Are operators describing $b \to s\gamma$? *

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Abstract: The operators of $b \to s\gamma$, $b \to sl^+l^-$ are usually regarded as being sufficient to describe $b \to s\gamma\gamma$, $b \to sl^+l^-\gamma$ with the statement that contributions from diagrams without an effective vertex $b \to s\gamma$ to processes $b \to s\gamma\gamma$ and $b \to sl^+l^-\gamma$ are negligible. In this work we present a comprehensive analysis of the transition $b \to s\gamma\gamma$ and find that 1) Effects due to off-shell quarks in vertex $b \to s\gamma$ on $b \to s\gamma\gamma$ are large; 2) Contributions from diagrams without an effective vertex $b \to s\gamma$ to $b \to s\gamma\gamma$ are not negligible compared with others; 3) These effects cancel each other out exactly, so the operators of $b \to s\gamma$ can safely be used to describe $b \to s\gamma\gamma$.

Key words: off-shell quarks effect, new Feynman diagrams, Ward identity

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1 Introduction

As is well known, the flavor-changing neutral-currents (FCNC) induced B-meson rare decays provide an ideal opportunity for extracting information about the fundamental parameters of the Standard Model (SM) and some hadronic parameters in QCD, such as the CKM matrix elements. The meson decay constant f_B providing information about heavy meson wave functions. Since these decays occur in the SM only through loops, they also play an important role in testing higher-order effects in the SM and in searching for physics beyond the SM [1, 2].

Theoretical predictions for inclusive decays $B \to X_s\gamma(\gamma)$, $B \to X_s\, l^+l^-(\gamma)$ and exclusive decays $B_s \to \gamma\gamma$, $B_s \to l^+l^-(\gamma)$ and $B \to K(K^*)l^+l^-$ have been studied extensively by many authors in the framework of the SM and new physics [1–3]. Among these works, obtaining effective Hamiltonians for $b \to s\gamma$, $b \to s\, l^+l^-$ is regarded as the fundamental research, with the following assumptions [2, 4]: i) Effective Hamiltonians for $b \to s\gamma$, $b \to s\, l^+l^-$ can be applied directly to the processes as $B_s \to \gamma\gamma$, $B_s \to l^+l^-(\gamma)$ and ii) the operators included in effective Hamiltonians are sufficient to describe the processes in models that are without particles much lighter than W boson. Our arguments are:

1) Effective Hamiltonians are obtained for on-shell quarks. Assumption i) seems inconsistent unless the off-shell quarks' effects on $b \to s\gamma\gamma$, $b \to s\, l^+l^-$ are small and can be neglected;

2) Some diagrams without an effective vertex $b \to s\gamma$ also contribute to $b \to s\gamma\gamma$ and $b \to s\, l^+l^-\gamma$. Even contributions coming from diagrams with another photon attached to internal lines are strongly suppressed by a factor m_b^2/m_W^2 as stated in some previous works [2]; contributions from the diagrams with $WW\gamma\gamma$ interaction in the SM are surely comparable to others.

In this paper, we present a comprehensive calculation for $b \to s\gamma\gamma$ at a matching scale. Based on the calculation, we will prove the arguments and point out that the off-shell quarks' effect and the contributions from diagrams without a $b \to s\gamma$ vertex cancel out each other exactly so the operators of $b \to s\gamma$ can be used safely to describe $b \to s\gamma\gamma$. Although the conclusion can be drawn by using Low's low energy theorem [4], this work is useful in deepening our understanding of B physics and is thus valuable.

2 Effective Hamiltonian for $b \to s\gamma$

Let us start with the calculation of an effective

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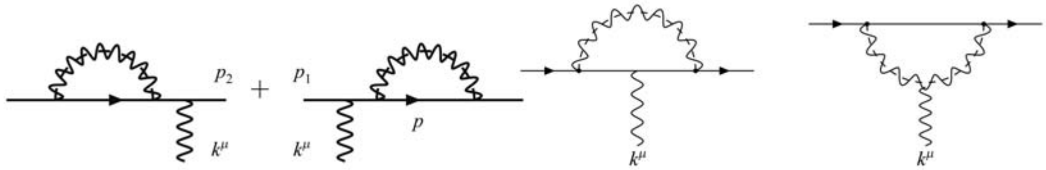

Fig. 1. Self energy (left) and triangle (right) Feynman diagrams for $\mathrm{b}\to\mathrm{s}\gamma$. The wavy and dashed lines in the figure stand for the $\mathrm{W}^\pm$ boson and corresponding $\mathrm{G}^\pm$ in R_ξ with the $\xi=1$ gauge, respectively.

vertex of $\mathrm{b}\to\mathrm{s}\gamma$ at a leading order at matching scale in general. We will adopt a naive dimensional regularization with an anticommuting γ_5 scheme and the non-linear R_ξ with a $\xi=1$ gauge for simplification [5]. This special gauge-fixing term guarantees explicit electromagnetic gauge invariance throughout the calculation, not just at the end, because the choice eliminates the $\gamma\mathrm{W}^\pm\mathrm{G}^\pm$ vertex in the Lagrangian where G is the charged Goldstone particle.

We first considered the self-energy diagrams with W and G in loops and expressed them as

$$-\mathrm{i}\Sigma^{\mathrm{S}} = \frac{g^2}{2}\frac{\mathrm{i}}{16\pi^2}\sum_{\mathrm{j=u,c,t}} V_{\mathrm{jb}}V_{\mathrm{js}}^*[A_1(p)\not{p}P_{\mathrm{L}} - m_{\mathrm{b}}A_2(p)P_{\mathrm{R}}], \qquad (1)$$

where

$$\delta_{\mathrm{j}} = \frac{m_{\mathrm{j}}^2}{m_{\mathrm{W}}^2},$$

$A_1(p)=(2+\delta_{\mathrm{j}})B_1(p)$, $A_2(p)=B_0(p)$ and $B_{\mathrm{i}}\ (p)=B_{\mathrm{i}}$ $(p,\ m_{\mathrm{j}},\ m_{\mathrm{W}})$ are loop functions [6]. The effective vertex of $b(p_1)\to s(p_2)\gamma(k)$ from self-energy diagrams is then written in a general form:

$$\Gamma^{\mathrm{S},\mu}_{\mathrm{b\to s\gamma}} = \frac{g^2}{2}\frac{1}{16\pi^2}e\sum_{\mathrm{j}} V_{\mathrm{jb}}V_{\mathrm{js}}^*\Big[f_1^{\mathrm{S}}\gamma^\mu P_{\mathrm{L}} + f_2^{\mathrm{S}}(\not{p}_1\gamma^\mu\not{p}_1 + \not{p}_2\gamma^\mu\not{p}_2)P_{\mathrm{L}} + f_3^{\mathrm{S}}\not{p}_1\gamma^\mu\not{p}_2P_{\mathrm{L}} + f_4^{\mathrm{S}}\not{p}_2\gamma^\mu\not{p}_1P_{\mathrm{L}} + f_5^{\mathrm{S}}m_{\mathrm{b}}p_1^\mu P_{\mathrm{R}} + f_6^{\mathrm{S}}m_{\mathrm{b}}\not{p}_2\gamma^\mu P_{\mathrm{R}} + f_7^{\mathrm{S}}m_{\mathrm{b}}\gamma^\mu\not{p}_1P_{\mathrm{R}}\Big], \qquad (2)$$

For on-shell quarks, Eq. (2) has a simple form:

$$V^{\mathrm{S}}_{\mathrm{b\to s\gamma}} = -Q_{\mathrm{d}}\frac{g^2}{2}\frac{1}{16\pi^2}\sum_{\mathrm{j}} V_{\mathrm{jb}}V_{\mathrm{js}}^*\gamma^\mu P_{\mathrm{L}} \times[A(p_1)+A_2(p_2)-A_2(p_1)], \qquad (3)$$

where $Q_{\mathrm{d}}=e_{\mathrm{d}}e$ is the down-type quark charge, and s quark mass is neglected.

Now we calculate the triangle diagrams' contribution with the same expression as Eq. (2) except that the coefficients $f_{\mathrm{i}}^{\mathrm{S}}$ are replaced by $f_{\mathrm{i}}^{\mathrm{T}}$. All coefficients can be found in Table 1.

Table 1. Coefficients f_{i} in the expressions of Eq. (2) and Eq. (6) with $C=C\ (p_1,\ p_2,\ m_{\mathrm{W}},\ m_{\mathrm{j}},\ m_{\mathrm{j}})$ and $\hat{C}=C\ (p_1,\ p_2,\ m_{\mathrm{i}},\ m_{\mathrm{W}},\ m_{\mathrm{W}})$.

f_1^{S}	$-e_{\mathrm{d}}\left[(2+\delta_{\mathrm{j}})B_1(p_1)+\frac{B_1(p_2)p_2^2-\delta_{\mathrm{j}}B_0(p_2)m_{\mathrm{b}}^2}{(p_2^2-m_{\mathrm{b}}^2)}\right]$	f_1^{T}	$(2+\delta_{\mathrm{j}})\left[\frac{1}{2}e_{\mathrm{u}}(-2m_{\mathrm{j}}^2C_0+C_{24}-1)-\frac{1}{2}\hat{C}_{24}+(\hat{C}_{11}-3\hat{C}_{23})(p_1^2+p_2^2)\right]$
f_2^{S}	0	f_2^{T}	$(2+\delta_{\mathrm{j}})[e_{\mathrm{u}}(C_{11}-2C_{23})+(\hat{C}_{11}-2\hat{C}_{23})]$
f_3^{S}	0	f_3^{T}	$e_{\mathrm{u}}[4C_{11}-2C_0-(2+\delta_{\mathrm{j}})C_{23}]-4\hat{C}_{11}$
f_4^{S}	0	f_4^{T}	$e_{\mathrm{u}}[\delta_{\mathrm{j}}(2C_{11}-C_0)-(2+\delta_{\mathrm{j}})C_{23}]+4\hat{C}_{11}$
f_5^{S}	0	f_5^{T}	$-2\delta_{\mathrm{j}}(2e_{\mathrm{u}}C_{11}+\hat{C}_0-2\hat{C}_{11})$
f_6^{S}	$-e_{\mathrm{d}}\frac{(2+\delta_{\mathrm{j}})B_1(p_2)-\delta_{\mathrm{j}}B_0(p_2)}{(p_2^2-m_{\mathrm{b}}^2)}$	f_6^{T}	$e_{\mathrm{u}}\delta_{\mathrm{j}}C_0$
f_7^{S}	$e_{\mathrm{d}}\delta_{\mathrm{j}}B_0(p_1)/p_1^2$	f_7^{T}	$e_{\mathrm{u}}\delta_{\mathrm{j}}C_0$

In B physics the loop functions can be expanded order by order as

$$B(p;m_1,m_2) = B^{(0)}+\frac{p^2}{m_{\mathrm{W}}^2}B^{(1)}+\cdots$$

$$C(p_1,p_2;m_1,m_2,m_2) = C^{(0)}+\frac{p_1^2+p_2^2}{m_{\mathrm{W}}^2}C^{1,(1)}+\frac{2p_1\cdot p_2}{m_{\mathrm{W}}^2}C^{2,(1)}+\cdots$$

with functions $B^{(n)}$, $C^{n,(1)}$ being independent of momenta. The definations and corresponding expansions can be found in Ref. [6].

For a cross check, it is necessary to check our result to see whether the Ward identity in on-shell $\mathrm{b}\to\mathrm{s}\gamma$ is guaranteed. Firstly, we check the leading term in effective terms, which are unsuppressed by a factor p^2/m_{W}. In this case, $p_1^2=p_{\mathrm{b}}^2=m_{\mathrm{b}}^2$, $p_2^2=p_{\mathrm{s}}^2=0$. With the aid of the loop function expansions listed in Ref. [6], it can be proven that

$$\frac{1}{3}B_1^{(0)}+\left[\frac{2}{3}\left(-m_{\mathrm{j}}^2C_0^{(0)}+\frac{1}{2}C_{24}^{(0)}-\frac{1}{2}\right)-\frac{1}{2}\hat{C_{24}^{(0)}}\right]=0. \qquad (4)$$

From Eq. (4) and Table 1, we have

$$\Gamma^{\mu,\text{Leading}}_{\text{b}\to\text{s}\gamma} = \frac{g^2}{2}e\frac{\text{i}}{16\pi^2}\gamma^\mu P_\text{L}\sum_\text{j} V_\text{jb}V^*_\text{js}[f_1^{\text{S},(0)}+f_1^{\text{T},(0)}] = 0, \quad (5)$$

where the first term comes from a self-energy contribution and the second term from a triangle contribution.

Secondly, we check the subleading terms:

$$\Gamma^{\mu,\text{Subleading}}_{\text{b}\to\text{s}\gamma} = \frac{g^2}{2}\frac{1}{16\pi^2}\sum_\text{j} V_\text{jb}V^*_\text{js} \times\left\{\left[f_2^{(0)}+f_3^{(0)}+\frac{1}{2}f_5^{(0)}\right]m_\text{b}\not{k}\gamma^\mu P_\text{R} + \left[\frac{f_1^{(1)}}{m_\text{W}^2}+\frac{1}{2}f_5^{(0)}+f_7^{(0)}\right]m_\text{b}^2\gamma^\mu P_\text{L}\right\}, \quad (6)$$

where the coefficient f is a sum of f^S and f^T.

In obtaining the above equation, we have used the motion equation for $\text{b}\to\text{s}\gamma$ and on-shell conditions, and

$$\bar{s}\not{p}_1\gamma^\mu\not{p}_1 P_\text{L}b = m_\text{b}\bar{s}\not{k}\gamma^\mu P_\text{R}b,$$
$$2m_\text{b}\bar{s}p_1^\mu P_\text{R}b = m_\text{b}\bar{s}\not{k}\gamma^\mu P_\text{R}b+m_\text{b}^2\bar{s}\gamma^\mu P_\text{L}b$$
$$m_\text{b}(\gamma^\mu\not{p}_1+\not{p}_2\gamma^\mu) = m_\text{b}^2\bar{s}\gamma^\mu P_\text{L}b. \quad (7)$$

Note that the last term in Eq. (6) receives contributions from both the self-energy and triangle diagrams. Since

$$\frac{f_1^{(1)}}{m_\text{W}^2}+\frac{1}{2}f_5^{(0)}+f_7^{(0)}=0, \quad (8)$$

we can see that at order $O(m_\text{b}^2/m_\text{W}^2)$, the on-shell effective vertex of $\text{b}\to\text{s}\gamma$ only consists of a $\not{k}\gamma^\mu$ term, which naturally satisfies the Ward identity and will contribute to the operator O_7.

We would like to point out here that it is still necessary to check the coefficient of operator O_7, $C_7(m_\text{W})$. Again, using the loop function expansions in Ref. [6], we obtain

$$C_7(m_\text{W}) = \frac{1}{2}\left[f_2^{(0)}+f_3^{(0)}+\frac{1}{2}f_5^{(0)}\right]$$
$$= \frac{1}{2}\{e_\text{u}[-2C_0-3(2+\delta_\text{j})C_{23}+(6-\delta_\text{j})C_{11}] +(-2+3\delta_\text{j})\hat{C}_{11}-3(2+\delta_\text{j})\hat{C}_{23}-\delta_\text{j}\hat{C}_0\}$$
$$= \frac{23}{36}-\frac{7\delta_\text{j}-5\delta_\text{j}^2-8\delta_\text{j}^3}{24(1-\delta_\text{j})^3}-\frac{3\delta_\text{j}^2(2-3\delta_\text{j})}{4(1-\delta_\text{j})^4}\ln\delta_\text{j}, \quad (9)$$

which is the same as that in Refs. [7, 8]. Note here that j is not yet summed and that the constant can be omitted using the unitarity of the CKM matrix V.

3 A complete calculation for $\text{b}\to\text{s}\gamma\gamma$

3.1 Effect due to off-shell quarks in vertex $\text{b}\to\text{s}\gamma$ on $\text{b}\to\text{s}\gamma\gamma$

Now we focus attention on the effect of off-shell quarks in vertex $\text{b}\to\text{s}\gamma$ on $\text{b}\to\text{s}\gamma\gamma$. As mentioned in Section 1, one of the two quarks is off-shell, while vertex $\text{b}\to\text{s}\gamma$ is used to describe $\text{b}\to\text{s}\gamma\gamma$, as shown in Fig. 2.

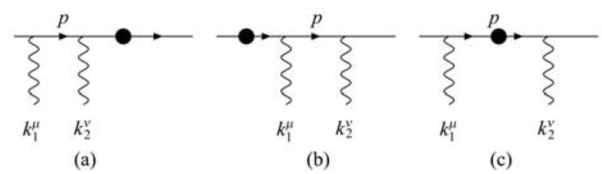

Fig. 2. Self-energy Feynman diagrams for $\text{b}\to\text{s}\gamma\gamma$. Diagrams with $\mu(k_1)\leftrightarrow\nu$, $p\to p'$ exchanges are omitted. The black circle stands for self-energy correction.

Using the effective vertex of $\text{b}\to\text{s}\gamma$ in general given in Eq. (2), with corresponding coefficients in Table 1, we express the contribution as follows:

1) Contributions from self-energy diagrams

At the lowest order, the unsuppressed terms from self-energy diagrams are independent of momentum, and thus canceled out by corresponding terms from the triangle diagrams, the same as those of on-shell $\text{b}\to\text{s}\gamma$; however, at a high order the situation changes. By combining the six pieces we obtain

$$\Delta W^\text{S}_{\mu\nu} = (-e_\text{d})^2\frac{g^2}{2}\frac{e^2}{16\pi^2}\sum_\text{j}V_\text{jb}V^*_\text{js}P_\text{R}\left\{A_1^{(1)}[\gamma_\nu\not{p}\gamma_\mu+\gamma_\mu\not{p}'\gamma_\nu]+2(A^1-A_2^{(1)})m_\text{b}g_{\mu\nu}\right\}$$
$$= e_\text{d}^2\frac{g^2}{2}\frac{e^2}{16\pi^2}\sum_\text{j}V_\text{jb}V^*_\text{js}P_\text{R}\left\{(A_1^{(1)}-2A_2^{(1)})m_\text{b}g_{\mu\nu}+A_1^{(1)}[(p_\text{b}+p_\text{s})_\mu\gamma_\nu+(p_\text{b}+p_\text{s})_\nu\gamma_\mu]-A_1^{(1)}\epsilon_{\mu\nu\alpha\beta}\gamma^\beta\right\}, \quad (10)$$

where $p=p_\text{s}+k_2$, $p'=p_\text{s}+k_1$ and A_i defined in Eq. (1). The unsuppressed terms are not presented in this equation. Note that the photons are assumed to be on-shell, quarks are off-shell and relation

$$\gamma_\mu\gamma_\nu\gamma_\alpha = g_{\mu\nu}\gamma_\alpha-g_{\mu\alpha}\gamma_\nu+g_{\nu\alpha}\gamma_\mu+\text{i}\epsilon_{\mu\nu\alpha\lambda}\gamma^\lambda\gamma^5 \quad (11)$$

has been used. From Eq. (10), it is clear that the off-shell effect can be expanded with three bases.

2) Contributions from triangle diagrams

In the following, for simplification the globe coefficient $\frac{g^2}{2}\frac{e^2}{16\pi^2}\sum_\text{j}V_\text{jb}V^*_\text{js}$ in Eq. (10) will be dropped.

After some straight calculation, we find that there is an asymmetry with a $\mu \leftrightarrow \nu$ exchange in off-shell effect. In fact, the coefficient of term $m_{\rm b}\gamma_\nu\gamma_\mu P_{\rm R}$ from Fig. 3(a), $2f_2^{\rm T}+f_3^{\rm T}+f_4^{\rm T}+\frac{1}{2}f^5+f_6^{\rm T}$, seems to be different from that of term $m_{\rm b}\gamma_\mu\gamma_\nu P_{\rm R}$ from Fig. 3(b), $\frac{f_1^{\rm T,(1)}}{m_{\rm W}}+\frac{1}{2}f^5+f_6^{\rm T}$. However, since relation

$$\frac{f_1^{\rm T,(1)}}{m_{\rm W}} = 2f_2^{\rm T,(0)}+f_3^{\rm T,(0)}+f_4^{\rm T,(0)}, \tag{12}$$

the asymmetry does not indeed exist so we can replace them by the basis of $m_{\rm b}g_{\mu\nu}P_{\rm R}$.

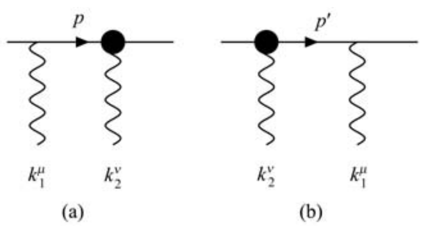

Fig. 3. Triangle Feynman diagrams for b → sγγ. Diagrams with $\mu(k_1)\leftrightarrow\nu$, $p\to p'$ exchanges are omitted.

In summing up all of the triangle diagrams' contributions, we obtain:

$$\begin{aligned}\Delta W_{\mu\nu}^{\rm T} \;=\; & -2e_{\rm d}P_{\rm R}\left\{\left[\frac{f_1^{\rm T,(1)}}{m_{\rm W}}+f_5^{\rm T}+2f_6^{\rm T}\right]m_{\rm b}g_{\mu\nu}\right.\\ & +\frac{f_1^{\rm T,(1)}}{m_{\rm W}}[(p_{\rm b}+p_{\rm s})_\mu\gamma_\nu+(p_{\rm b}+p_{\rm s})_\nu\gamma_\mu]\\ & \left.-\frac{f_1^{\rm T,(1)}}{m_{\rm W}}{\rm i}\epsilon_{\mu\nu\alpha\beta}\gamma^\alpha(k_2-k_1)^\beta\right\}. \end{aligned}\tag{13}$$

The total off-shell effect is then obtained by summing up the contributions from the self-energy and triangle diagrams. Using Eq. (12) and

$$\begin{aligned} C_0-2C_{11} \;&=\; \hat{C}_0-2\hat{C}_{11} = -B_0^{(1)},\\ f_5^{\rm T}+2f_6^{\rm T} \;&=\; 2\delta_{\rm j}[(C_0-2C_{11})e_{\rm u}-(\hat{C}_0-2\hat{C}_{11})]\\ &=\; -2e_{\rm d}A_2^{(1)}, \end{aligned}\tag{14}$$

we can write the total off-shell contribution as

$$\begin{aligned}\Delta W_{\mu\nu}^{\text{off-shell}} \;=\; & \Delta W_{\mu\nu}^{\rm S}+\Delta W_{\mu\nu}^{\rm T} = -e_{\rm d}^2P_{\rm R}\left\{[A_1^{(1)}\right.\\ & -2A_2^{(1)}]m_{\rm b}g_{\mu\nu}+A_1^{(1)}[(p_{\rm b}+p_{\rm s})_\mu\gamma_\nu\\ & +(p_{\rm b}+p_{\rm s})_\nu\gamma_\mu]+[A_1^{(1)}+6(f_2^{\rm T}\\ & \left.+f_3^{\rm T})]{\rm i}\epsilon_{\mu\nu\alpha\beta}\gamma^\alpha(k_2-k_1)^\beta\right\}. \end{aligned}\tag{15}$$

3.2 Contribution from diagrams without vertex b → sγ to b → sγγ

Compared with the contributions from diagrams with vertex b → sγ, those from the diagrams shown in Fig. 4 to b → sγγ are not neglected, at least the part from diagrams 4(a) is unsuppressed.

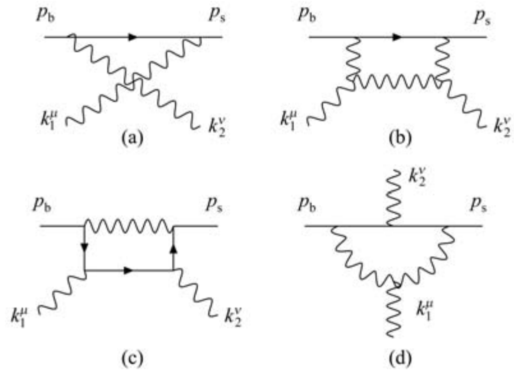

Fig. 4. Feynman diagrams without an effective vertex of b → sγ contribute to b → sγγ. Fig. (e) to (h), which are not shown here, stand for the corresponding diagrams (a) to (d) with W replaced by G, respectively. Diagrams with $\mu(k_1)\leftrightarrow\nu(k_2)$ are also omitted.

Before going into a detail calculation, it is expected that as with the calculations for the off-shell effects, the effective vertex of b → sγγ from diagrams shown in Fig. 4 can be expanded using a set of bases in Eq. (13):

$$\begin{aligned} W_{\mu\nu} \;=\; & P_{\rm R}\{a_1m_{\rm b}g_{\mu\nu}+a_2[(p_{\rm b}+p_{\rm s})_\mu\gamma_\nu+(p_{\rm b}+p_{\rm s})_\nu\gamma_\mu]\\ & +a_3{\rm i}\epsilon_{\mu\nu\alpha\beta}\gamma^\alpha(k_2-k_1)^\beta\}. \end{aligned}\tag{16}$$

We have extracted the coefficients for each diagram and list them in Table 2 so that they may be checked step by step. The total result is a sum of the contributions.

Keeping the functions up to order $O(m_{\rm b}^2/m_{\rm W}^2)$ for consistency in our calculation, we use the loop function D in the expression and denote $D=D(m_{\rm j}, m_{\rm W}, m_{\rm W}, m_{\rm W})$, $\hat{D}=D(m_{\rm W}, m_{\rm j}, m_{\rm j}, m_{\rm j})$ and $\tilde{D}=D(m_{\rm j}, m_{\rm j}, m_{\rm W}, m_{\rm W})$. It is noticeable that the contributions from Fig. 4(d) and the corresponding one with W replaced by G, i.e., (h) seem to be asymmetric for $p_{\rm b}\leftrightarrow p_{\rm s}$ exchanges. Indeed, we find that the coefficients of the term $\gamma^\mu p_{\rm b}^\nu+\gamma^\nu p_{\rm b}^\mu$ from Fig. 4(d) and (h) are

$$-e_{\rm u}[16\tilde{D}_{311}+24\tilde{D}_{312}-3\tilde{D}_{27}+4m_{\rm j}^2(\tilde{D}_0+\tilde{D}_{11}-2\tilde{D}_{12})]$$

and

$$\delta_{\rm j}e_{\rm u}\left[-8\tilde{D}_{311}-12\tilde{D}_{312}+\frac{1}{2}\tilde{D}_{27}-2m_{\rm j}^2(\tilde{D}_0-\tilde{D}_{11}-2\tilde{D}_{12})\right],$$

respectively. We can prove that

$$8(\tilde{D}_{311}+\tilde{D}_{312}) \;=\; \tilde{D}_{27}, \tag{17}$$

$$\tilde{D}_{11}+\tilde{D}_{12} \;=\; \tilde{D}_0. \tag{18}$$

Table 2. Coefficients a_i in the expression of an effective vertex $b\to\gamma\gamma$. The first four lines correspond to the contributions from the diagrams displayed in Fig. 4, and the last four represent the contributions from the diagrams with W replaced by G, respectively.

diagram	a_1	a_2	a_3
(a)	$-4C_{11}$	0	0
(b)	$24D_{311}$	$24D_{311}-2D_{27}$	$-4D_{27}$
(c)	$-e_u^2[-12\hat{D}_{311}+\hat{D}_{27}-2m_j^2(\hat{D}_0+\hat{D}_{11})]$	$-e_u^2[-12\hat{D}_{311}+\hat{D}_{27}-2m_j^2(\hat{D}_0-3\hat{D}_{11})]$	$e_u^2[12\hat{D}_{311}-\hat{D}_{27}+2m_j^2(\hat{D}_0-3\hat{D}_{11})]$
(d)	$16e_u\tilde{D}_{312}$	$e_u[16\tilde{D}_{311}+8\tilde{D}_{312}-\tilde{D}_{27}-4m_j^2\tilde{D}_{11}]$	$e_u\tilde{D}_{27}$
(e)	$2\delta_j[\hat{C}_0-\hat{C}_{11}]$	0	0
(f)	$\delta_j[12D_{311}-2D_{27}]$	$\delta_j[12D_{311}-D_{27}]$	0
(g)	$\delta_j e_u^2\left[6\hat{D}_{311}-\frac{1}{2}\hat{D}_{27}-m_j^2(\hat{D}_0+3\hat{C}_{11})\right]$	$\delta_j e_u^2\left[6\hat{D}_{311}-\frac{1}{2}\hat{D}_{27}+m_j^2(\hat{D}_0-3\hat{D}_{11})\right]$	$-\delta_j e_u^2\left[6\hat{D}_{311}-\frac{1}{2}\hat{D}_{27}-m_j^2(\hat{D}_0+\hat{C}_{11})\right]$
(h)	$\delta_j e_u[8\tilde{D}_{312}-2\tilde{D}_{27}]$	$\delta_j e_u\left[8\tilde{D}_{311}+4\tilde{D}_{312}-\frac{1}{2}\tilde{D}_{27}-2m_j^2\tilde{D}_{11}\right]$	$\delta_j e_u\left[\frac{1}{2}\tilde{D}_{27}-4m_j^2\tilde{D}_0\right]$

This relation ensures that the bases introduced above are enough for the result from diagrams with vertex $b\to s\gamma$.

4 Discussions

We would like to make some remarks here regarding our results:

1) The effects due to off-shell quarks in vertex $b\to s\gamma$ on $b\to s\gamma\gamma$ in Eq. (15) are large ;

2) From Eq. (16) it can clearly be seen that the contributions from diagrams without an effective vertex $b\to s\gamma$ to $b\to s\gamma\gamma$ are not negligible compared with others.

We now need to check the result to see whether the Ward identity in $b\to s\gamma$ is guaranteed. This implies that the coefficients of the first two bases should be zero. If the coefficient of the last bases does not disappear, to describe $b\to s\gamma\gamma$ the last operator should be added to a set of bases for $b\to s\gamma$ without violating the Ward identity. From Eqs. (15) and (16), one can check that

$$a_1-e_d^2[A_1^{(1)}-2A_1^{(1)}] = 0, \quad (19)$$

$$a_2-e_d^2A_1^{(1)} = 0, \quad (20)$$

$$a_3-e_d^2[A_1^{(1)}+6(f_2^T+f_3^T)] = 0, \quad (21)$$

where a_i as a sum of the corresponding values in Table 2 is understood. Therefore we can draw a conclusion:

3) The off-shell effect and contribution from diagrams without vertex $b\to s\gamma$ cancel each other out exactly, so the operators of $b\to s\gamma$ can safely be used to describe $b\to s\gamma\gamma$.

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