Quantitative conditions for the formation of *p*-wave neutron halos*

GUO Yan-Qing(郭艳青)[1)] SONG Jie(宋捷)

Department of Physics and Electronic Engineering, Hanshan Normal University, Chaozhou 521041, China

Abstract: With the experimental binding energies and configurations, the root-mean-square radii of p-wave valence neutron distributions for nuclei up to the second p-shell have been calculated in the framework of the single-particle potential model. By analyzing the relations between the radii and the binding energies, the scaling laws of p-wave valence neutron distributions are obtained. The quantitative conditions for the formation of p-wave neutron halos are deduced from these scaling laws. Based on the investigation on the probability of finding the valence nucleon outside the range of the interaction potential, a $2p_{3/2}$ halo state in ^{47}S is anticipated for the first time. These obtained results might provide reference for searching for p-wave neutron halos in nuclei up to the second p-shell.

Key words: scaling law, p-wave neutron halo, single-particle potential model

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1 Introduction

With the development of radioactive nuclear beam techniques, exotic structures have been observed in nuclei far from stability. One of the most amazing discoveries is the neutron halo. Up to now, a number of light neutron-rich nuclei have been identified as halo nuclei, such as the one-neutron halo nuclei ^{11}Be and ^{15}C [1–3], and the two-neutron halo nuclei ^{6}He and ^{11}Li [4, 5]. However, due to the model dependence of the experimental results, there is no well-accepted definition of the halo state. Halo characters are still controversial for some halo candidates [6, 7].

Although the nature of the halo is only partly understood, some common features of halos have been recognized. The halo structure prefers to develop in the weak-bound state with low angular momentum. The density distribution of this state usually has large spatial extension as compared with the core density distribution. Jensen et al. [8–12] ever formulated these conditions of halo occurrence as the general scaling laws. The scaling laws exhibit the universal scaling relations between the radii and the binding energies of the single-particle states. Quantitative conditions for halo occurrence can be derived from the scaling laws. These conditions provide a quick evaluation of possible halo candidates. In previous work [8–14], the scaling laws were mainly extracted from light nuclei. That is because, the most likely territory for the halo formation is the vicinity of the light dripline nuclei. So far, the observed halo states are s-states or p-states in light exotic nuclei. Recently, some research has revealed the possibility of medium halo nuclei [15]. As Jensen and Riisager [11] proposed that the search scope for medium halos should aim at the neutron dripline in the second p-shell. To provide a clue for searching for the unknown medium halos, it is meaningful to extend the scaling laws to nuclei up to the second p-shell.

In this work, the scaling laws of p-wave valence neutron distributions are extracted from the nuclei up to the second p-shell. With the binding energies and configurations determined experimentally, the root-mean-square radii of the single-particle states are calculated in the framework of the single-particle potential model. By analyzing the relations between the radii and the binding energies, we deduce the scaling laws and the quantitative conditions for p-wave halo formation. In addition, we also investigate the probability of finding the valence nucleon outside the

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1) E-mail: yqguo126@126.com

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range of the interaction potential. Based on the probabilities, a $2p_{3/2}$ halo state in ^{47}S is anticipated for the first time. These obtained results might provide useful clues for searching for p-wave neutron halos in nuclei up to the second p-shell.

2 Theoretical framework

By using the binding energies and configurations determined experimentally, the root-mean-square (rms) radii for the single-particle states of valence neutrons are calculated by numerically solving the following Schrödinger equation,

$$\left[-\frac{\hbar^2}{2m}\nabla^2+V_0f(r)+V_{\rm so}\frac{1}{r}\frac{{\rm d}f({\rm r})}{{\rm d}r}(\vec{l}\cdot\vec{s})\right]\phi_{nlj}(\vec{r}) = E_{nlj}\ \phi_{nlj}(\vec{r}). \quad (1)$$

In the above equation, $f(r)$ is the Woods-Saxon shape function with the standard parameters $r_0=1.25$ fm and $a_0=0.65$ fm [14, 16–18]. The nuclear potential depth V_0 is determined by the binding energy. The spin-orbit potential parameter $V_{\rm so}$ takes the form $V_{\rm so}=(\lambda/45.2)V_0$ with Thomas form factor $\lambda=25$ [14, 17].

In our calculations, the rms radius of the valence nucleon distribution is defined as

$$\langle r_v^2\rangle^{1/2}=\left[\frac{\int_0^\infty r^4\phi_{cv}^2(r){\rm d}r}{\int_0^\infty r^2\phi_{cv}^2(r){\rm d}r}\right]^{1/2}, \quad (2)$$

where ϕ_{cv} is the single-particle wave function derived from the Schrödinger Eq. (1).

In the present work, the occurrence conditions of neutron halo states are derived from the definition of a halo state proposed by Hansen et al. [8]. They proposed that a nucleus with its valence nucleon having more than 50% probability of being outside the range of the interaction potential is a halo nucleus. The interaction between valence nucleon and core can be described by the radius of the interaction potential $R_{cv}=r_0(A_c^{1/3}+A_v^{1/3})$ (A_c and A_v are the mass numbers of the core and valence nucleon, respectively.) [14]. Therefore, the necessary condition for halo occurrence is $\langle r_v^2\rangle/R_{cv}^2>2$ [14].

In addition, we also investigate the probability of finding the valence nucleon outside the range of the interaction potential. Its definition is [19]

$$D(R_{cv})=\frac{\int_{R_{cv}}^\infty r^2\phi_{cv}^2(r){\rm d}r}{\int_0^\infty r^2\phi_{cv}^2(r){\rm d}r}. \quad (3)$$

According to the above definition of halos, these states with $D>50\%$ are halo states.

3 Numerical results and discussions

In the framework of the single-particle potential model, we perform a systematic calculation for the rms radii of the p-wave valence neutron distributions for even-Z nuclei up to the second p-shell. The detailed numerical results are given in Tables 1–4. In these tables, "J^π" is the spin and parity of the calculated nucleus. "Level" represents the occupied level of the valence neutron. "S_v" (in units of MeV) denotes the separation energy of the valence nucleon. The separation energy, angular momentum and spin are assigned by the experimental results [20, 21]. "$\langle r_v^2\rangle^{1/2}$" (in units of fm) is the rms radius of the valence nucleon distribution. "D", which is defined in Eq. (3), represents the probability of finding the valence nucleon outside the range of the interaction potential.

In Tables 1 and 2, $N=3-6$ isotones and $N=7-8$ isotones are calculated for the $1p_{3/2}$-state and $1p_{1/2}$-state valence neutron distributions, respectively. Based on the values of D, the nuclei ^{6}He and ^{10}He are shown to be halo nuclei. ^{8}He and ^{9}Be are shown to have large neutron skins. In Table 3, the even-Z nuclei with their neutron numbers being 29,

Table 1. Theoretical results for the valence neutron in $1p_{3/2}$ state. The even-Z nuclei with $N=3-6$ are calculated.

nuclei	J^π	valence neutron(s)	level	S_v/ MeV	$\langle r_v^2\rangle^{1/2}$/ fm	D(%)
^{6}He	0^+	n	$1p_{3/2}$	1.86	3.77	40.13
		2n	$1p_{3/2}$	0.49	4.94	54.87
^{8}He	0^+	n	$1p_{3/2}$	2.57	3.62	32.29
		2n	$1p_{3/2}$	1.07	4.26	44.23
^{7}Be	$3/2^-$	n	$1p_{3/2}$	10.68	2.73	14.67
^{8}Be	0^+	n	$1p_{3/2}$	18.90	2.49	7.29
		2n	$1p_{3/2}$	14.79	2.56	10.67
^{9}Be	$3/2^-$	n	$1p_{3/2}$	1.67	3.98	36.38
^{10}Be	0^+	n	$1p_{3/2}$	6.81	3.08	16.77
		2n	$1p_{3/2}$	4.24	3.32	24.13
^{9}C	$3/2^-$	n	$1p_{3/2}$	14.26	2.66	9.13
^{10}C	0^+	n	$1p_{3/2}$	21.28	2.51	5.05
		2n	$1p_{3/2}$	17.77	2.55	7.00
^{11}C	$3/2^-$	n	$1p_{3/2}$	13.12	2.77	8.59
^{12}C	0^+	n	$1p_{3/2}$	18.72	2.64	5.18
		2n	$1p_{3/2}$	15.92	2.68	6.80
^{13}O	$3/2^-$	n	$1p_{3/2}$	17.01	2.71	5.43
^{14}O	0^+	n	$1p_{3/2}$	23.18	2.61	3.25
		2n	$1p_{3/2}$	20.09	2.64	4.27

Table 2. Theoretical results for the valence neutron in $1p_{1/2}$ state. The even-Z nuclei with $N=7$ and 8 are calculated.

nuclei	J^{π}	valence neutron(s)	level	S_v/ MeV	$\langle r_v^2\rangle^{1/2}$/ fm	$D(\%)$
^{10}He	0^+	n	$1p_{1/2}$	0.20	6.21	53.11
^{12}Be	0^+	n	$1p_{1/2}$	3.17	3.56	23.94
		2n	$1p_{1/2}$	1.84	3.93	31.86
^{13}C	$1/2^-$	n	$1p_{1/2}$	4.95	3.30	17.20
^{14}C	0^+	n	$1p_{1/2}$	8.18	3.04	10.80
		2n	$1p_{1/2}$	6.56	3.13	13.74
^{15}O	$1/2^-$	n	$1p_{1/2}$	13.22	2.82	5.88
^{16}O	0^+	n	$1p_{1/2}$	15.66	2.76	4.40
		2n	$1p_{1/2}$	14.44	2.77	5.21
^{17}Ne	$1/2^-$	n	$1p_{1/2}$	15.61	2.79	4.23
^{18}Ne	0^+	n	$1p_{1/2}$	19.22	2.71	2.87
		2n	$1p_{1/2}$	17.41	2.73	3.58
^{20}Mg	0^+	n	$1p_{1/2}$	23.54	2.67	1.79

Table 3. Theoretical results for the valence neutron in $2p_{3/2}$ state. The even-Z nuclei with $N=29$, 31 and 33 are calculated.

nuclei	J^{π}	valence neutron	level	S_v/ MeV	$\langle r_v^2\rangle^{1/2}$/ fm	$D(\%)$
^{45}S	$3/2^-$	n	$2p_{3/2}$	2.21	5.45	36.05
^{47}S	$3/2^-$	n	$2p_{3/2}$	0.77	6.58	50.37
^{47}Ar	$3/2^-$	n	$2p_{3/2}$	4.26	4.92	25.03
^{49}Ar	$3/2^-$	n	$2p_{3/2}$	2.50	5.39	32.94
^{51}Ar	$3/2^-$	n	$2p_{3/2}$	1.37	6.00	41.58
^{49}Ca	$3/2^-$	n	$2p_{3/2}$	5.15	4.80	21.40
^{51}Ca	$3/2^-$	n	$2p_{3/2}$	4.36	4.96	23.57
^{53}Ca	$3/2^-$	n	$2p_{3/2}$	3.46	5.17	26.78
^{51}Ti	$3/2^-$	n	$2p_{3/2}$	6.37	4.67	17.72
^{53}Ti	$3/2^-$	n	$2p_{3/2}$	5.44	4.81	19.69
^{55}Ti	$3/2^-$	n	$2p_{3/2}$	4.15	5.05	23.42
^{53}Cr	$3/2^-$	n	$2p_{3/2}$	7.94	4.54	14.21
^{55}Cr	$3/2^-$	n	$2p_{3/2}$	6.25	4.74	17.21
^{57}Cr	$3/2^-$	n	$2p_{3/2}$	5.31	4.88	19.16
^{55}Fe	$3/2^-$	n	$2p_{3/2}$	9.30	4.46	11.80
^{59}Fe	$3/2^-$	n	$2p_{3/2}$	6.58	4.75	15.81
^{57}Ni	$3/2^-$	n	$2p_{3/2}$	10.25	4.42	10.30
^{59}Ni	$3/2^-$	n	$2p_{3/2}$	9.00	4.53	11.67
^{61}Ni	$3/2^-$	n	$2p_{3/2}$	7.82	4.65	13.08
^{59}Zn	$3/2^-$	n	$2p_{3/2}$	13.03	4.30	7.55
^{61}Zn	$3/2^-$	n	$2p_{3/2}$	10.23	4.47	9.82
^{63}Zn	$3/2^-$	n	$2p_{3/2}$	9.11	4.57	10.96
^{61}Ge	$3/2^-$	n	$2p_{3/2}$	14.03	4.27	6.60
^{63}Ge	$3/2^-$	n	$2p_{3/2}$	12.74	4.36	7.35
^{65}Ge	$3/2^-$	n	$2p_{3/2}$	10.14	4.52	9.41

31, 33 are calculated for the $2p_{3/2}$-state valence neutron distributions. There appears one-neutron halo structure in the $2p_{3/2}$ ground state of ^{47}S. The nuclei ^{45}S and ^{51}Ar seem to have large neutron skin structures. The results of even-Z nuclei with $N=39$, 41, 43 are presented in Table 4. Their valence neutrons are predicted to stay in $2p_{1/2}$ states from shell models. The results in Table 4 manifest that there is no halo nucleus but a skin nucleus ^{63}Ti.

Table 4. Theoretical results for the valence neutron in $2p_{1/2}$ state. The even-Z nuclei with $N=39$, 41 and 43 are calculated.

nuclei	J^{π}	valence neutron	level	S_v/ MeV	$\langle r_v^2\rangle^{1/2}$/ fm	$D(\%)$
^{61}Ti	$1/2^-$	n	$2p_{1/2}$	2.07	5.68	31.64
^{63}Ti	$1/2^-$	n	$2p_{1/2}$	1.62	5.94	35.06
^{63}Cr	$1/2^-$	n	$2p_{1/2}$	3.18	5.31	24.64
^{65}Cr	$1/2^-$	n	$2p_{1/2}$	2.72	5.47	26.55
^{67}Cr	$1/2^-$	n	$2p_{1/2}$	2.32	5.64	28.68
^{65}Fe	$1/2^-$	n	$2p_{1/2}$	4.18	5.11	19.98
^{67}Fe	$1/2^-$	n	$2p_{1/2}$	4.19	5.13	19.65
^{69}Fe	$1/2^-$	n	$2p_{1/2}$	3.34	5.34	22.74
^{67}Ni	$1/2^-$	n	$2p_{1/2}$	5.81	4.88	14.99
^{71}Ni	$1/2^-$	n	$2p_{1/2}$	4.13	5.19	19.25
^{69}Zn	$1/2^-$	n	$2p_{1/2}$	6.48	4.82	13.28
^{71}Zn	$1/2^-$	n	$2p_{1/2}$	5.83	4.92	14.40
^{73}Zn	$1/2^-$	n	$2p_{1/2}$	5.35	5.00	15.30
^{71}Ge	$1/2^-$	n	$2p_{1/2}$	7.42	4.74	11.36
^{75}Ge	$1/2^-$	n	$2p_{1/2}$	6.51	4.88	12.61
^{77}Se	$1/2^-$	n	$2p_{1/2}$	7.42	4.81	10.78
^{79}Kr	$1/2^-$	n	$2p_{1/2}$	8.33	4.74	9.25
^{81}Sr	$1/2^-$	n	$2p_{1/2}$	9.29	4.69	8.02
^{83}Zr	$1/2^-$	n	$2p_{1/2}$	10.34	4.64	6.82
^{85}Mo	$1/2^-$	n	$2p_{1/2}$	11.37	4.59	5.91

An unexpected finding of our calculations is the $2p_{3/2}$ halo state in ^{47}S. So far, the known p-wave neutron halos mainly develop in the $1p_{1/2}$ states. Research into the p-wave halos in the second p-shell is rare. As far as we know, there is no experiment about the halo content of ^{47}S. In order to provide reference for the search of halo state in ^{47}S, it is necessary to investigate the density distributions of this nucleus. Therefore, in Fig. 1, we plot the density distribution of the $2p_{3/2}$ state in ^{47}S. The density distributions of the p-wave last neutron in ^{45}S and ^{46}S are also given for comparison. It is clear that the curve of ^{47}S($2p_{3/2}$) has a long tail as compared with that of ^{46}S($2p_{3/2}$). This indicates the one-neutron halo structure in ^{47}S($2p_{3/2}$) which deserves further investigation in future experiments. Compared with the even-even nucleus ^{46}S, the spatial extension of the density distribution for ^{45}S($2p_{3/2}$) manifests the neutron skin structure in the ground state of ^{45}S.

Based on the calculated data listed in Tables 1–4, the results as the relation between $\langle r_v^2\rangle/R_{cv}^2$ and $S_vR_{cv}^2$ are presented in Fig. 2. In this figure, the circles and triangles denote the calculated data. The lines are the fitting results using the fitting function

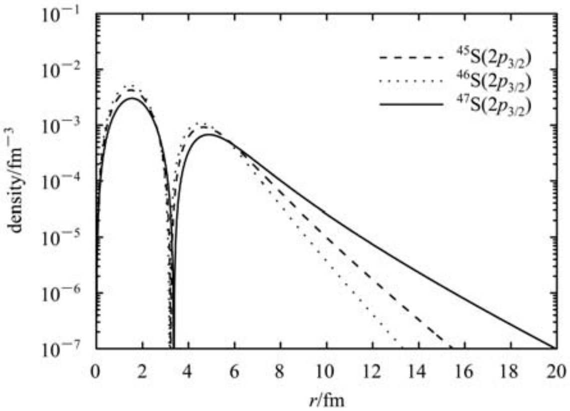

Fig. 1. The density distributions for the $2p_{3/2}$ states in ^{45}S, ^{46}S and ^{47}S. The dashed, dotted and solid curves are the density distributions of the last neutron in ^{45}S, ^{46}S and ^{47}S, respectively.

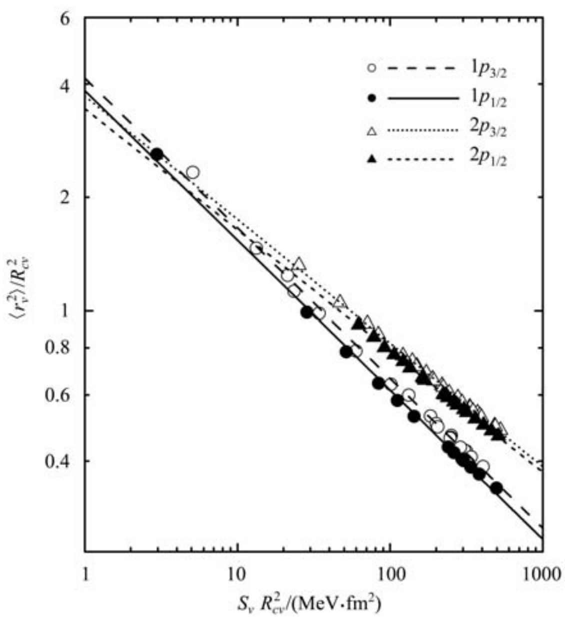

Fig. 2. The rms radii of p-wave valence neutrons vary with the separation energies. The circles and triangles represent the calculated data. The lines are the fitting results.

$y= a/x^b$, where a and b are the fitting parameters. The scaling laws for the valence neutrons in the first p-shell are

$$\begin{cases} \dfrac{\langle r_v^2\rangle}{R_{cv}^2} \approx \dfrac{4.15}{(S_v R_{cv}^2)^{0.398}}, & \text{for } 1p_{3/2} \text{ states,} \\[2ex] \dfrac{\langle r_v^2\rangle}{R_{cv}^2} \approx \dfrac{3.83}{(S_v R_{cv}^2)^{0.396}}, & \text{for } 1p_{1/2} \text{ states.} \end{cases} \tag{4}$$

According to the halo definition that the states with $\langle r_v^2\rangle/R_{cv}^2 > 2$ are halo states, the necessary conditions for the formation of p-wave neutron halos in the first p-shell are

$$\begin{cases} S_v R_{cv}^2 < 6.24 \ \ \text{MeV}\cdot\text{fm}^2, & \text{for } 1p_{3/2} \text{ states,} \\[2ex] S_v R_{cv}^2 < 5.17 \ \ \text{MeV}\cdot\text{fm}^2, & \text{for } 1p_{1/2} \text{ states.} \end{cases} \tag{5}$$

These quantitative conditions are consistent with $S_v R_{cv}^2 < 5.79$ MeV fm^2 which is extracted from ^{4}He to ^{22}C by Lin et al.[14].

Similarly, the scaling laws for the valence neutrons in the second p-shell are

$$\begin{cases} \dfrac{\langle r_v^2\rangle}{R_{cv}^2} \approx \dfrac{3.71}{(S_v R_{cv}^2)^{0.327}}, & \text{for } 2p_{3/2} \text{ states,} \\[2ex] \dfrac{\langle r_v^2\rangle}{R_{cv}^2} \approx \dfrac{3.43}{(S_v R_{cv}^2)^{0.321}}, & \text{for } 2p_{1/2} \text{ states.} \end{cases} \tag{6}$$

Accordingly, the necessary conditions for the formation of p-wave neutron halos in the second p-shell are

$$\begin{cases} S_v R_{cv}^2 < 6.60 \ \ \text{MeV}\cdot\text{fm}^2, & \text{for } 2p_{3/2} \text{ states,} \\[2ex] S_v R_{cv}^2 < 5.38 \ \ \text{MeV}\cdot\text{fm}^2, & \text{for } 2p_{1/2} \text{ states.} \end{cases} \tag{7}$$

4 Summary

The rms radii of p-wave valence neutron distributions for nuclei up to the second p-shell have been calculated by numerically solving the Schrödinger equations. By analyzing the relations between the radii and the binding energies, the scaling laws of p-wave valence neutron distributions are obtained. In the light of these scaling laws, the necessary conditions for neutron halo occurring in $1p_{3/2}$, $1p_{1/2}$, $2p_{3/2}$ and $2p_{1/2}$ states are deduced, respectively. In addition, the values of "D", which represents the probability of finding the valence nucleon outside the range of the interaction potential, are also calculated. Based on the "D" values, a $2p_{3/2}$ halo state in ^{47}S is predicted for the first time. These results might provide reference for searching for p-wave neutron halos in nuclei up to the second p-shell.

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