary universe model.

Appendix A

Proof of the validity of $mt \gg 1$ given in Eq. (36)

In this appendix, we estimate the size of mt in order to demonstrate the validity of the assumption in Eq. (36). According to the GUTs, the energy scale at the beginning of reheating is 10^{14} GeV, while the time scale at that moment is $t_{\rm r} \simeq 10^{-34}$ s (for instance, see Refs. [57, 58]). A suitable choice of constant b with a consideration of the theoretically known boundary condition may enable us to describe the time evolution for the expectation value of $\hat{\rho}$. If we take the single scalar field cosmology with $\xi = 0$, Eq. (6) exactly recovers the Hamiltonian of the familiar simple harmonic oscillator at $t = b$. In this case, $\omega(t)$ becomes m. We therefore see that m can be estimated as the scale of the vacuum energy at a boundary time b. This fact can also be confirmed from Eq. (45), which just shows that m is roughly the same as the energy scale at time b.

For the radiation-dominated era, it may be favorable to choose $b = t_{\rm r}$. Then, we can take $m \simeq 10^{14}$ GeV at $t = t_{\rm r} (\simeq 10^{-10}\ \text{GeV}^{-1})$. From this, we have

$$mt \simeq (10^{14}\ \text{GeV})(10^{-10}\ \text{GeV}^{-1}) = 10^4, \tag{A1}$$

at the moment of the time when reheating begins. It is very large compared with unity. On the other hand, the time behavior of the vacuum energy density during the matter-dominated era is somewhat different from that of the radiation-dominated era. For this reason, we need to rechoose the boundary time b for the case of the matter-dominated era. For convenience, we choose $b = t_{\rm e}$ for the problem of the matter-dominated era where $t_{\rm e}$ is the matter-radiation equality time. It is possible to calculate mt at $t_{\rm e}$ by making use of Eq. (46) owing to the fact that the boundary condition at $t_{\rm r}$ is known. However, at this stage, since we would like to merely prove Eq. (36), let us use the previously known data at $t_{\rm e}$. If we refer to the figure of page 73 of Ref. [36], the matter-radiation equality time is $t_{\rm e} \simeq 10^{11}$ s and the energy scale at that time is 9×10^{-9} GeV. Thus, under these choices as a boundary condition, we have

$$\begin{aligned} mt &\simeq (9 \times 10^{-9}\ \text{GeV})(1.5 \times 10^{35}\ \text{GeV}^{-1}) \\ &= 1.4 \times 10^{27}, \end{aligned} \tag{A2}$$

at the moment of the matter-radiation equality time. This is also extremely larger than unity. Moreover, the size of mt increases as time goes by since m is constant once it is determined in each era according to the boundary condition. We therefore can readily conclude that

$$mt \gg 1, \tag{A3}$$

at least in the region $t \geqslant t_{\rm r}$.

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