Evidence for special relativity with de Sitter space-time symmetry*

YAN Mu-Lin(闫沐霖)[1)]

Interdisciplinary Center for Theoretical Study, Department of Modern Physics,
University of Science and Technology of China, Hefei 230026, China

Abstract: I show the formulation of de Sitter Special Relativity (dS-SR) based on Dirac-Lu-Zou-Guo's discussions. dS-SR quantum mechanics is formulated, and the dS-SR Dirac equation for hydrogen is suggested. The equation in the earth-QSO framework reference is solved by means of the adiabatic approach. It's found that the fine-structure "constant" α in dS-SR varies with time. By means of the t-z relation of the ΛCDM model, α's time-dependency becomes redshift z-dependent. The dS-SR's predictions of $\Delta\alpha/\alpha$ agree with data of spectra of 143 quasar absorption systems, the dS-space-time symmetry is $SO(3,2)$ (i.e., anti-dS group) and the universal parameter R (de Sitter ratio) in dS-SR is estimated to be $R \approx 2.73\times10^{12}$ ly. The effects of dS-SR become visible at the cosmic space-time scale (i.e., the distance $\geqslant 10^9$ ly). At that scale, dS-SR is more reliable than Einstein SR. The α-variation with time is evidence of SR with de Sitter symmetry.

Key words: special relativity, de Sitter spece-time symmetry, quasar, varying fine-structure constant

PACS: 03.30.+p, 03.65.Ge, 32.10.Fn **DOI:** 10.1088/1674-1137/35/3/004

1 Introduction

Einstein's Special Relativity (E-SR) is the cornerstone of physics, and any discovery beyond E-SR would be very significant. E-SR indicates the space-time metric is $\eta_{\mu\nu} = \mathrm{diag}\{+,-,-,-\}$. The most general transformation to preserve metric $\eta_{\mu\nu}$ is the Poincaré group. It is well known that the Poincaré group is the limit of the de Sitter group with sphere radius $R \to \infty$. Thus people could pursue whether there exists another type of de Sitter transformation with $R \to$finite which also leads to a Special Relativity theory (SR). In 1935, P.A.M. Dirac presented an electron wave equation in de Sitter space, and suggested the study of atomical physics in the equation based on such a kind of special relativity, i.e., the Special Relativity with de Sitter symmetry (dS-SR) [1]. Differing from General Relativity (GR), SRs rely on two principles: 1) the inertial motion law for free particle must hold; and 2) there must exist a specific space-time symmetry in the frameworks. Both E-SR and dS-SR satisfy these two principles (see below). To address the difference between GR and SR, Dirac pointed out [1] that de Sitter space-time is associated "with no local gravitational fields" (just like the case in Minkowski space).

In this paper, I will study dS-SR, and solve the dS-SR Dirac equation of hydrogen atom by means of adiabatic approximation, and show that the time-variation of the fine-structure constant reported by [2–5] is evidence of dS-SR, and hence an effect beyond E-SR. In other words, the true SR for the real world is dS-SR with $SO(3,2)$ dS-space-time symmetry (or anti-dS group) and dS sphere radius $R \approx 2.73\times10^{12}$ly instead of E-SR.

Spectroscopic observations of gas clouds seen in absorption against a background quasi-stellar object (QSO) (see Fig. 1) have been used to search for time variation of $\alpha \equiv e^2/(\hbar c)$. Comparing the observations with the corresponding atomic spectra measured in the laboratory, the results clearly show the first experimental evidence of the fundamental physics constant variations [2–5]. Even though there are some debates on the results [2], this discovery is very significant, and has greatly stimulated the various theoretical discussions during the last decade (e.g., see

Received 12 June 2010, Revised 17 August 2010

* Supported by NSF (10975128), 973 Program of China (2007CB815401)

1) E-mail: mlyan@ustc.edu.cn

Refs. [6, 7]). The BSBM model [8–10] is one of them. This theory models the variation in α by means of a scalar field that obeys a Euler-Lagrangian equation derived from an action. Combining the scalar field theory with General Relativity (GR) and adjusting the model's parameters, one can get suitable results describing the α-variation and evolutions along with z (redshift). However, the price paid for the successes of BSBM is that an unknown matter field (i.e., scalar field) has to be introduced. Some authors called the force propagated by the quanta of such an unknown field as the "fifth force" [11], which breaks the electric charger conservation law [12], and violates the weak equivalence principle [9]. There is not yet any experimental evidence to show the existence of such material scalar field so far besides explaining the time-variation of α. In this case, therefore, searching for an alternative scenario without any unknown particle to explain the α-variation with time would be more conservative, and hence more reliable for solving the puzzle.

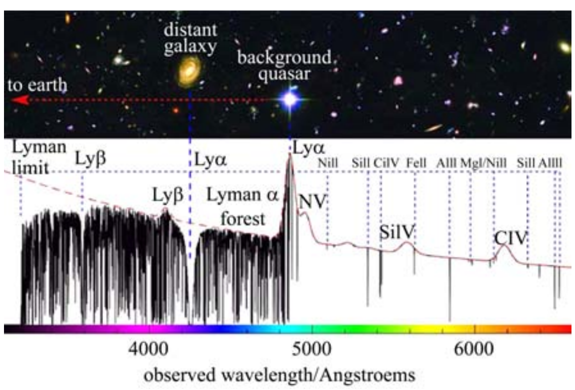

Fig. 1. A sketch map showing an example of a spectrum of gas clouds seen in absorption against a background quasi-stellar object (QSO) (download from M. T. Murphy's slide file (2009)).

Moreover, the absorption spectra observations resulting in declaration of α-variation with time reported by [2–5] rely on the measurements of the spectrum's fine-structures of atoms and ions at gas clouds near QSO. So, if possible, Quantum Mechanics (QM) calculations of atomic spectra for atoms in the distance in some suitable model would be a direct answer to the puzzle. For example, the dS-SR atomic physics scheme suggested by Dirac [1] should be considered seriously. As is well known, the spectra fine-structures in atomic physics represent E-SR corrections to levels, which are in principle derived from the E-SR Dirac equation in QM. In particular, the E-SR Dirac equation of hydrogen in QM has an exact solution, and the calculations of such corrections are sound. These corrections are space-time independent, and hence α is a constant due to the space-time translation invariant symmetry of E-SR. Thus, it should be very interesting to pursue what the dS-SR corrections to the levels of atoms in distance are in QM by means of solving the dS-SR Dirac equation of hydrogen. Because the time translations of dS-SR are significantly different from those of E-SR, one could expect that dS-SR QM may yield time-dependent α, and lead to solving the puzzle. In the following, I pursue this topic.

2 Solutions of hydrogen's dS-SR Dirac equation

In order to precisely formulate the dS-SR space-time theory and dynamics, in 1970–1974, LU, ZOU and GUO[1)] [13] (for the English version, see Refs. [14, 15]) proved two theorems, as follows.

Lemma Ⅰ: Inertial motion law for free particles holds to be true in the de Sitter space characterized by Beltrami metric

$$B_{\mu\nu}(x)=\frac{\eta_{\mu\nu}}{\sigma(x)}+\frac{\lambda}{R^2\sigma(x)^2}\eta_{\mu\lambda}\eta_{\nu\rho}x^\lambda x^\rho, \tag{1}$$

where

$$\sigma(x)\equiv 1-\frac{\lambda}{R^2}\eta_{\mu\nu}x^\mu x^\nu,$$

$R^2>0$, and $\lambda=1$ or -1, which corresponds to dS symmetries $SO(4,1)$ or $SO(3,2)$, respectively. And the constant R is the radius of the pseudo-sphere in dS-space. This means that in dS space characterized by $B_{\mu\nu}$, the velocity of a free particle is constant, i.e.,

$$\dot{\boldsymbol{x}}=\overrightarrow{v}=\text{cnstant},\qquad \text{for free particle} \tag{2}$$

which is exactly the counterpart of E-SR's inertial law in Minkowski space characterized by $\eta_{\mu\nu}$ (see Refs. [14, 15] for the English version of proof to Eq. (2)).

Lemma Ⅱ: The de Sitter space-time transformation preserving $B_{\mu\nu}(x)$ is

$$\begin{aligned}
x^\mu \longrightarrow \tilde{x}^\mu &= \pm\sigma(a)^{1/2}\sigma(a,x)^{-1}(x^\nu-a^\nu)D^\mu_\nu,\\
D^\mu_\nu &= L^\mu_\nu+\lambda R^{-2}\eta_{\nu\rho}a^\rho a^\lambda(\sigma(a)+\sigma^{1/2}(a))^{-1}L^\mu_\lambda,\\
L: &= (L^\mu_\nu)\in SO(1,3),\\
\sigma(x) &= 1-\frac{\lambda}{R^2}\eta_{\mu\nu}x^\mu x^\nu,\\
\sigma(a,x) &= 1-\frac{\lambda}{R^2}\eta_{\mu\nu}a^\mu x^\nu,
\end{aligned} \tag{3}$$

1) LOOK K H (LU Qi-Keng), Why the Minkowski metric must be used ?, (1970), unpublished.

where x^μ is the coordinate in an initial Beltrami frame, and $\tilde{x}^\mu$ is in another Beltrami frame whose origin is a^μ in the original one. There are 10 parameters in the transformations between them. Under transformation (3), we have the equation preserving $B_{\mu\nu}$ as follows,

$$B_{\mu\nu}(x) \longrightarrow \widetilde{B}_{\mu\nu}(\widetilde{x}) = \frac{\partial x^\lambda}{\partial \widetilde{x}^\mu}\frac{\partial x^\rho}{\partial \widetilde{x}^\nu} B_{\lambda\rho}(x) = B_{\mu\nu}(\widetilde{x}) \quad (4)$$

(see Appendix of Ref. [15] for the English version of proof to Eq. (4)). Eq. (4) will yield conservation laws for the energy, momenta, angular momenta and boost chargers of particles in dS-SR mechanics [15].

Based on those two lemmas, Yan, Xiao, Huang and Li formulated the Lagrangian-Hamiltonian formulism for dS-SR dynamics with two universal constants c and R, and the dS-SR Dirac equation has been proved to be [15–18]

$$\left(\mathrm{i}e^\mu_{\mathrm{a}}\gamma^{\mathrm{a}} D_\mu - \frac{m_0 c}{\hbar}\right)\psi = 0, \quad (5)$$

where e^μ_{a} is the tetrad satisfying $e^\mu_{\mathrm{a}} e^\nu_{\mathrm{b}}\eta^{\mathrm{ab}} = B^{\mu\nu}$, and

$$D_\mu = \partial_\mu - \frac{\mathrm{i}}{4}\omega^{\mathrm{ab}}_\mu \sigma_{\mathrm{ab}}$$

is the covariant derivative with Lorentz spin connection ω^{ab}_μ derived from $B_{\mu\nu}$ of Eq. (1). Furthermore, by gauge principle, $D_\mu \to \mathcal{D}_\mu = D_\mu - \mathrm{i}e/(c\hbar)A_\mu$) with $A_\mu = B_{\mu\nu}A^\nu$, $A^\nu = (\phi, \boldsymbol{A} = 0)$ and

$$-B^{ij}\partial_i\partial_j\phi = \frac{-4\pi e}{\sqrt{-\det(B_{ij})}}\delta^{(3)}(\boldsymbol{x})$$

, where ϕ is the proton's electric Coulomb potential, one has the dS-SR Dirac equation for the electron in hydrogen atom as follows

$$(\mathrm{i}e^\mu_{\mathrm{a}}\gamma^{\mathrm{a}}\mathcal{D}^{\mathrm{L}}_\mu - \frac{\mu c}{\hbar})\psi = 0, \quad (6)$$

where

$$\mu = m_{\mathrm{e}} \Big/ \left(1 + \frac{m_{\mathrm{e}}}{m_{\mathrm{p}}}\right)$$

is the reduced mass of electron. In this formulism, the conserved measurable 4-momentum operator is [15]

$$p^\mu = \mathrm{i}\hbar\left[\left(\eta^{\mu\nu} - \frac{\lambda x^\mu x^\nu}{R^2}\right)\partial_\nu + \frac{5\lambda x^\mu}{2R^2}\right]. \quad (7)$$

The observation results reported by Refs. [2–5] are the absorption spectra of gas clouds against background QSO. We briefly call the gas-QSO system QSO for simplicity. We are interested in the atoms, typically the hydrogen atom, at QSO that locates on the light-cone in de Sitter space with Beltrami metrics because only this kind of QSO can be observed by earth-observers. As illustrated in Fig. 2, the earth locates at the origin of the frame, the proton (nucleus of hydrogen atom) locates at $Q = \{Q^0 \equiv ct,\ Q^1 = ct,\ Q^2 = 0,\ Q^3 = 0\}$, which is on QSO-light-cone $B_{\mu\nu}(Q)\ Q^\mu Q^\nu = \eta_{\mu\nu}Q^\mu Q^\nu = 0$. The metric of the space-time near Q is $B_{\mu\nu}(Q) = \eta_{\mu\nu} + \frac{\lambda}{R^2}\eta_{\mu\lambda}Q^\lambda\eta_{\nu\rho}Q^\rho$, and hence $B_{ij}(Q) = \eta_{ij} + \frac{\lambda c^2t^2}{R^2}\delta_{i1}\delta_{j1}$. Electron-coordinates are $L = \{L^0 \equiv ct_{\mathrm{L}},\ L^1,\ L^2,\ L^3\}$, and the relative space coordinates between the proton and electron are $x^i = L^i - Q^i$. The magnitude of $r \equiv \sqrt{-\eta_{ij}x^ix^j} \sim a$ (where $a \approx 0.5\times10^{-10}$ m is Bohr radius), and $|x^i| \sim a$. Another scale is the Compton wave length of electron $a_{\mathrm{c}} = \hbar/(m_{\mathrm{e}}c) \approx 0.3\times10^{-12}$ m. Noting R is cosmologically large and $R \gg ct$, so the calculations for our purpose will be accurate up to $\mathcal{O}(c^2t^2/R^2)$. The terms proportional to $\mathcal{O}(c^4t^4/R^4)$, $\mathcal{O}(cta_{\mathrm{c}}/R^2)$, $\mathcal{O}(cta/R^2)$, etc. will be ignored. Note also that Eq. (7) indicates that the energy eigenstate equation is

$$E\psi = \mathrm{i}\hbar\left[\partial_t - \frac{\lambda c^2t^2}{R^2}\partial_t + \frac{5\lambda ct}{2R^2}\right]\psi \approx \mathrm{i}\hbar\left(1 - \frac{\lambda c^2t^2}{R^2}\right)\partial_t\psi.$$

Then Eq. (6) becomes

$$E\psi = \left[-\mathrm{i}\hbar c\left(1 - \frac{\lambda c^2t^2}{2R^2}\right)\vec{\alpha}\cdot\nabla_{\mathrm{B}} + \left(1 - \frac{\lambda c^2t^2}{2R^2}\right)\mu c^2\beta - \frac{e^2}{r_{\mathrm{B}}}\right]\psi, \quad (8)$$

where $r_{\mathrm{B}} = \sqrt{(\tilde{x}^1)^2 + (x^2)^2 + (x^3)^2}$ with $\tilde{x}^1 = (1 - \lambda c^2t^2/(2R^2))x^1$ and

$$\nabla_{\mathrm{B}} = \boldsymbol{i}\frac{\partial}{\partial\tilde{x}^1} + \boldsymbol{j}\frac{\partial}{\partial x^2} + \boldsymbol{k}\frac{\partial}{\partial x^3}.$$

Eq. (8) is a time-dependent quantum Hamiltonian equation. It is somewhat difficult to deal with the time-dependent problems in quantum mechanics. Fortunately, comparing (8) with the usual E-SR Dirac equation for hydrogen, all correction terms due to dS-SR are proportional to (c^2t^2/R^2). since $R \gg ct$, those factors make the time-evolution of the system so slow that the adiabatic approximation [19] will legitimately work (see Chapter XVII of Vol II of Ref. [20], and Appendix B in Ref. [18]). Thus, rewriting (8) as

$$E\psi = \left[-\mathrm{i}\hbar_{\mathrm{t}}c\vec{\alpha}\cdot\nabla_{\mathrm{B}} + \mu_{\mathrm{t}}c^2\beta - \frac{{e_{\mathrm{t}}}^2}{r_{\mathrm{B}}}\right]\psi$$

with

$$\hbar_{\mathrm{t}} = \left(1 - \frac{\lambda c^2t^2}{2R^2}\right)\hbar,\ \ \mu_{\mathrm{t}} = \left(1 - \frac{\lambda c^2t^2}{2R^2}\right)\mu,\ \ e_{\mathrm{t}} = e,$$

we obtain the predictions

$$\alpha_{\mathrm{t}} \equiv \frac{e_{\mathrm{t}}^2}{\hbar_{\mathrm{t}}c} = \left(1 + \frac{\lambda c^2t^2}{2R^2}\right)\alpha, \quad \text{or} \quad \frac{\Delta\alpha}{\alpha} = \frac{\lambda c^2t^2}{2R^2}, \quad (9)$$

$$\omega_{\rm t} = E/\hbar_{\rm t} = \frac{\mu_{\rm t}}{\hbar_{\rm t}}c^2\left[1+\frac{\alpha_{\rm t}^2}{(\sqrt{K^2-\alpha_{\rm t}^2}+n_{\rm r})^2}\right]^{-1/2}$$

$$= \frac{\mu}{\hbar}c^2\left[1+\frac{\alpha_{\rm t}^2}{(\sqrt{K^2-\alpha_{\rm t}^2}+n_{\rm r})^2}\right]^{-1/2}. \qquad (10)$$

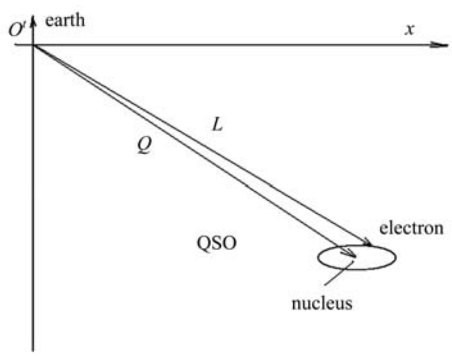

Fig. 2. Sketch of the earth-QSO reference frame. The earth locates at the origin. The position vector for the nucleus of the atom on QSO is Q, and for the electron is L. The distance between the nucleus and electron is r.

3 Comparison between theory predictions and observation data

Murphy and collaborators [3] studied the spectra of 143 quasar absorption systems over the redshift range $0.2 < z_{\rm abs} < 4.2$. Their most robust estimate is a weighted mean,

$$\frac{\Delta\alpha}{\alpha} = (-0.57\pm0.11)\times10^{-5}. \qquad (11)$$

Compared with the prediction (9), we conclude that

$$\lambda = -1. \qquad (12)$$

This means that the space-time symmetry for dS-SR is de Sitter-$SO(3,2)$ instead of anti-de Sitter-$SO(4,1)$. Substituting Eq. (12) into (9), we predict as follows,

$$\frac{\Delta\alpha}{\alpha} = -\frac{c^2t^2}{2R^2}. \qquad (13)$$

The 134 data points are assigned three epochs in Ref. [21] (see Table 1), and the redshift z-dependence of $\Delta\alpha/\alpha$ is shown roughly in Ref. [21]. In the following, I further test the prediction of (9) in terms of these z-dependent data of $\Delta\alpha/\alpha$. In order to transfer the t-dependence of $\Delta\alpha/\alpha$ in (9) to a z-dependence prediction, a relation of t-z is needed. For this aim, an appropriate cosmological model is necessary. In this paper, we treat t as comoving time t in the ΛCDM model [22, 23]. In the model, the t-z relation is as follows,

$$t = \int_0^z \frac{{\rm d}z'}{H(z')(1+z')}, \qquad (14)$$

where

$$H(z') = H_0\sqrt{\Omega_{\rm m0}(1+z')^3+1-\Omega_{\rm m0}},$$

$$H_0 = 100\ {\rm h} \approx 100\times0.705\ {\rm km\cdot s^{-1}}/M{\rm pc},$$

$$\Omega_{\rm m0} \approx 0.274.$$

The t-z relation is shown in Fig. 3(a). Substituting this relation into (9), we obtain a desirable z-dependence prediction of $\frac{\Delta\alpha}{\alpha}(z)$, where R is a free parameter. By using the observation data $\frac{\Delta\alpha}{\alpha}(z=1.47)=-0.58\times10^{-5}$, we get $R\approx2.73\times10^{12}$ ly (which is consistent with the estimation in Ref. [24]). Then the theory predictions are $\frac{\Delta\alpha}{\alpha}(z=0.65)=-0.24\times10^{-5}$ and $\frac{\Delta\alpha}{\alpha}(z=2.84)=-0.87\times10^{-5}$, which are in agreement with the corresponding data in Refs. [3] and [21]. The results are listed in Table 1, and the curve of $\frac{\Delta\alpha}{\alpha}(z)$ is shown in Fig. 3(b). The comparison indicates that the dS-SR theory predictions of (9) agree with the observation data within the error band.

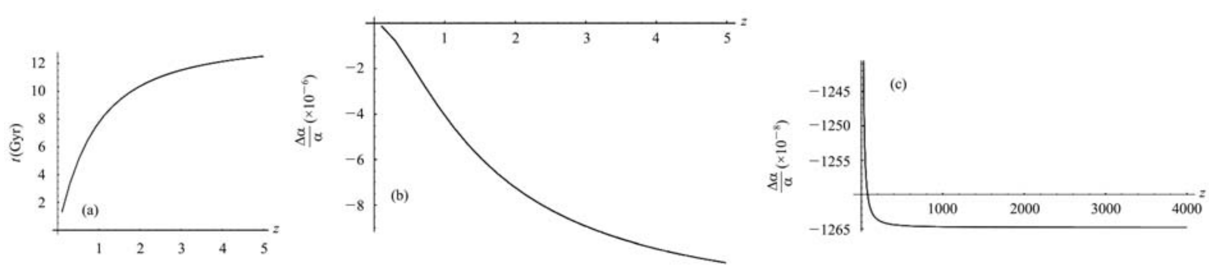

Fig. 3. (a) The t-z relation in the ΛCDM model; (b) the $\Delta\alpha/\alpha$ as function of the red shift z; (c) the evolution of α-variations $\frac{\Delta\alpha}{\alpha}(z)$ along with z. By Eq. (13) with $R\approx2.73\times10^{12}$ ly and the ΛCDM model's t-z relation, a $\frac{\Delta\alpha}{\alpha}(z)$ curve is plotted in the region of $(0\leqslant z\leqslant4000)$.

Table 1. Time variations of $\Delta\alpha/\alpha$: the first two columns are quoted from Ref. [21]. Eq. (13) with $R\approx 2.73\times10^{12}$ ly, and the ΛCDM model's t-z relation (14) are used.

redshift $\langle z\rangle$ and (t)	$(\Delta\alpha/\alpha)_{\rm expt}$	results of (13)
0.65 (6.04 Gyr)	$(-0.29\pm0.31)\times10^{-5}$	-0.24×10^{-5}
1.47 (9.29 Gyr)	$(-0.58\pm0.13)\times10^{-5}$	-0.58×10^{-5}
2.84 (11.39 Gyr)	$(-0.87\pm0.37)\times10^{-5}$	-0.87×10^{-5}

Next, we turn to discuss the evolution of α-variations $\frac{\Delta\alpha}{\alpha}(z)$ along with z, and plot a $\frac{\Delta\alpha}{\alpha}(z)$ curve in the region of $(0\leqslant z\leqslant 4000)$ in Fig. 3(c). We can see that $z<10$, $\frac{\Delta\alpha}{\alpha}(z)$ changes relatively sharply, and then the changes become slow. When $z\geqslant 10^3$, $\frac{\Delta\alpha}{\alpha}(z)$ is almost independent of z, i.e., α-variation ceases in that very high z region. Fig. 3(c) shows that the lower bound of $\frac{\Delta\alpha}{\alpha}(z)$ is about $\sim -1.3\times10^{-5}$. This result coincides with other considerations (e.g., BSBM model) [9], which suggests a negligible change in α in the radiation epoch of the universe, that epoch roughly corresponds to $z\geqslant 3\times10^3$.

4 Conclusion

In summary, in this paper, I have shown the formulation of de Sitter Special Relativity (dS-SR) based on Dirac-Lu-Zou-Guo's discussions, formulated the dS-SR quantum mechanics, and then determined the dS-SR Dirac equation for hydrogen. In order to discuss the spectra of atoms on (or near) QSO, I solved it in the earth-QSO framework reference by means of the adiabatic approach. Aspects of de Sitter spacetime geometry described by the Beltrami metric are taken into account. The dS-SR Dirac equation of hydrogen turns out to be a time dependent quantum Hamiltonian system. Since the radius of de Sitter sphere R is cosmologically large, it makes the time-evolution of the system so slow that the adiabatic approximation legitimately works with high accuracy. Consequently, it is revealed that all those facts yield important conclusions that the electromagnetic fine-structure "constant" α varies with time. By means of the t-z relation of the ΛCDM model, the α's time-dependent becomes redshift z-dependent. The dS-SR's predictions of $\Delta\alpha/\alpha$ are in agreement with the data, the dS-space-time symmetry is $SO(3,2)$ (i.e., anti-dS group) and the universal parameter R (the de Sitter ratio) in the theory is estimated to be $R\approx 2.73\times10^{12}$ ly. This fact indicates that the effects of dS-SR become visible at the cosmic spacetime scale (i.e., the distance $\geqslant 10^9$ ly). At that scale, de Sitter Special Relativity is more reliable than Einsteinian Special Relativity, and the latter is the former's approximation for the distance, which is much less than R, or much less than $\sim 10^9$ ly. I conclude that the α-variation with time is evidence of SR with de Sitter symmetry.

References

1 Dirac P A M. Annals of Mathmatics, 1935, **36**(3): 657–669
2 Murphy M T, Webb J K, Flambaum V V. Phys. Rev. Lett., 2007, **99**: 239001-239001 arXiv:0708.3677 [astro-ph]; astro-ph/0612407; astro-ph/0911.4512
3 Murphy M T, Flambaum V V, Webb J K, Dzuba V V, Prochaska J X, Wolfe A M. Lec. Notes in Phys., 2004, **648**: 131
4 Webb J K, Murphy M T, Flambaum V V, Dzuba V A, Barrow J D, Churchill C W, Pochaska J X, Wolfe A M. Phys. Rev. Lett., 2001, **87**: 091301–091304
5 Webb J K, Flambaum V V, Churchill C W, Drinkwater M J, Barrow J D. Phys. Rev. Lett., 1999, **82**: 884–888
6 Uzan J P. Rev. of Mod. Phys., 2003, **75**: 403–455
7 Flambaum V V. Int. J. Mod. Phys. A, 2007, **22**: 4937–4950
8 Bekenstein J D. Phys. Rev. D, 1982, **25**: 1527–1539
9 Sandvik H B, Barrow J D, Magueijo J. Phys. Rev. Lett., 2002, **88**: 031302-1–031302-4
10 Barrow J D, Phys Rev. D, 2005, **71**: 083520-1–083520-7; Barrow J D, Sandvik H B, Magueijo J. Phys. Rev. D, 2002, **65**: 063504-1–063504-9; D, 2002, **66**: 043515-1–043515-6
11 Dvali G, Zaldarriaga M. Phys Rev Lett., 2002, **88**: 091303-1–091303-4
12 Landau S, Sisterna P, Vucetich H. Phys. Rev. D, 2001, **63**: 081303(R)-1–081303-4
13 LOOK K H (LU Qi-Keng), TSOU C L (ZOU Zhen-Long), KUO H Y (GUO Han-Ying). Acta Physica Sinica, 1974, **23**: 225–237 (in Chinese)
14 GUO Han-Ying, HUANG Chao-Guang, XU Zhan, ZHOU Bin. Phys. Lett. A, 2004, **331**: 1–7, hep-th/0403171
15 YAN Mu-Lin, XIAO Neng-Chao, HUANG Wei, LI Si. Commun. Theor. Phys., 2007, **48**: 27–36, hep-th/0512319.
16 Utiyama R. Phys. Rev., 1956, **101**: 1597–1607
17 NIEH H T, YAN Mu-Lin. Ann. Phys., 1982, **138**: 237–259
18 YAN Mu-Lin. Int. Mod. Phys A (to appear), "Hydrogen Atom in de Sitter Special Relativity and Time Variation of Fine-Structure Constant", arXiv: 1004.3023 [physics.gen-ph]
19 Born M, Fock V. Z. Phys., 1928, **51**: 165–175
20 Messiah A. Quantum Mechanics Ⅰ, Ⅱ. North-Holland Publishing Company, 1970
21 Dent T, Stern T, Wetterich C. Phys. Rev. D, 2008, **78**: 103518-1–103508-17
22 Weinberg S. Rev. Mod. Phys., 1989, **61**: 1–23; Padmanabhan T. Phys. Rep., 2003, **380**: 235–320
23 Komatsu E et al. Astrophys. J. Suppl., 2009, **180**: 330–376
24 CHEN Shao-Xia, XIAO Neng-Chao, YAN Mu-Lin. Chinese Physics C, 2008, **32**: 612–616, astro-ph/0703110