One-loop QCD contribution to the potential of $Q\bar{Q}$ *

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Abstract: Without the non-relativistic approximation in one-loop function, the dominating one-loop contribution to the quark-antiquark potential is studied numerically in terms of perturbative Quantum Chromo Dynamics (QCD). For Coulomb-like potential, the ratio of the one-loop correction to the tree diagram contribution is presented, whose absolute value is about 20%. Our result is consistent with the analysis that the one-loop contribution should be suppressed by a factor $\frac{\alpha_s}{\pi}$ to the leading order contribution. This work can deepen the comprehension of α_s in Cornel potential.

Key words: one-loop, potential, Coulomb

PACS: 12.38.Aw, 12.38.Bx, 12.39.Hg **DOI:** 10.1088/1674-1137/35/2/003

1 Introduction

The strong interactions of quarks and gluons making up hadrons are governed by quantum chromodynamics (QCD) ultimately. In terms of QCD, it is satisfactory to study hadron physics at the quark-gluon level. However, obtaining the analytic interquark potential from QCD is very difficult, because QCD is a non-Abelian theory.

The quark-antiquark potential is composed of two parts. One part is the Coulomb-like potential, which belongs to short-distance effects. The other part is the linear confinement term produced from large-distance behavior of QCD. It is important to compute the perturbative QCD potential as exactly as possible. The authors [1, 2] present the Coulomb term from one-gluon-exchange diagram, together with the relevant linear confinement term. The one-loop correction including the spin-dependent term to the heavy quark-antiquark potential is obtained analytically with the non-relativistic approximation in the center-of-mass system [3]. Considering the relativistic correction to the effective potential of the $Q\bar{Q}$ system, the authors [4] study the heavy quark mesons. Recently, the quark-antiquark system has been studied in the potential model including a linear potential, a relativistic kinetic term and one-loop QCD correction, and they get satisfactory results [5]. About ten years ago, based on the quasipotential approach, D. Ebert and R. N. Faustov [6] developed the relativistic quark model. Taking account of the relativistic and retardation effects and the one-loop radiative correction, they calculated the charmonium and bottomonium mass spectra, and their results for the fine splittings of quarkonium are better.

Using lattice QCD, the authors [7–9] analyse the Coulomb plus linear-quark confinement potential for two quarks and three quarks particularly. The formula of quark-antiquark potential though sum of the Coulomb term by perturbative one-gluon-exchange process and the linear confinement term in Refs. [7, 10–12] reads as,

$$V_{Q\bar{Q}}(r) = -\frac{A_{Q\bar{Q}}}{r} + \sigma_{Q\bar{Q}}r + C_{Q\bar{Q}}. \tag{1}$$

Considering the quark confinement as the result of strong interaction between the quarks in hadron, the correction of the potential from the one-loop diagram in the quark-antiquark system is very important.

We don't use the non-relativistic approximation in the one-loop function, and obtain the correction to the Coulomb-like potential numerically, because the

Received 12 April 2010, Revised 23 August 2010

* Supported by National Natural Science Foundation of China (10947167, 11047002, 11047120) and Fund of Education Department of Hebei Province (2007409)

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result is too difficult to be transformed. For the three heavy flavor quarks (c, b and t), we give out the numerical one-loop correction to the tree diagram contribution with LoopTools [13, 14]. From this work we can understand α_s in Cornel potential deeper.

The paper is organized as follows. After this introduction, in Section 2, we give the tree diagram and the corresponding amplitude in purterbative QCD. In Section 3, a detailed analysis of the one-loop diagram is presented. The numerical result along with all the input parameters are shown in Section 4. The last section is devoted to simple discussion and conclusion.

2 The contribution of the tree diagram to the potential

In order to get the general QCD contribution for the $Q\bar{Q}$ system potential, we compute the tree diagram analytically and those one-loop diagrams numerically. The tree diagram is presented in Fig. 1.

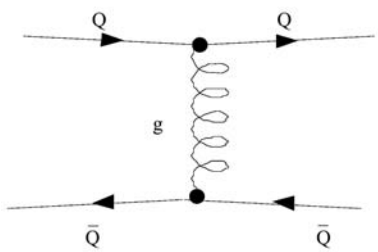

Fig. 1. The tree diagram for the $Q\bar{Q}$ system.

The amplitude of Fig. 1 can be written as:

$$\bar{u}_\beta(p_1^{'})(-\mathrm{i}g_s\gamma^\mu T^a_{\beta\alpha})u_\alpha(p_1)\frac{-\mathrm{i}}{q^2}\bar{v}_\theta(p_2)(-\mathrm{i}g_s\gamma_\mu T^a_{\theta\rho}) \times v_\alpha(p_2^{'}), \quad (2)$$

where $p_1, p_2, q_1^{'}$ and $q_2^{'}$ stand for the four-momentum of the initial states and the final particles respectively. Here, q is the gluon four-momentum $q = p_1^{'} - p_1 = p_2 - p_2^{'}$. After simplification, Eq. (2) reads:

$$(\mathrm{i}g_s^2 T^a_{\beta\alpha}T^a_{\theta\rho})\bar{u}_\beta(p_1^{'})\gamma^\mu u_\alpha(p_1)\frac{1}{q^2}\bar{v}_\theta(p_2)\gamma_\mu v_\alpha(p_2^{'}) \rightarrow (\mathrm{i}g_s^2 T^a_{\beta\alpha}T^a_{\theta\rho})\bar{u}_\beta(p_1^{'})\gamma^\mu u_\alpha(p_1)\bar{v}_\theta(p_2)\gamma_\mu v_\alpha(p_2^{'})\frac{1}{r}, \quad (3)$$

with the Fourier transform for the gluon propagator,

$$\int \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}}\frac{4\pi}{\boldsymbol{q}^2}\frac{\mathrm{d}^3 q}{(2\pi)^3} = \frac{1}{r}. \quad (4)$$

Here, the Coulomb-like potential is our investigative object.

After simple deduction, the Coulomb-like potential for the $Q\bar{Q}$ system is gotten analytically, with which we study the Schrödinger equation for the $Q\bar{Q}$ system:

$$\hat{H}\psi = \left[\frac{\mathrm{i}}{2m}(\hat{\boldsymbol{p}}_1^2 + \hat{\boldsymbol{p}}_1^2) + \hat{U}(r)\right]\psi = E_n\psi, \quad (5)$$

$$\hat{U}(r) = -\frac{A\alpha_s^2(M_Q)}{r}, \quad (6)$$

where $A = \frac{4}{3}$, and $\alpha_s(M_Q)$ is the effective quark-gluon strong coupling constant. Dealing with a H atom in the same way, the energy level formula of the $Q\bar{Q}$ system is obtained:

$$E_{qn} = -\frac{A^2\mu_q\alpha_s^2}{2n^2}, \quad n = 1, 2, 3\ldots, \quad (7)$$

where $\mu_q = \frac{m_q}{2}$ is the reduced mass. It is well known that the energy level of a H atom is,

$$E_{en} = -\frac{\mu_e\alpha_e^2}{2n^2}, \quad (8)$$

where μ_e is the mass of electron and α_e is the fine structure constant $\left(\frac{1}{137}\right)$. For three heavy flavor quarks, we obtain the ratio of ground-state energy of the $Q\bar{Q}$ system to that of a H atom from Eqs. (7), (8). Thus the ground energy of $Q\bar{Q}$ is about 0.1 GeV, which is deep.

3 One-loop diagrams of quark anti-quark interactions in QCD

In this section, on the general principle of perturbative QCD, we study the dominant one-loop diagrams with two or more gluons exchange. These one-loop diagrams contributing to the potential are shown in Fig. 2.

For the convenience of readers, we carry out the calculation of diagram (a) in Fig. 2 with the software LoopTools. The amplitude can be written as:

$$\mathcal{M}_a = \bar{u}_\beta(p_1^{'})(-\mathrm{i}g_s\gamma^\mu T^a_{\beta\alpha})u_\alpha(p_1)\frac{-\mathrm{i}}{q^2}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\left[\bar{v}_\theta(p_2)(-\mathrm{i}g_s\gamma^\nu T^b_{\theta\omega})\frac{\mathrm{i}}{\not{k} - \not{p}_2 - m}(-\mathrm{i}g_s\gamma_\mu T^a_{\omega\eta})\frac{\mathrm{i}}{\not{k} - \not{p}_2^{'} - m} \times(-\mathrm{i}g_s\gamma_\nu T^b_{\eta\rho})v_\alpha(p_2^{'})\frac{-\mathrm{i}}{k^2}\right] = (g_s^4 T^a_{\beta\alpha}T^b_{\theta\omega}T^a_{\omega\eta}T^b_{\eta\rho})\bar{u}_\beta(p_1^{'})\gamma^\mu u_\alpha(p_1)\frac{1}{q^2}\int\frac{\mathrm{d}^4k}{(2\pi)^4} \times\left[\bar{v}_\theta(p_2)\gamma^\nu(\not{k} - \not{p}_2 + m)\gamma_\mu(\not{k} - \not{p}_2^{'} + m)\gamma_\nu v_\alpha(p_2^{'}) \times \frac{1}{[(k-p_2)^2 - m^2][(k-P_2^{'})^2 - m^2]k^2}\right]. \quad (9)$$

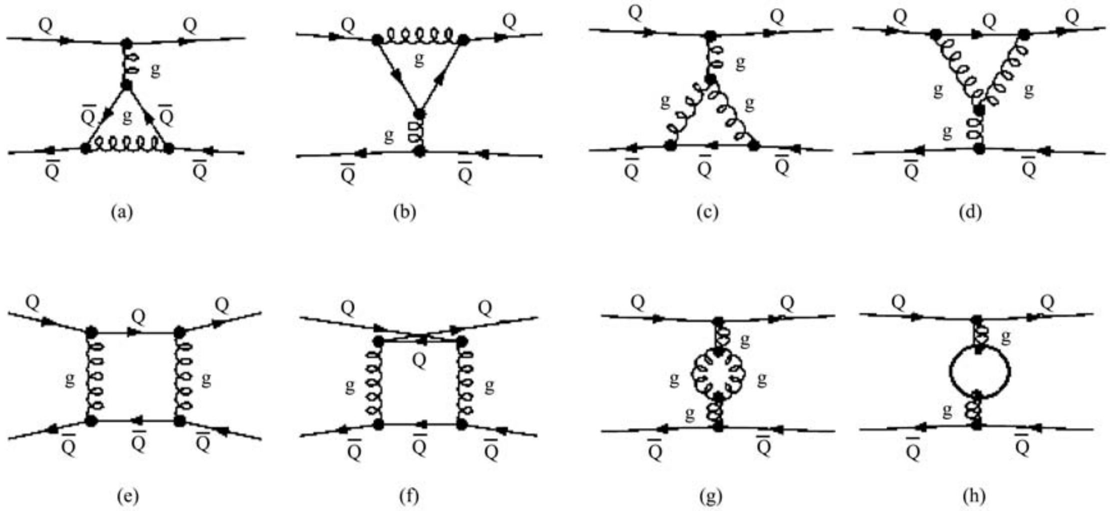

Fig. 2. The one-loop diagrams for the $Q\bar{Q}$ system.

Considering the Fourier transform for the gluon propagator, we calculate the one-loop integrals and apply the Dirac equation for the outside Fermions to simplify the analytic results. Then, the amplitude reads:

$$\begin{aligned}\mathcal{M}_{\rm a} =\ & {\rm i}\pi^2(g_{\rm s}^4 T^{\rm a}_{\beta\alpha}T^{\rm b}_{\theta\omega}T^{\rm a}_{\omega\eta}T^{\rm b}_{\eta\rho})\frac{1}{q^2}\Big\{\bar{u}_\beta(p_1^{'})\gamma^\mu u_\alpha(p_1)\\ & \times\bar{v}_\theta(p_2)\gamma_\mu v_\alpha(p_2^{'})\Big[2B_0+4C_0 p_2.p_2^{'}+2m^2C_0\\ & -2C_0p_2^{'2}+2p_2^2C_1+4p_2.p_2^{'}C_2+2p_2^{'2}C_2-4C_{00}\\ & +2m^2C_1+2m^2C_2\Big]+4m\bar{u}_\beta(p_1^{'})\gamma_\mu u_\alpha(p_1)\\ & \times\bar{v}_\theta(p_2)v_\alpha(p_2^{'})\Big[p_2^{'\mu}C_1+p_2^\mu C_2+p_2^\mu C_{12}\\ & +p_2^{'\mu}C_{22}+p_2^\mu C_{11}+p_2^{'\mu}C_{12}\Big]\Big\},\end{aligned}\qquad(10)$$

where [15]

$$\begin{aligned}& T^N_{\mu_1\dots\mu_{\rm p}}(p_1,\dots,p_{N-1},m_0,\dots,m_{N-1})\\ =\ & \frac{(2\pi\mu)^{4-D}}{{\rm i}\pi^2}\int{\rm d}^Dq\frac{q_{\mu_1}\dots q_{\mu_1}}{D_0D_1\dots D_{N-1}};\end{aligned}\qquad(11)$$

with the denominator factor

$$\begin{aligned}D_0 &= q^2-m^2+{\rm i}\varepsilon,\\ D_i &= (q+p_i)^2-m^2+{\rm i}\varepsilon,\ i=1,\dots,N-1,\end{aligned}\qquad(12)$$

originating from the propagators in the Feynman diagram. Furthermore, we introduce

$$p_{i0}=p_i,\quad p_{ij}=p_i-p_j,\qquad(13)$$

$$\begin{aligned}C_0 &= C_0\Big[p_2^2,p_2^{'2},p_2^2-2p_2.p_2^{'}+p_2^{'2},m_1^2,0,m_2^2\Big],\\ C_1 &= C\Big[1,\{p_2^2,p_2^{'2},p_2^2-2p_2.p_2^{'}+p_2^{'2}\},\{m_1^2,0,m_2^2\}\Big],\\ C_{11} &= C\Big[1,1,\{p_2^2,p_2^{'2},6p_2^2-2p_2.p_2^{'}+p_2^{'2}\},\{m_1^2,0,m_2^2\}\Big].\end{aligned}$$

The three-point one-loop functions C_2, C_{12} and C_{22} are similar to C_1 and C_{11} respectively. Those one-loop functions can be calculated numerically by LoopTools. Then, the next-to-leading order contribution is obtained.

4 Numerical results

In Fig. 2 all one-loop diagrams for three heavy flavor quarks ($Q\bar{Q}$), Q = c, b, t are taken into account. In order to get the last results, we have to calculate the one-loop functions numerically. Using the weak-binding approximation [16], i.e. $p_{\rm q}=p_{\bar{\rm q}}, p_{\rm q}^2=m_{\rm q}^2$ and the relation $q=p_1^{'}-p_1=p_2-p_2^{'}$, we get

$$\begin{aligned}& p_1.p_1^{'}=\frac{p_1^2+p_1^{'2}-q^2}{2},\quad p_2.p_2^{'}=\frac{p_2^2+p_2^{'2}-q^2}{2},\\ & p_1.p_2^{'}=\frac{p_1^2-p_1.p_1^{'}}{2},\quad p_1^{'}.p_2^{'}=\frac{p_1.p_1^{'}-p_1^{'2}}{2}.\end{aligned}\qquad(14)$$

The input parameters are taken as follows [16–18]: $\alpha_{\rm s}(m_{\rm c})=0.26$, $\alpha_{\rm s}(m_{\rm b})=0.17$, $\alpha_{\rm s}(m_{\rm t})=0.09$, $m_{\rm c}=1.25$ GeV, $m_{\rm b}=4.70$ GeV, $m_{\rm t}=174.20$ GeV.

Without non-relativistic approximation, q^2 in the one-loop function is very difficult to Fourier transform. Fortunately, it is a tiny parameter, which makes a very small difference to the one-loop functions. Therefore, it is reasonable to treat q^2 as a tiny value in the one-loop integral. For convenience, q^2 varies from 10^{-10} GeV^2 to 0.1 GeV^2. After tedious calculation, we obtain the one-loop correction for the Coulomb-like term, and get the last numerical result in the end. For Coulomb-like potential, the ratio of the one-loop diagram correction to that of the tree diagram is presented in Table 1.

From Table 1, it is easy to see that the one-loop diagram contribution to the Coulomb-like potential

Table 1. The ratio of one-loop diagram correction to that from tree diagram varying with q^2 for heavy quark.

q^2/GeV^2	$V_{\text{one-loop}}\left(\frac{1}{r}\right)/V_{\text{tree}}\left(\frac{1}{r}\right)$		
	$c\bar{c}$	$b\bar{b}$	$t\bar{t}$
10^{-11}	−0.2904	−0.2525	−0.1568
10^{-10}	−0.2758	−0.2378	−0.1131
10^{-9}	−0.2612	−0.2232	−0.1074
10^{-8}	−0.2467	−0.2086	−0.0942
10^{-7}	−0.2321	−0.1940	−0.0795
10^{-6}	−0.2172	−0.1795	−0.0649
10^{-5}	−0.2029	−0.1649	−0.0510
10^{-4}	−0.1883	−0.1503	−0.0360
10^{-3}	−0.1638	−0.1351	−0.0170
10^{-2}	−0.0706	−0.1211	0.0324
0.1	0.0296	0.0066	0.0697

is about −0.2 times that from the tree diagram. Generally speaking, the rate is stable for the varying q^2. When q^2 is not smaller than 0.01 GeV^2, the one-loop contribution can be positive. Otherwise, the next-to-leading order correction weakens the Coulomb-like term. For the $c\bar{c}$ and $b\bar{b}$ systems the ratios are around −0.2, for a $t\bar{t}$ system the ratio is probably −0.1. Comparing the results for three heavy flavor quarks, it implies when quark mass becomes heavier, the absolute value of one-loop correction turns smaller. From the analysis, the numerical result with q^2 varying from (10^{-10} GeV^2 to 10^{-4} GeV^2) is more reasonable.

5 Discussion and conclusion

About thirty years ago, people studied the quark-antiquark potential at next-to-leading order with non-relativistic approximation, and they obtained analytic results. In this work, we study the one-loop QCD contribution to the $Q\bar{Q}$ system with two or more gluons exchange with the help of the software LoopTools. Non-relativistic approximation is not used in the one-loop integrals which are computed numerically because of complexity.

The obtained ratio of the one-loop correction to the tree contribution for Coulomb-like term is at the order of −20%. For the charm quark, with the varying q^2 the ratio can reach −0.28. For bottom quark and top quark the ratios achieve −0.24 and −0.11 respectively. From Table 1, we can see the absolute values of the ratios become large with the quark masses turning small for the same q^2, which is consistent with HQET.

Because the quarks are very heavy, q^2 varying from 10^{-10} GeV^2 to 10^{-4} GeV^2 is more reasonable and can be treated as a tiny value in the one-loop functions, which makes the corresponding results quite believable. In the view of analysis from perturbative QCD, the one-loop contribution is suppressed by a factor $\frac{\alpha_s}{\pi}$ compared with that of tree diagram. It is contented that our numerical result does not break the rule. Our numerical result is also consistent with the previous analytic result [4]. Though α_s can not be obtained from field theory essentially, this work for the one-loop correction to the contribution of the tree diagram is in favor of the study of α_s in Cornel potential.

References

1 Eichten E, Gottfried K. Phys. Lett. B, 1977, **66**: 286
2 Celmaster W, Georgi H, Machacek M. Phys. Rev. D, 1978, **17**: 879
3 Gupta S N, Radford S F. Phys. Rev. D, 1981, **24**: 2309–2323
4 Titard S, Yndurain F J. Phys. Rev. D, 1994, **49**: 6007–6025; Phys. Rev. D, 1995, **51**: 6348–6363; Pineda A, Yndurain F J. Phys. Rev. D, 1998, **58**: 094022; Phys. Rev. D, 2000, **61**: 077505.7
5 Radford S F, Repko W W. Phys. Rev. D, 2007, **75**: 074031; Radford S F, Repko W W, Saelim M J. Phys. Rev. D, 2009, **80**: 034012
6 Ebert D, Faustov R N, Galkin V O. Talk given at the Vth International Workshop "Heavy Quark Physics". Dubna, Russia, 6–8 April 2000, hep-ph/0006186
7 Takahashi T T, Suganuma H, Nemoto Y et al. Phys. Rev. D, 2002, **65**: 114509
8 Compean C B, Kirchbach M. Eur. Phys. J. A, 2007, **33**: 1
9 Takahashi T T, Matsufuru H, Nemoto Y et al. Phys. Rev. Lett., 2001, **86**: 18–21
10 Takahashi T T, Matsufuru H, Nemoto Y et al. Phys. Rev. Lett., 2001, **86**: 18
11 Matsufuru H, Nemoto Y, Suganuma H et al. Nucl. Phys. B, 2001, **94**: 554
12 Bali G S, Schlichter C, Schilling K. Phys. Rev. D, 1995, **51**: 5165
13 Hooft G, Veltman M. Nucl. Phys. B, 1979, **153**: 365–401; Denner A, Nierste U, Scharf R. Nucl. Phys. B, 1991, **367**: 637–656
14 Hahn T, Victoria M P. Comput. Phys. Commun, 1999, **118**: 153
15 Denner A, Dittmaier S. Nucl. Phys. B, 2003, **658**: 175–202 Denner A. Nucl. Phys. B, 1998, **519**: 39–84
16 LI Gang, LI Tong, LI Xue-Qian et al. Nucl. Phys. B, 2005, **727**: 301
17 Eidelman S et al. Partical Data Group, Phys. Lett. B, 2004, **592**: 1; YAO Wei-Ming et al. PARTICLE PHYSICS BOOKLIET, 2006, 24
18 Fritzsch H, ZHOU Yu-Feng. Phys. Rev. D, 2003, **68**: 034015