Hadrons from a hard wall AdS/QCD model*

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Abstract We review a recent work on masses of mesons and nucleons within a hard wall model of holographic QCD in a unified approach. In order to treat meson and nucleon properties on the same footing, we introduced the same infrared (IR) cut for both sectors. In framework of present approach, the best fit to the experimental data is achieved introducing the anomalous dimension for baryons.

Key words AdS/QCD, nucleons, mesons

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1 Introduction

After the realization that a string theory in anti-de Sitter (AdS) space corresponds to the strongly coupled conformal field theory (CFT) in its boundary [1–3], several models inspired by this AdS/CFT correspondence have been developed to investigate hadron properties in the low-energy region. In general, these models can be divided into two categories: In the first approach, so-called "top-down" approach, one starts from the fundamental (string/M) theory living on $\mathrm{AdS}_{d+1}\times\mathcal{C}$ (where $\mathcal{C}$ is a compact manifold) and tries to formulate an effective theory which is supposed to describe low energy phenomena in strongly interacting systems. In the second approach, so-called "bottom-up" approach, one starts from the 5 dimensional (5D) phenomenological Lagrangian, which incorporates as much as possible properties of the fundamental theory (QCD) and known low-energy phenomenological facts, and attempts to extend the model by constructing its dual (which include the extra dimensions too). For the detailed descriptions of mesons in gauge/gravity duality in both approaches, refer to a recent review(for example, see Ref. [4]).

It is well known that QCD is not a conformal theory. Consequently, the different approaches have been developed in the "bottom-up" approach, where one has some free parameters in a constructed effective model. Clearly, these free parameters are usually related to phenomenologically well known quantities. One of such parameters is the size of the extra dimension (compactification scale), which is fixed and related to the QCD scale Λ_{QCD}. Naturally, the compactification breaks the conformal invariance and these types of models are coined as hard-wall models [5, 6].

2 Lagrangian of the hard-wall model

We start from a hard-wall model for mesons developed in Refs. [5, 6] and study its applications to nucleons [7, 8]. The model has a geometry of 5D AdS

$$\mathrm{d}s^2 = g_{MN}\mathrm{d}x^M\mathrm{d}x^N = \frac{1}{z^2}(\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu - \mathrm{d}z^2), \qquad (1)$$

where $\eta_{\mu\nu}$ stands for the 4D Minkowski metric: $\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$. The 5D AdS space is compactified by two different boundary conditions, i.e. the IR boundary at $z = z_m$ and the UV one at $z = \epsilon \to 0$. Considering the global chiral symmetry $SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}}$ of QCD, we need to introduce 5D

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local gauge fields $A_{\rm L}$ and $A_{\rm R}$ of which the values at $z=0$ play a role of external sources for $SU(2)_{\rm L}$ and $SU(2)_{\rm R}$ currents respectively. Since chiral symmetry is known to be broken to $SU(2)_{\rm V}$ spontaneously as well as explicitly, we introduce a bi-fundamental field X with respect to the local gauge symmetry $SU(2)_{\rm L}\times SU(2)_{\rm R}$, in order to realize the spontaneous and explicit breakings of chiral symmetry in the AdS side. Considering these two, we can construct the bi-fundamental 5D bulk scalar field X in terms of the current quark mass $m_{\rm q}$ and the quark condensate σ

$$X_0(z)=\langle X\rangle=\frac{1}{2}(m_{\rm q}z+\sigma z^3), \quad (2)$$

with isospin symmetry assumed.

The 5D gauge action in AdS space with the scalar bulk field and the vector field can be expressed as

$$\begin{aligned} S_{\rm M} &= \int {\rm d}^4x\int {\rm d}z\,\mathcal{L}_{\rm M}\,, \\ \mathcal{L}_{\rm M} &= \frac{1}{z^5}\,{\rm Tr}\left[|DX|^2+3|X|^2-\frac{1}{2g_5^2}(F_{\rm L}^2+F_{\rm R}^2)\right], \quad (3)\end{aligned}$$

where covariant derivative and field strength tensors are defined as $DX=\partial X-{\rm i}A_{\rm L}X+{\rm i}XA_{\rm R}$ and $F^{\rm MN}_{\rm L,R}=\partial^{\rm M}A_{\rm L,R}-\partial^{\rm N}A_{\rm L,R}-{\rm i}[A^{\rm M}_{\rm L,R},A^{\rm N}_{\rm L,R}]$. The 5D gauge coupling g_5 is fixed by matching the 5D vector correlation function to that from the operator product expansion (OPE): $g_5^2=12\pi^2/N_c$. The 5D mass of the bulk gauge field $A_{\rm L,R}$ is determined by the relation $m_5^2=(\Delta-p)(\Delta+p-4)$ [2, 3] where Δ denotes the canonical dimension of the corresponding operator with spin p and turns out to be $m_5^2=0$ because of gauge symmetry. The effective action (3) describes the mesonic sector [5, 6] completely apart from exotic mesons [9].

To consider baryons in the flavor-two ($N_{\rm F}=2$) sector, one needs to introduce a bulk Dirac field corresponding to the nucleon at the boundary [7, 8]. This hard-wall model was applied to describe the neutron electric dipole moment [10] and holographic nuclear matter [11]. In this model, the nucleons are the massless chiral isospin doublets $(p_{\rm L},n_{\rm L})$ and $(p_{\rm R},n_{\rm R})$ which satisfy the 't Hooft anomaly matching. Then the spontaneous breakdown of chiral symmetry induces a chirally symmetric mass term for nucleons

$$\mathcal{L}_{\chi{\rm SB}}\sim -M_{\rm N}\begin{pmatrix}\bar p_{\rm L}\\ \bar n_{\rm L}\end{pmatrix}\Sigma\,(p_{\rm R},n_{\rm R})+{\rm h.c.}, \quad (4)$$

where $\Sigma=\exp(2{\rm i}\pi^{\rm a}\tau^{\rm a}/f_\pi)$ is the nonlinear pseudo-Goldstone boson field that transforms as $\Sigma\to U_{\rm L}\Sigma U_{\rm R}^\dagger$ under $SU(2)_{\rm L}\times SU(2)_{\rm R}$. The $\tau^{\rm a}$ and f_π represent the $SU(2)$ Pauli matrices and the pion decay constant, respectively. Thus, we have to consider the following mass term in the AdS side

$$\mathcal{L}_{\rm I}=-g\begin{pmatrix}\bar p_{\rm L}\\ \bar n_{\rm L}\end{pmatrix}X\,(p_{\rm R},n_{\rm R})+{\rm h.c.}, \quad (5)$$

where g denotes the mass coupling (or Yukawa coupling) between X and nucleon fields, which is usually fitted by reproducing the nucleon mass $M_{\rm N}=940$ MeV. In this regard, we can introduce two 5D Dirac spinors N_1 and N_2 of which the Kaluza-Klein (KK) modes should include the excitations of the massless chiral nucleons $(p_{\rm L},n_{\rm L})$ and $(p_{\rm R},n_{\rm R})$, respectively. By this requirement, one can fix the IR boundary conditions for N_1 and N_2 at $z=z_m$.

Note that the 5D spinors $N_{1,2}$ do not have chirality. However, one can resolve this problem in such a way that the 4D chirality is encoded in the sign of the 5D Dirac mass term. For a positive 5D mass, only the right-handed component of the 5D spnior remains near the UV boundary $z\to 0$, which plays the role of a source for the left-handed chiral operator in 4 dimension. It is vice versa for a negative 5D mass. The 5D mass for the $(d+1)$ bulk dimensional spinor is determined by the AdS/CFT expression

$$(m_5)^2=\left(\Delta-\frac{d}{2}\right)^2, \quad (6)$$

and turns out to be $m_5=5/2$. However, since QCD does not have conformal symmetry in the low-energy regime, the 5D mass might acquire an anomalous dimension due to a 5D renormalization flow. Though it is not known how to derive it, we will introduce some anomalous dimension to see its effects on the spectrum of the nucleon.

Summarizing all these facts, we are led to the 5D gauge action for the nucleons

$$\begin{aligned} S_{\rm N} &= \int {\rm d}^4x\int {\rm d}z\,\frac{1}{z^5}\,{\rm Tr}\left[\mathcal{L}_{\rm K}+\mathcal{L}_{\rm I}\right], \\ \mathcal{L}_{\rm K} &= {\rm i}\bar N_1\Gamma^{\rm M}\nabla_{\rm M}N_1+{\rm i}\bar N_2\Gamma^{\rm M}\nabla_{\rm M}N_2- \\ &\quad \frac{5}{2}\bar N_1N_2+\frac{5}{2}\bar N_2N_1, \\ \mathcal{L}_{\rm I} &= -g\left[\bar N_1XN_2+\bar N_2X^\dagger N_1\right], \quad (7)\end{aligned}$$

where

$$\nabla_{\rm M}=\partial_{\rm M}+\frac{\rm i}{4}\,\omega^{\rm AB}_{\rm M}\Gamma_{\rm AB}-{\rm i}A^{\rm L}_{\rm M}\,. \quad (8)$$

The non-vanishing components of the spin connection are $w^{\rm 5A}_{\rm M}=-w^{\rm A5}_{\rm M}=\delta^{\rm A}_{\rm M}/z$ and $\Gamma_{\rm AB}=\frac{1}{2{\rm i}}[\Gamma^{\rm A},\Gamma^{\rm B}]$ are the Lorentz generators for spinors. The Γ matrices are related to the ordinary γ matrices as $\Gamma^{\rm M}=(\gamma^\mu,-{\rm i}\gamma_5)$.

The details of the calculations within the present approach can be found in Ref. [8].

3 Results

Presenting our results we note, that the most of input parameters of the model such as $m_{\rm q}$, σ and $z_{\rm m}$ are can be well fitted from the data in the mesonic sector [5]. In order to reproduce the data in the baryonic sector we have only one free parameter g. Our calculations showed that this additional parameter alone, which is bounded to some region $|g|<g_{\rm crit}$ [8], is not enough to reproduce the experimental data.

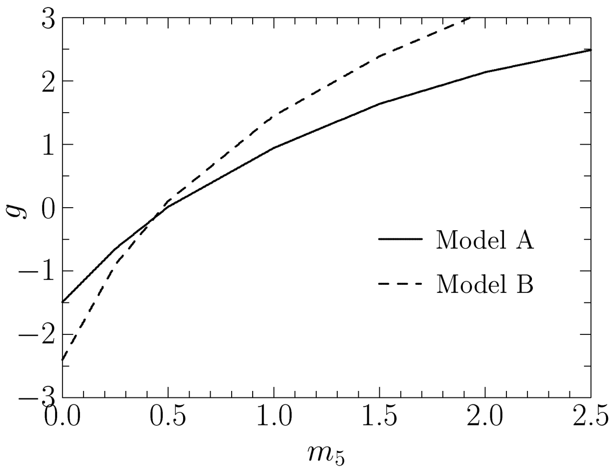

Fig. 1. Mass coupling g dependence on the renormalized 5D mass m_5. The parameters for models A (solid curve) and B (dashed one) are taken from Ref. [5].

For example, with the given sets of input parameter sets A and B taken from the mesonic sector [5], the reproduced baryon masses are to high: low lying nucleon state mass is $m_{\rm N}$ = 2050 MeV (Model A) and $m_{\rm N}$ = 2196 MeV (Model B) in contrast to the experimental value 940 MeV. One has to look for additional possibilities in order to reproduce experimental results and one of such possibilities is to introduce the anomalous dimension for baryons as we mentioned above. Although it is not obvious to calculate an anomalous dimension, it is known that it renormalizes the 5D mass. Taking this assumption into account, one can consider m_5 as a free parameter.

The results are drawn in Fig. 1 and Fig. 2. One can see that, when anomalous dimension is set to zero (i.e. $m_5=5/2$ is fixed), the experimental data is badly reproduced. The inclusion of an additional parameter (i.e. considering m_5 as a free parameter) improves the output data well but leads to larger values of the possible anomalous dimension.

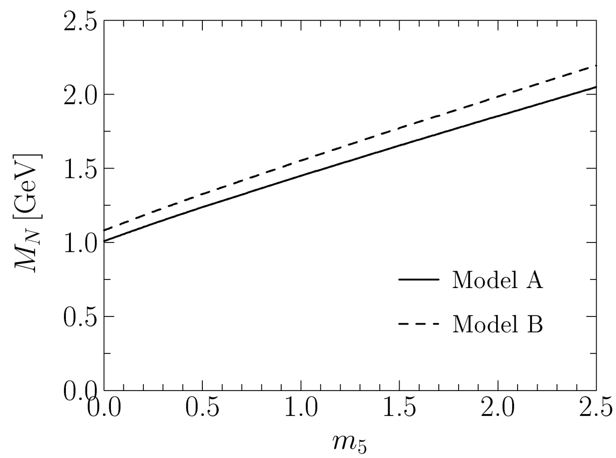

Fig. 2. The mass of the lowest-lying nucleon (in GeV) as a function of the renormalized 5D mass m_5. The notations are same as in Fig. 1.

The best fit to experimental data in the chiral limit is reproduced with the values of input parameters $z_m^{-1}=285$ MeV, $\sigma^{1/3}=227$ MeV, and $g=-9.6$, as listed in Table 1.

Table 1. The best fit to the experimental data. 5D mass is equal to zero due to the large anomalous dimension.

	$(\rm p,n)^+$	$\rm N^+$	$\rm N^-$	ρ	ρ
experiment	939	1440	1535	776	1475
this model	930	1826	1856	685	1573

One can note that the masses of nonstrange baryons are well reproduced, within (10~20) % deviations from the experimental data.

References

1 Maldacena J M. Adv. Theor. Math. Phys., 1998, **2**: 231–252
2 Gubser S S, Klebanov I R, Polyakov A M. Phys. Lett. B, 1998, **428**: 105–114
3 Witten E. Adv. Theor. Math. Phys., 1998, **2**: 253–291
4 Erdmenger J, Evans N, Kirsch I, Threlfall E. Eur. Phys. J. A, 2008, **35**: 81–133
5 Erlich J, Katz E, Son D T, Stephanov M A. Phys. Rev. Lett., 2005, **95**: 261602(4)
6 Rold D L, Pomarol A. Nucl. Phys. B, 2005, **721**: 79–97
7 Hong D K, Inami T, Yee H U. Phys. Lett. B, 2007, **646**: 165–171
8 Kim H Ch, Kim Y, Yakhshiev U T. JHEP, 2009, **0911**: 034(15)
9 Kim H Ch, Kim Y. JHEP, 2009, **0901**: 034(10)
10 Hong D K, Kim H Ch, Siwach S, Yee H U. JHEP, 2007, **0711**: 036(19)
11 Kim Y, Lee H C, Yee H U. Phys. Rev. D, 2008, **77**: 085030(8)