Exotic pentaquarks as Gamov–Teller resonances[*]

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Abstract If the number of colors N_c is taken large, baryons and their excitations can be considered in a mean-field approach. We argue that the mean field in baryons breaks spontaneously the spherical and $SU(3)$ flavor symmetries, but retains the $SU(2)$ symmetry of simultaneous rotations in space and isospace. The one-quark and quark-hole excitations in the mean field, together with the $SU(3)$ rotational bands about them determine the spectrum of baryon resonances, which turns out to be in good accordance with reality when one puts $N_c = 3$. A by-product of this scheme is a confirmation of the light pentaquark Θ^+ baryon uudd$\bar{s}$ as a typical Gamov–Teller resonance long known in nuclear physics. An extension of the same large-N_c logic to charmed baryons leads to a prediction of a anti-decapenta ($\overline{15}$)-plet of charmed pentaquarks, two of which, $\mathcal{B}_c^{++} = $ cuud$\bar{s}$ and $\mathcal{B}_c^{+} = $ cudd$\bar{s}$, may be light and stable with respect to strong decays.

Key words mean field, baryon resonances, exotics, charmed baryons

PACS 12.39.Ki, 14.20.Dh, 14.20.Jn

1 Relativistic mean field

It has been argued 30 years ago by Witten [1] that if the number of colors N_c is large, the N_c quarks of a baryon can be viewed as moving in a mean field. It is helpful to understand how baryons look like in the large-N_c limit, before $1/N_c$ corrections are considered.

At the microscopic level quarks experience only color interactions, however large N_c do not suppress gluon fluctuations: the mean field can be only 'colorless'. An example how originally color interactions are Fiertz-transformed into interactions of quarks with mesonic fields are provided by the instanton liquid model [2].

We shall thus assume that quarks in the large-N_c baryon obey the Dirac equation in a background mesonic field since there are no reasons to expect quarks to be non-relativistic, especially in excited baryons. In a most general case the background field couples to quarks through all five Fermi variants. If the background field is stationary in time, it leads to the eigenvalue equation for the u, d, s quarks in the background field:

$$H\psi = E\psi,$$

$$H = \gamma^0\Big(-i\partial_i\gamma^i + S(x) + P(x)i\gamma^5 +$$
$$V_\mu(x)\gamma^\mu + A_\mu(x)\gamma^\mu\gamma^5 + T_{\mu\nu}(x)\frac{i}{2}[\gamma^\mu\gamma^\nu]\Big), \quad (1)$$

where S, P, V, A, T are the mean fields that are matrices in flavor. In fact, the one-particle Dirac Hamiltonian (1) is generally nonlocal, however that does not destroy symmetries in which we are primarily interested. We include dynamically-generated quarks masses into the scalar term S.

The key issue is the symmetry of the mean field. From the large-N_c point of view, the current strange quark mass is very small, $m_s = \mathcal{O}(1/N_c^2)$ [4], therefore a good starting point is exact $SU(3)$ flavor symmetry. A natural assumption, then, would be that the mean field is flavor-symmetric, and spherically symmetric. This assumption, however, leads to too many "missing resonances" in the spectrum. In addition, we know that baryons are strongly coupled to pseudoscalar mesons ($g_{\pi NN} \approx 13$). It means that there is a large pseudoscalar field inside baryons; at large N_c it is a classical mean field. There is no way of writing down the pseudoscalar field that would be compatible with the $SU(3)_{\text{flav}} \times SO(3)_{\text{space}}$ symmetry. The minimal extension of spherical symmetry is to write the "hedgehog" *Ansatz* "marrying" the isotopic and

space axes:

$$\pi^a(x) = \begin{cases} n^a\, F(r),\ n^a = \dfrac{x^a}{r}, & a = 1,2,3, \\ 0, & a = 4,5,6,7,8. \end{cases} \quad (2)$$

This *Ansatz* breaks the $SU(3)_{\text{flav}}$ symmetry. Moreover, it breaks the symmetry under independent space $SO(3)_{\text{space}}$ and isospin $SU(2)_{\text{iso}}$ rotations, and only a simultaneous rotation in both spaces remains a symmetry, since a rotation in the isospin space labeled by a, can be compensated by the rotation of the space axes. Therefore, the *Ansatz* (2) breaks spontaneously the original $SU(3)_{\text{flav}} \times SO(3)_{\text{space}}$ symmetry down to the $SU(2)_{\text{iso+space}}$ symmetry. It is analogous to the spontaneous breaking of spherical symmetry by the ellipsoid form of many nuclei.

2   Quarks in the 'hedgehog' mean field

We shall call the $SU(2)_{\text{iso+space}}$ symmetry of the mean field the "hedgehog symmetry". What mesonic fields S, P, V, A, T in Eq. (1) are compatible with this symmetry? Since $SU(3)$ symmetry is broken, all fields can be divided into three categories:

I. Isovector fields acting on u, d quarks

$$\begin{aligned} \text{pseudoscalar}: P^a(x) &= n^a\, P_0(r), & (3) \\ \text{vector}: V_i^a(x) &= \epsilon_{aik}\, n_k\, P_1(r), \\ \text{axial}: A_i^a(x) &= \delta_{ai}\, P_2(r) + n_a n_i\, P_3(r), \\ \text{tensor}: T_{ij}^a(x) &= \epsilon_{aij}\, P_4(r) + \epsilon_{bij}\, n_a n_b\, P_5(r). \end{aligned}$$

II. Isoscalar fields acting on u, d quarks

$$\begin{aligned} \text{scalar}: S(x) &= Q_0(r), & (4) \\ \text{vector}: V_0(x) &= Q_1(r), \\ \text{tensor}: T_{0i}(x) &= n_i\, Q_2(r). \end{aligned}$$

III. Isoscalar fields acting on s quarks

$$\begin{aligned} \text{scalar}: S(x) &= R_0(r), & (5) \\ \text{vector}: V_0(x) &= R_1(r), \\ \text{tensor}: T_{0i}(x) &= n_i\, R_2(r). \end{aligned}$$

All the rest fields and components are zero as they do not satisfy the $SU(2)$ symmetry and/or the needed discrete C, P, T symmetries. The 12 'profile' functions $P_{0,1,2,3,4,5}$, $Q_{0,1,2}$ and $R_{0,1,2}$ should be eventually found self-consistently from the minimization of the mass of the ground-state baryon. However, even if we do not know those profiles, there are important consequences of this *Ansatz* for the baryon spectrum.

Given the *Ansatz*, the Hamiltonian (1) actually splits into two: one for s quarks and the other for u, d quarks. The former commutes with the angular momentum of s quarks, $J = L + S$, and with the inversion

of spatial axes, hence all energy levels are characterized by half-integer J^P and are $(2J+1)$-fold degenerate. The latter commutes only with the 'grand spin' $K = T + J$ and with inversion, hence the u, d quark levels have definite integer K^P and are $(2K+1)$-fold degenerate. The energy levels for u, d quarks on the one hand and for s quarks on the other are completely different, even in the chiral limit $m_s \to 0$.

All energy levels, both positive and negative, are probably discrete owing to confinement. Indeed, a continuous spectrum would correspond to a situation when quarks are free at large distances from the center, which contradicts confinement. [One can model confinement by forcing the effective quark masses to grow at infinity, e.g. $Q_0(x) \sim R_0(x) \sim \sigma r$.]

According to the Dirac theory, all negative-energy levels, both for s and u, d quarks, have to be fully occupied, corresponding to the vacuum. It means that there must be exactly N_c quarks antisymmetric in color occupying all (degenerate) levels with J_3 from $-J$ to J, or K_3 from $-K$ to K; they form closed shells that do not carry quantum numbers. Filling in the lowest level with $E > 0$ by N_c quarks makes a baryon [3, 4], see Fig. 1.
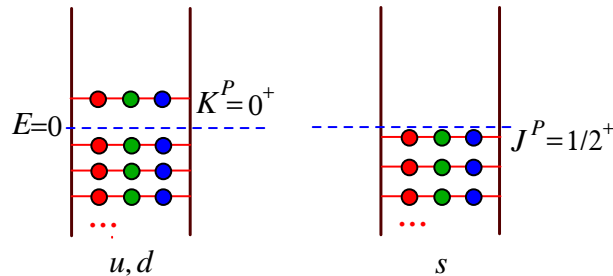

Fig. 1. (color online). Filling u, d, s shells for the ground-state baryons: $(8, 1/2^+)$, $(10, 3/2^+)$.

The mass of a baryon is the aggregate energy of all filled states, and being a functional of the mesonic field it is proportional to N_c since all quark levels are degenerate in color. Therefore quantum fluctuations of mesonic field in baryons are suppressed as $1/N_c$ so that the mean field is indeed justified.

Quantum numbers of the lightest baryons are determined from the quantization of the rotations of the mean field, leading to specific $SU(3)$ multiplets that reduce at $N_c = 3$ to the octet with spin $\dfrac{1}{2}$ and the decuplet with spin $\dfrac{3}{2}$, see e.g. [5]. Witten's quantization condition $Y' = \dfrac{N_c}{3}$ [6] follows trivially from the fact that there are N_c u, d valence quarks each with

the hypercharge $\frac{1}{3}$ [7]. Therefore, the ground state shown in Fig. 1 entails in fact 56 rotational states. The splitting between the centers of the multiplets $(8, \frac{1}{2}^{+})$ and $(10, \frac{3}{2}^{+})$ is $\mathcal{O}(1/N_c)$, and the splittings inside multiplets can be determined as a perturbation in m_s [7].

3   Excited states in the mean field

The lowest baryon resonance beyond the rotational excitations of the ground state is the singlet $\Lambda(1405, \frac{1}{2}^{-})$. Apparently, it can be obtained only as an excitation of the s quark, and its quantum numbers must be $J^P = \frac{1}{2}^{-}$ [4], see transition 1 in Fig. 2.

The existence of an $\frac{1}{2}^{-}$ level for s quarks automatically implies that there is a particle-hole excitation of this level by an s quark from the $\frac{1}{2}^{+}$ level. We identify this transition 2 with N(1535, $\frac{1}{2}^{-}$) [4]. It is predominantly a pentaquark state u(d)uds$\bar{s}$ (at $N_c = 3$). This explains its large branching ratio in the ηN decay [8], a long-time mystery. We also see that, since the highest filled level for s quarks is lower than the highest filled level for u, d quarks, N(1535, $\frac{1}{2}^{-}$) must be heavier than $\Lambda(1405, \frac{1}{2}^{-})$: the opposite prediction of the nonrelativistic quark model has been always of some concern. Subtracting $1535 - 1405 = 130$, we find that the $\frac{1}{2}^{+}$ s-quark level is approximately 130 MeV lower in energy than the valence 0^+ level for u, d quarks.

The low-lying Roper resonance N(1440, $\frac{1}{2}^{+}$) requires an excited one-particle u, d state with $K^P = 0^+$ [4], see transition 3. Just as the ground state nucleon, it is part of the excited $(8', \frac{1}{2}^{+})$ and $(10', \frac{3}{2}^{+})$ split as $1/N_c$.

Given that there is an excited 0^+ level for u, d quarks, one can put there an s quark as well, taking it from the s-quark $\frac{1}{2}^{+}$ shell, see transition 4. It is a particle-hole excitation with the valence u, d level left untouched, its quantum numbers being $S = +1$, $T = 0$, $J^P = \frac{1}{2}^{+}$. At $N_c = 3$ it is a pentaquark state uudd$\bar{s}$, precisely the exotic Θ^+ baryon predicted in Ref. [9] from other considerations. The quantization of its rotations produces the antidecuplet $(\overline{10}, \frac{1}{2}^{+})$. In our original prediction the $\mathcal{O}(1)$ gap between Θ^+ and the nucleon was due to the rota-

tional energy only, whereas here the main $\mathcal{O}(1)$ part of that gap is due to the one-particle levels, while the rotational energy is $\mathcal{O}(1/N_c)$. Methodologically, it is more satisfactory.
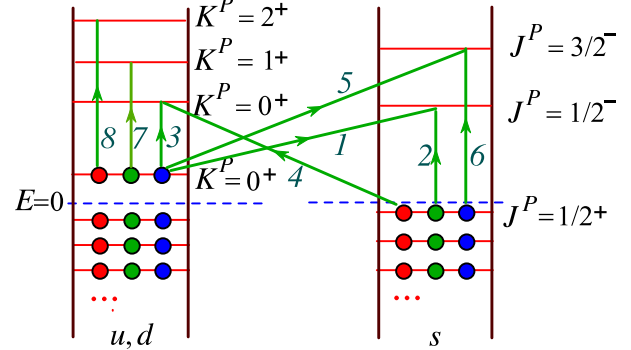

Fig. 2. (color online). All baryon resonances below 2 GeV follow from this scheme of one-quark levels. The transitions shown by arrows correspond to: 1: $\Lambda(1405, 1/2^-)$, 2: N(1535, $1/2^-$), 3: N(1440, $1/2^+$), 4: $\Theta^+(1530, 1/2^+)$, 5: $\Lambda(1520, 3/2^-)$, 6: N(1650, $1/2^-$?), 7: N(1710, $3/2^+$), 8: N(1680, $5/2^+$). Other resonances belong to $SU(3)$ multiplets obtained as rotational excitations of these one-particle and particle-hole excitations.

In nuclear physics, excitations generated by the axial current $j_{\mu 5}^{\pm}$, when a neutron from the last occupied shell is sent to an unoccupied proton level or v.v. are known as Gamov–Teller transitions. Thus our interpretation of the Θ^+ is that it is a Gamov–Teller-type resonance long known in nuclear physics.

An unambiguous feature of our picture is that the exotic pentaquark is a consequence of the three well-known resonances and must be light. Indeed, the Θ^+ mass can be estimated from the sum rule [4]: $m_\Theta \approx 1440 + 1535 - 1405 \approx 1570$ MeV, however there are $\mathcal{O}(m_s)$ corrections to this equation.

To account for higher baryon resonances one has to assume that there are higher one-particle excitations, both in the u, d- and s-quark sectors, shown in Fig. 2. It is easy to obtain that order of levels under mild assumptions about the profile functions (3)–(5).

The original $SU(3)_{\text{flav}} \times SO(3)_{\text{space}}$ symmetry is restored when flavor and space rotations are accounted for. Each transition in Fig. 2 generally entails "rotational bands" of $SU(3)$ multiplets with definite spin and parity. The short recipe of getting them is: Find the hypercharge Y' from the number of u, d, s quarks involved; only those multiplets are allowed that contain this Y'. Take an allowed multiplet and read off the isospin(s) T' of particles at this value of Y'. The allowed spin of the multiplet obeys the angular momentum addition law: $\boldsymbol{J = T' + J_1 + J_2 + K_1 + K_2}$

where $J_{1,2}$ and $K_{1,2}$ are the initial and final momenta of the s and u, d shells involved in the transition, respectively. The mass of the center of a multiplet does not depend on J but only on T' according to the relation [10]

$$\mathcal{M} = \mathcal{M}_0 + \frac{C_2(p,q) - T'(T'+1) - \frac{3}{4}Y'^2}{2I_2} + \frac{T'(T'+1)}{2I_1},$$

where $C_2(p,q) = \frac{1}{3}(p^2 + q^2 + pq) + p + q$ is the quadratic Casimir eigenvalue of the multiplet, $I_{1,2} = \mathcal{O}(N_c)$ are moments of inertia. After the rotational band for a given transition is constructed, one has to check if the rotational energy of a particular multiplet is $\mathcal{O}(1/N_c)$ and not $\mathcal{O}(1)$, and if it is compatible with Fermi statistics at $N_c = 3$: some a priori possible multiplets drop out. One gets a satisfactory description of all baryon resonances up to about 2 GeV, to be published elsewhere.

4   Charmed and beauty baryons

If one of the u, d quarks in a light baryon is replaced by a heavy b or c quark, there are still $N_c - 1$ u, d quarks left. At large N_c, they form the same mean field as in light baryons, with the same sequence of Dirac levels (up to $1/N_c$ corrections). The heavy quark contributes to the mean $SU(3)$-symmetric field but it is a $1/N_c$ correction, too.

The filling of Dirac levels for the ground-state c (or b) baryon is shown in Fig. 3: there is a hole in the 0^+ shell for u, d quarks. Quantizing rotations of this state leads to the following $SU(3)$ multiplets: $(\bar{3}, 1/2^+)$, $(6, 1/2^+)$ and $(6, 3/2^+)$. The last two are degenerate whereas the first is split from the rest by $\mathcal{O}(1/N_c)$. The splitting inside multiplets is $\mathcal{O}(m_s N_c)$.
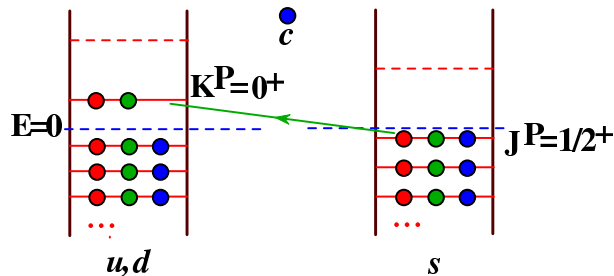
Fig. 3. (color online). Filling u, d, s shells for the ground-state charmed baryons, $(\bar{3}, 1/2^+)$, $(6, 1/2^+)$ and $(6, 3/2^+)$. The arrow shows the Gamov–Teller excitation leading to charmed pentaquarks forming $(\overline{15}, 1/2^+)$.

There are good candidates for those ground-state multiplets: $\Lambda_c(2287)$ and $\Xi_c(2468)$ for $(\bar{3}, 1/2^+)$; $\Sigma_c(2455)$, $\Xi_c(2576)$ and $\Omega_c(2698)$ for $(6, 1/2^+)$; finally $\Sigma_c(2520)$, $\Xi_c(2645)$ and $\Omega_c(2770)$ presumably

form $(6, 3/2^+)$. There are $\bar{3}$'s and 6's with parity minus arising from exciting the $1/2^-$ s-quark level. The lightest are the degenerate singlets, presumably $\Lambda_c(2595, 1/2^-)$ and $\Lambda_c(2625, 3/2^-?)$.

Our new observation is that there is a Gamov–Teller-type transition of an s quark from the $\frac{1}{2}^+$ shell to a u or d quark in the 0^+ shell, like in the case of the Θ^+. In heavy baryons it is even more simple as there is a hole in the u, d 0^+ valence shell from the start. Filling in this hole means making charmed pentaquarks which we call "beta baryons". $\mathcal{B}_c^+ = \mathrm{cudd\bar{s}}$ and $\mathcal{B}_c^{++} = \mathrm{cuud\bar{s}}$. These pentaquarks are members of the anti-decapenta-plet $(\overline{15}, 1/2^+)$, Fig. 4. In fact there must be two additional nearly degenerate $\overline{15}$-plets with spin $1/2^+$ and spin $3/2^+$.

Charmed pentaquarks have been considered by Wu and Ma in another approach [11]; however, they get far larger masses and in addition pentaquarks with $\bar{c}$ quarks appear almost degenerate with those made of c quarks. In our picture the lightest $\bar{c}$ pentaquarks Θ_c arise from putting the fourth (s) quark at the $\frac{1}{2}^-$ level; they form a quadruplet, have parity minus, and are much heavier.

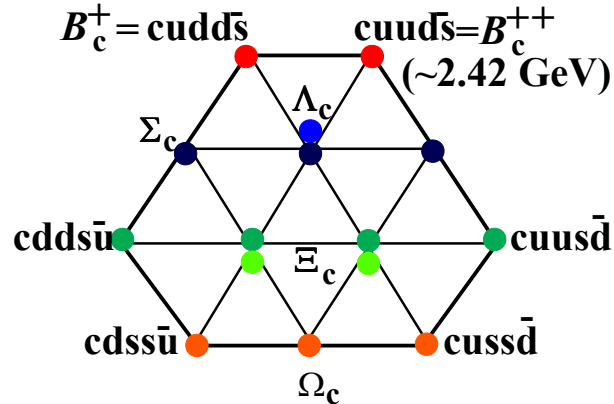Fig. 4. (color online). $\overline{15}$-plet of charmed pentaquarks.

Since we know the separation between the $1/2^+$ level for s quarks and the 0^+ level for u, d quarks from fitting the light baryon resonances, and assuming that it does not change for heavy baryons (as it would be at $N_c \to \infty$), we estimate the mass of the $\mathcal{B}_c^{++,+}$ pentaquarks at about 2420 MeV! The corresponding bottom pentaquark $\mathcal{B}_b^{+,0}$ is about 5750 MeV. Such light pentaquarks have no strong decays. Their weak decays, for example $\mathcal{B}_c^+ \to \mathrm{pK^+K^-}$, have clear signatures especially in a vertex detector, and should be looked for at LHC, Fermilab and B-factories. A cautionary remark, though, is that the production rate is expected to be quite low. More details are

given in Ref. [12].

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