New quantization conditions for field theory without divergence

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Abstract: Quantum field theory is a fundamental tool in particle and nuclear physics. Elemental particles are assumed to be point particles and, as a result, the loop integrals are divergent in many cases. Regularization and renormalization are introduced in order to get the physical finite results from the infinite, divergent loop integrations. We propose new quantization conditions for the fields whose base is very natural, i.e., any particle is not a point particle but a solid one with three dimensions. With this solid quantization, divergence could disappear.

Key words: quantization conditions, elemental particle, divergence

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1 Introduction

Quantum field theory is the fundamental theory for nuclear and particle physics. The first method developed for quantization of field theories was canonical quantization. Canonical quantization of a field theory is analogous to the construction of quantum mechanics from classical mechanics. The classical field is treated as a dynamical variable called the canonical coordinate, and its time-derivative is the canonical momentum. One introduces a commutation relation between these quantities which is exactly the same as the commutation relation between a particle's position and momentum in quantum mechanics. The procedure is also called second quantization.

With the quantized field theory, one can study the micro process with Feynman rules. When high order terms are included, the loop contribution will appear. These integrals are often divergent, i.e., they become infinite when momentum integration goes to infinity. This ultraviolet divergence is a short-distance phenomenon. We will see that the divergence is caused by the assumption that the particles are point ones.

Many kinds of methods are introduced in quantum field theory to deal with the divergence. One of the most popular methods is the dimensional regularization, invented by Gerardus 't Hooft and Martinus J. G. Veltman, which tames the integrals by carrying them into a space with a fictitious fractional number of dimensions [1]. Another is Pauli-Villars regularization, which adds fictitious particles to the theory with large masses, so that loop integrations involving the massive particles cancel out the existing loops at large momentum [2].

The above quantum field theory is very standard and widely accepted. However, the assumption that the elemental particles are point ones is not justified and could only be an approximation of real particles. It is difficult to imagine that an existing physical particle is like a mathematical point one. Is there anything in the world which exists as a dimensionless point? One may agree that the elemental particles are not point ones, but think it is a good approximation to treat them as point-like, since we can treat a physical object as a point particle even in Newton dynamics. In Newton dynamics, mass-point approximation simplifies the calculation and the volume effect can be added in the high order correction. However, for quantum field theory, the point-like treatment will cause not only quantitative but also qualitative differences, which is different from that in Newton dynamics. It causes infinity in field theory.

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2 Solid quantization

Let's start with the traditional canonical quantization for the simplest scalar field. The traditional commutation relations are

$$[\phi(\vec{x},t),\phi(\vec{y},t)] = [\pi(\vec{x},t),\pi(\vec{y},t)]=0,$$
$$[\phi(\vec{x},t),\pi(\vec{y},t)] = \mathrm{i}\delta^{(3)}(\vec{x}-\vec{y}). \quad (1)$$

The δ function in the above equation means that a point particle and anti-particle can only be created at the same position point. This is natural since the particles are assumed to be point ones. The δ function guarantees the "causality".

The field and its conjugate partner can be expanded in momentum space, expressed as

$$\phi(\vec{x},t)=\int\widetilde{\mathrm{d}p}\left[a(\vec{p})\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}-\mathrm{i}\omega_{\mathrm{p}}t}+a^{\dagger}(\vec{p})\mathrm{e}^{-\mathrm{i}\vec{p}\cdot\vec{x}+\mathrm{i}\omega_{\mathrm{p}}t}\right], \quad (2)$$

$$\pi(\vec{x},t)=\int\widetilde{\mathrm{d}p}(-\mathrm{i})\omega_{\mathrm{p}}\left[a(\vec{p})\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}-\mathrm{i}\omega_{\mathrm{p}}t}-a^{\dagger}(\vec{p})\mathrm{e}^{-\mathrm{i}\vec{p}\cdot\vec{x}+\mathrm{i}\omega_{\mathrm{p}}t}\right], \quad (3)$$

where

$$\widetilde{\mathrm{d}p}=\frac{\mathrm{d}^3p}{(2\pi)^3 2\omega_{\mathrm{p}}}. \quad (4)$$

It is straightforward to obtain the commutation relations between creation and annihilation operators,

$$[a(\vec{p}),a(\vec{q})] = \left[a^{\dagger}(\vec{p}),a^{\dagger}(\vec{q})\right]=0,$$
$$\left[a(\vec{p}),a^{\dagger}(\vec{q})\right] = (2\pi)^3 2\omega_{\mathrm{p}}\delta^{(3)}(\vec{p}-\vec{q}). \quad (5)$$

The creation operator creates a momentum state $|p\rangle=a^{\dagger}(\vec{p})|0\rangle$, which is normalized as

$$\int\widetilde{\mathrm{d}p}|p\rangle\langle p|=1. \quad (6)$$

Because the particle is assumed to be a point particle (behaves like δ function in position space), when expanded in momentum space, it has the same possibility for different momenta. However, the real particle could be like a wavepacket. It is partially localized in both position and momentum space. The possibility of the particle with high momentum is small. With high-momentum suppression, the divergence in the loop integral may not appear.

Therefore, we propose new quantization conditions (solid quantization),

$$[\phi(\vec{x},t),\phi(\vec{y},t)] = [\pi(\vec{x},t),\pi(\vec{y},t)]=0,$$
$$[\phi(\vec{x},t),\pi(\vec{y},t)] = \mathrm{i}\Phi(\vec{x}-\vec{y}). \quad (7)$$

The function $\Phi(\vec{x}-\vec{y})$ describes the correlation between $\vec{x}$ and $\vec{y}$. Due to the fact that the particle is not a dimensionless point particle, but a solid one, particles at different positions could be partially superimposed, which means that there exists some possibility that particle and antiparticle are created in different positions. For point particles, it is impossible. If a point particle is created at position A, an antiparticle should be created at the same time at position B due to the conservation laws, such as the conservation of baryon number, energy, etc. Therefore, the information transfers from A to B with infinite speed. This is why the commutation function for point particles has to be a delta function. For solid particles, the situation is different. Though the particles are created in different positions, their wave- functions are partially superimposed. There is no violation of "causality". The solid particle system satisfies the conservation laws, which means that the total energy, baryon number, etc are conserved within a finite volume.

One can also expand the field as Eq. (2) (in this case, we use capital letter A instead of a),

$$\phi(\vec{x},t)=\int\widetilde{\mathrm{d}p}\left[A(\vec{p})\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}-\mathrm{i}\omega_{\mathrm{p}}t}+A^{\dagger}(\vec{p})\mathrm{e}^{-\mathrm{i}\vec{p}\cdot\vec{x}+\mathrm{i}\omega_{\mathrm{p}}t}\right]. \quad (8)$$

As a result, the creation and annihilation operators satisfy the following relations,

$$[A(\vec{p}),A(\vec{q})] = \left[A^{\dagger}(\vec{p}),A^{\dagger}(\vec{q})\right]=0, \quad (9)$$

$$\left[A(\vec{p}),A^{\dagger}(\vec{q})\right] = (2\pi)^3\delta^{(3)}(\vec{p}-\vec{q})\Psi(\vec{p}). \quad (10)$$

$\Phi(\vec{x})$ and $\Psi(\vec{p})$ obey the following relations,

$$\Phi(\vec{x}) = \int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{\Psi(\vec{p})}{2}(\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}}+\mathrm{e}^{-\mathrm{i}\vec{p}\cdot\vec{x}}), \quad (11)$$

$$\Psi(\vec{p}) = \int\mathrm{d}^3x\frac{\Phi(\vec{x})}{2}(\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}}+\mathrm{e}^{-\mathrm{i}\vec{p}\cdot\vec{x}}). \quad (12)$$

The above two equations generate two normalization formulas,

$$\Phi(0) = \int\frac{\mathrm{d}^3p}{(2\pi)^3}\Psi(\vec{p}), \quad (13)$$

$$\Psi(0) = \int\mathrm{d}^3x\Phi(\vec{x})=1. \quad (14)$$

Compared with the traditional commutation relation where $\Phi(\vec{x})=\delta^{(3)}(\vec{x})$, $\Phi(\vec{x})$ is normalized to be 1, while $\Psi(\vec{p})$ is normalized to be $\Phi(0)$.

With the new quantization, the field can be written in terms of traditional creation and annihilation operators as

$$\phi(\vec{x},t)=\int\widetilde{\mathrm{d}p}\sqrt{\Psi(\vec{p})}\left[a(\vec{p})\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}-\mathrm{i}\omega_{\mathrm{p}}t}+a^{\dagger}(\vec{p})\mathrm{e}^{-\mathrm{i}\vec{p}\cdot\vec{x}+\mathrm{i}\omega_{\mathrm{p}}t}\right]. \quad (15)$$

It is easy to get the Feynman propagator of the scalar field in the solid quantization. The propagator is defined as

$$\Delta_{\mathrm{F}}(x'-x)=\langle 0|T\phi(x')\phi(x)|0\rangle=\int\widetilde{\mathrm{d}k}[\theta(t'-t)\,\mathrm{e}^{\mathrm{i}k\cdot(x'-x)} +\theta(t-t')\,\mathrm{e}^{-\mathrm{i}k\cdot(x'-x)}].$$

The integral expression of the step function is

$$\theta(t)=\lim_{\epsilon\to 0^+}\int\frac{\mathrm{d}\tau}{2\pi\mathrm{i}}\frac{\mathrm{e}^{\mathrm{i}\tau t}}{\tau-\mathrm{i}\epsilon}. \quad (16)$$

With the help of the above equation, the Feynman propagator can be obtained as

$$\Delta_{\mathrm{F}}(x'-x)=\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\mathrm{i}\Psi(\vec{k})\mathrm{e}^{-\mathrm{i}k\cdot(x'-x)}}{k^2-m^2+\mathrm{i}\epsilon}. \quad (17)$$

For the other fields, the quantization condition is similar. For example, for spin 1/2 fermion, the nonzero anti-commutation relationship is

$$\{\psi_\alpha(\vec{x},t),\bar{\psi}_\beta(\vec{y},t)\}=\gamma^0_{\alpha\beta}\Phi(\vec{x}-\vec{y}). \quad (18)$$

Correspondingly, the field should be written as

$$\psi(\vec{x},t) = \sum_{\mathrm{s}=\pm}\int\widetilde{\mathrm{d}p}\sqrt{\Psi(\vec{p})}\left[b_{\mathrm{s}}(\vec{p})u_{\mathrm{s}}(\vec{p})\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}-\mathrm{i}\omega_{\mathrm{p}}t} +d^\dagger(\vec{p})v_{\mathrm{s}}(\vec{p})\mathrm{e}^{-\mathrm{i}\vec{p}\cdot\vec{x}+\mathrm{i}\omega_{\mathrm{p}}t}\right], \quad (19)$$

where b and $d^\dagger$ are normal annihilation and creation operators. $u_{\mathrm{s}}(\vec{p})$ and $v_{\mathrm{s}}(\vec{p})$ are Dirac spinors. The propagator of the spin 1/2 field can be obtained as

$$S_{\mathrm{F}}(x'-x)=\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\mathrm{i}\Psi(\vec{k})(k\cdot\gamma+m)\mathrm{e}^{-\mathrm{i}k\cdot(x'-x)}}{k^2-m^2+\mathrm{i}\epsilon}. \quad (20)$$

The vector field, say the photon field, can also be expanded as

$$A^\mu(\vec{x},t) = \sum_{\lambda=\pm}\int\widetilde{\mathrm{d}p}\sqrt{\Psi(\vec{p})}\left[a_\lambda(\vec{p})\epsilon^\mu(\vec{p},\lambda)\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}-\mathrm{i}\omega_{\mathrm{p}}t} +a^\dagger_\lambda(\vec{p})\epsilon^\mu(\vec{p},\lambda)\mathrm{e}^{-\mathrm{i}\vec{p}\cdot\vec{x}+\mathrm{i}\omega_{\mathrm{p}}t}\right], \quad (21)$$

where $\epsilon^\mu(\vec{p},\lambda)$ is the polarization vector. The photon propagator can be written as

$$D^{\mu\nu}_{\mathrm{F}}(x'-x)=\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{-\mathrm{i}\Psi(\vec{k})g^{\mu\nu}\mathrm{e}^{-\mathrm{i}k\cdot(x'-x)}}{k^2-m^2+\mathrm{i}\epsilon}. \quad (22)$$

We should mention that, in principle, the function $\Psi(\vec{p})$ or $\Phi(\vec{x}-\vec{y})$ is particle dependent. It describes the particle's property ("shape") in addition to the mass and width. Therefore, with the new quantization condition, the Feynman rules should be changed correspondingly. The new propagator of the field should be multiplied by a factor $\Psi(\vec{k})$. The external field should be multiplied by a factor $\sqrt{\Psi(\vec{k})}$.

A question that may arise here is how to connect the new propagator with the path integral formulation. The path integral for the free point-like field is defined as

$$Z_0(J)=\int\mathcal{D}\phi\mathrm{e}^{\mathrm{i}\int\mathrm{d}^4x[\mathcal{L}_0+J\phi]}, \quad (23)$$

where

$$\mathcal{L}_0=-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi-\frac{1}{2}m^2\phi^2 \quad (24)$$

is the Lagrangian density and J is the external current. For a solid particle, the free Lagrangian density is different. From Eq. (2), the density can be written in terms of a realistic physical field with three dimensions as

$$\mathcal{L}_0=\phi\frac{(\partial^\mu\partial_\mu-m^2)}{2\Psi(\vec{\partial})}\phi. \quad (25)$$

With the above Lagrangian density, the propagator of scalar field obtained in the path integral formulation is the same as that in solid canonical quantization. For the fermion and vector fields, the situation is the same.

The factor $\Phi(\vec{x}-\vec{y})$ is the correlation of two particles at $\vec{x}$ and $\vec{y}$. If we choose $\Phi(\vec{x}-\vec{y})=\delta^{(3)}(\vec{x}-\vec{y})$, $\Psi(\vec{p})$ will equal 1. All of the above propagators will be changed back to the conventional ones. As we explained previously, the particle could be a solid particle with three space dimensions. The particle and antiparticle can be created at small distance. Therefore, the function of $\Phi(\vec{x}-\vec{y})$ can be a function that decreases with increasing distance $|\vec{x}-\vec{y}|$. The smaller the particle, the closer the function to the δ function.

With the new propagator, the loop integration is convergent. From Eq. (14), one can see that our new approach does not affect the infrared behavior. It is known that infrared divergence cancels in any given order of perturbation theory. In practical calculations, it is helpful to use a non-zero photon (gluon) mass and to set its value to zero at last. The infrared divergence disappears when the soft photons are included. For details, see, for instance, Refs. [3, 4]. Here we discuss an example of how the new quantization deals with ultraviolet divergence. Let's look at the following integration, which appears in the photon self-energy at one-loop level,

$$I=\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\Psi(\vec{k})\Psi(\vec{k}+\vec{p})}{[k^2-m^2][(p+k)^2-m^2]}, \quad (26)$$

where p is the external momentum of a photon. k and $k+p$ are the internal momentum of two electron or quark propagators. After the integration of k_0, the above equation can be written as

$$I = \int\frac{\mathrm{d}^3k}{2(2\pi)^3}\left\{\frac{-\mathrm{i}\Psi(\vec{k})\Psi(\vec{k}+\vec{p})}{\omega(\vec{k})\left[(\omega(\vec{k})+\omega(\vec{p}))^2-\omega^2(\vec{k}+\vec{p})\right]} -\frac{\mathrm{i}\Psi(\vec{k})\Psi(\vec{k}+\vec{p})}{\omega(\vec{k}+\vec{p})\left[(\omega(\vec{k}+\vec{p})-\omega(\vec{p}))^2-\omega^2(\vec{k})\right]}\right\}, \quad (27)$$

where $\omega(\vec{q})=\sqrt{\vec{q}^2+m^2}$. Without the factor $\Psi(\vec{k})$ and $\Psi(\vec{k}+\vec{p})$, the above integration is log-divergent. Since the particle is a solid one with three dimensions, its wave-function is suppressed at high momentum. If we choose $\Psi(\vec{k})$ to be a dipole or Gauss function, the integration is convergent.

We should mention that due to the inclusion of the size of the particle, the fields as well as the propagators are not Lorentz covariant quantities. One may think how to get the Lorentz covariant formalism for this new quantization method. For example, the possible propagator could look like $\Psi(k^2)\Delta_{\rm F}(k)$ in momentum space, where k^2 is the Lorentz invariant scalar. However, it is interesting to study the possible CPT [5, 6] violation due to this size effect. This CPT violation is proportional to the size of the particle and it disappears when the particle is a point one.

Without the renormalization, the so-called running coupling constant can also be understood. With the new quantization condition, even at tree level, the coupling constant will be associated with a momentum dependent factor. For example, for the fermion-boson coupling, if the initial and final momentum of the fermion are $-\vec{q}/2$ and $\vec{q}/2$, the momentum of the absorbed boson is $\vec{q}$, and the momentum dependent factor of the coupling constant at tree level is $\sqrt{\Psi_{\rm f}(-\vec{q}/2)\Psi_{\rm f}(\vec{q}/2)\Psi_{\rm g}(\vec{q})}$. The label f and g are for fermion and gauge boson, respectively. The asymptotic free is a general property not only for strong interaction. It is because the size of the particle is not a point, the momentum is partially localized which favors at low value.

Let's now discuss the potential between two Fermion fields. For simplicity, the functions of $\Phi(\vec{x}-\vec{y})$ and $\Psi(\vec{p})$ can be written as $\Phi(|\vec{x}-\vec{y}|^2)$ and $\Psi(\vec{p}^2)$. In momentum space, one gauge field exchange will give the potential

$$V(Q^2)\sim\frac{\Psi_{\rm f}^2(Q^2/4)\Psi_{\rm g}(Q^2)}{Q^2}, \tag{28}$$

where Q^2 is the momentum transfer. Since $\Psi(Q^2)$ decreases with increasing Q^2, the above potential could be written as

$$V(Q^2)\sim\frac{1}{Q^2(1+aQ^2+bQ^4+\cdots)}. \tag{29}$$

When Q^2 is very small, the Q^2 term is dominant, which means that the potential behaves like $1/r$ at large r in position space. If Q^2 is very large, the higher order term of momentum in the expansion is more important. In position space, the potential behaves like r^n (n is positive) when r is small. When we separate two particles, the r dependence of the potential changes from r^n to r^{n-1}, r^{n-2}, $\cdots$, r^2, r, r^0 and eventually to $1/r$.

When the coupling constant is small and perturbation theory is valid, one boson exchange gives $1/r$ potential at large r. If the coupling constant is large, the non-perturbation behavior is dominant. It is difficult to get the potential in this case. However, we can understand (not prove) the linear confining potential. For example, when we separate a quark and an antiquark, at some distance, the string will be broken and a quark-antiquark pair will be produced. It is unknown at what distance the quark pairs will be produced. We assume that they appear before the potential behaves like r^0 or $1/r$. Therefore, there are many quark-antiquark pairs between a quark and an antiquark. The total potential between a quark and an antiquark at distance R ($R=Nr$) is $N(ar+br^2+\cdots)$, i.e. $(aR+bR^2/N+\cdots)$, since the potential between each quark pair is $(ar+br^2+\cdots)$. The linear potential survives when N is large. Color confinement is still an open question. It may be related to the non-Abelian properties of the strong interaction. In this work, we did not prove the confinement but tried to understand the linear confining potential.

This new quantization is for the elemental particles. It is very straightforward to apply this quantization to hadrons. For example, in the effective field theory or chiral perturbation theory, hadron fields are assumed to be point particles. To deal with the divergence, dimensional regularization was applied in the same way as for the elemental particles [7, 8]. Finite regularization in which a vertex regulator was introduced was also used to get rid of the divergence [9, 10]. In fact, in many model calculations, regulators or form factors are widely used to avoid divergence. Most of the regulators or form factors were introduced “by hand”. Our method is very fundamental and systematic.

3 Summary

In summary, we have proposed a new quantization - solid quantization for the elemental fields. The traditional point-like treatment created divergence, which needs to be taken care of with the regularization method. This solid quantization condition is very natural and based on the idea that a physical particle is not a mathematic point particle. Particles cannot exist in the world as dimensionless points. Each particle has its “shape”, which is another fundamental property of the particle as well as mass, spin, width, etc. The corresponding Feynman rules need to be changed for both external fields and internal propagators. The divergence of loop integration could be avoided from the beginning. We need not deal with infinity, which is caused by point particle assumption. The asymptotic free and confining potentials can be understood in our method. The small size of the elemental particles could be the source of CPT violation. This method can easily be applied to the effective theory, which is on the hadron level.

This method provides an interesting approach that is quite different from traditional quantum field theory. In this paper, we did not specify the function of $\Phi(\vec{x})$ or $\Psi(\vec{p})$. It could be dipole, monopole, Gauss or some other type of function. To get more information about the "shape" of the particle, it is important to do further calculations to compare with experiments and traditional results.

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