

# Binding energy effects in the decay properties of charmonia

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**Abstract** The di-gamma and di-gluon decay widths of  $P$ -wave  $c\bar{c}$  mesons are computed in nonrelativistic phenomenological quark-antiquark potential of the type  $V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + Ar^\nu$  with different choices of  $\nu$  using spectroscopic parameters. The numerically obtained radial solutions are employed to obtain the di-gamma and di-gluon decay widths. The computed decay widths are consistent with other model predictions as well as with the known experimental values in the range of potential index  $0.7 \leq \nu \leq 1.1$ .

**Key words** charmonia, potential models, decay widths,  $P$ -waves

**PACS** 12.39Jh, 12.40Yx, 13.20Gd

## 1 Introduction

Decay properties of heavy flavour hadrons have attracted considerable interest in recent years due to many experimental facilities like the Beijing Electron Positron Collider (BEPC), E835 at Fermilab, CLEO at the Cornell Electron Storage Ring (CESR) etc. At the hadronic scale the nonperturbative effects of QCD necessarily play an important role. But our limited knowledge about the nonperturbative QCD leads to a theoretical uncertainty in the  $Q\bar{Q}$  potential at large and intermediate distances [1]. The most commonly used potential is the coulomb plus linear power potential,  $V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r$ , with the string tension  $\sigma$ . However, for the higher excited mesonic states it is argued that the string tension  $\sigma$  must depend on the  $Q\bar{Q}$  separation [2, 3]. This corresponds to flattening of the confinement potential at larger  $r$  ( $r \geq 1$  fm). More over the analysis based on Regge trajectories for meson states suggests the confinement part of the potential to have the power  $\frac{2}{3}$  instead of 1 [4, 5]. Thus, in this paper we make an attempt to study the di-gamma and di-gluon decay widths of the  $P$ -wave  $c\bar{c}$  meson system based on a phenomenological coulomb plus power potential (CPP $_\nu$ ), with the po-

tential power index in the range  $0.1 < \nu < 1.5$ .

## 2 Di-gamma and di-gluon decay widths

It is well known that two-photon or two-gluon annihilation rate of heavy quarkonia is related to the radial wave function, so this process will be helpful to understand the form of inter-quark interactions, and can be a sensitive test of the potential models [6]. With the one-loop QCD radiative corrections, the di-gamma decays of  $^3P_0$  and  $^3P_2$  charmonia states are given by [7]

$$\Gamma_{\chi_{Q0}}^{\gamma\gamma} = \frac{27e_Q^4\alpha_e^2}{m_Q^4} |R'_{\chi_{Q0}}(r_0)|^2 \left[ 1 + \frac{\alpha_s}{\pi} \left( \frac{\pi^2}{3} - \frac{28}{9} \right) \right], \quad (1)$$

$$\Gamma_{\chi_{Q2}}^{\gamma\gamma} = \frac{36e_Q^4\alpha_e^2}{5m_Q^4} |R'_{\chi_{Q2}}(r_0)|^2 \left[ 1 - \frac{\alpha_s}{\pi} \frac{16}{3} \right]. \quad (2)$$

Here,  $\alpha_e = \frac{1}{137}$  is the electromagnetic coupling constant and  $e_Q$  corresponds to the charge content of the  $Q\bar{Q}$  meson in units of the electron charge. The two-gluons decay widths give the information on total width of the corresponding quarkonium [8]. The relevant theoretical expressions, including leading order

Received 19 January 2010

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QCD corrections are given by [9, 10]

$$\Gamma(^1S_0 \rightarrow gg) = \frac{2\alpha_s^2}{3m_Q^2} |R_{nS}(0)|^2 \times [1 + 4.8(\alpha_s/\pi)], \quad (3)$$

$$\Gamma(\chi_{Q0} \rightarrow gg) = \frac{6\alpha_s^2}{m_Q^4} |R'_{nP}(0)|^2 \times [1 + 9.5(\alpha_s/\pi)], \quad (4)$$

$$\Gamma(\chi_{Q2} \rightarrow gg) = \frac{8\alpha_s^2}{5m_Q^4} |R'_{nP}(0)|^2 \times [1 - 2.2(\alpha_s/\pi)]. \quad (5)$$

Within the potential confinement scheme, we consider the constituent quark mass  $m_c$  appeared in Eqns 1 to 5 as effective mass of the quark within the bound state of the  $c\bar{c}$  systems defined by

$$m_c^{\text{eff}} = m_c \left( 1 + \frac{\langle E_{\text{bind}} \rangle_{\text{nl}}}{m_c + m_{\bar{c}}} \right)$$

where,  $\langle E_{\text{bind}} \rangle_{\text{nl}} = M_{c\bar{c}} - (m_c + m_{\bar{c}})$  and the  $m_{c/\bar{c}}$  corresponds to the respective quark/antiquark mass parameter employed in the phenomenological model and  $M_{c\bar{c}}$  is the respective mass of the  $c\bar{c}$  mesonic state.

### 3 Results and discussion

For the binding energy calculations as well as for the radial wave functions, we have employed the potential model approach with an interquark potential of the form,  $V(r) = -\frac{4}{3} \frac{\alpha_s}{r} + Ar^\nu$  with  $\nu$  varying  $\nu$  from 0.1 to 1.5. The same model parameters such as the potential strength  $A(\nu)$  and the model quark mass parameter  $m_c = 1.28$  GeV employed to study the spectroscopic properties of the quarkonia are the same values used in the present study [15]. The Branching ratio of the  $\Gamma_{\chi_{Q0}}^{\gamma\gamma}$  and  $\Gamma_{\chi_{Q2}}^{\gamma\gamma}$  with  $m_Q$ , with  $m_Q^{\text{eff}}$  (BR with  $m_Q^{\text{eff}}$ ) and without the radiative corrections (BR with  $m_Q$ ) are obtained using the total widths of  $\Gamma_{\chi_{c0/2}}$  states from PDG [14]. They are shown graphically in Fig. 1 against the potential index  $\nu$ . The experimental branching ratio are drawn with error bar for comparison. Masses of  $P$ -wave agree with experimental mass within the potential index range  $1.0 \leq \nu \leq 1.1$  which are drawn by vertical net in Fig. 1. Our prediction for the masses of  $P$ -wave are given in [15].

The di-gluon decay widths of the  $1S$  to  $3S$  and  $1P$  to  $2P$  of  $c\bar{c}$  states are listed in Table 1 and 2

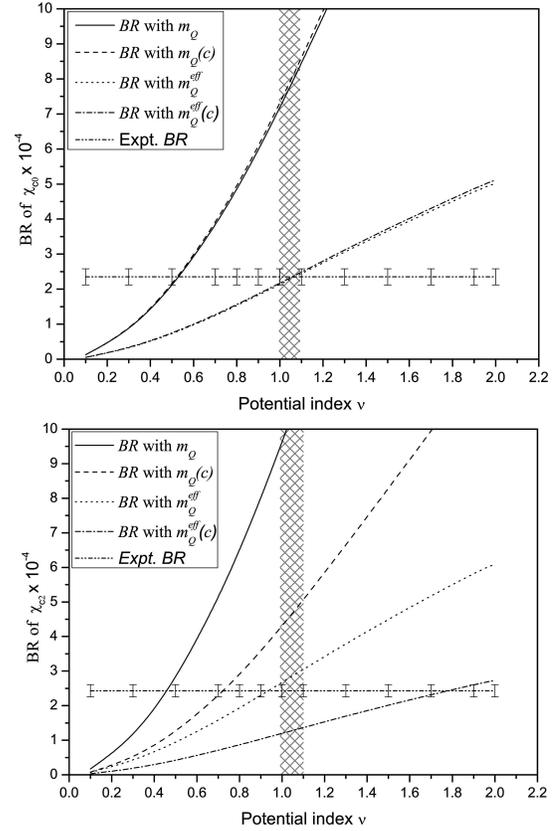


Fig. 1. (color online). Branching Ratio of  $\chi_{c0}$  and  $\chi_{c2}$  of  $c\bar{c}$  meson with using total width of 10.2 MeV and 2.03 MeV of PDG 2008 respectively.

respectively along with other theoretical and available experimental values. It is observed that the di-gluon widths with radiative correction but without binding energy effects are consistent with experimental values but at the lower potential index ( $0.5 \leq \nu \leq 0.8$ ). Accordingly, we predict width of  $\eta_c(3s \rightarrow gg)$  to be around 9.0 MeV to 11.0 MeV. Similarly the predicted di-gluon decay widths of the  $P$ -wave states agree with the known experimental values in the same interaction potential range without considering the binding energy effects, while with the binding energy effects, the agreement shifts towards higher interquark potential index towards  $\nu > 1.0$ . It is expected that future high luminosity experiments will be able to throw more light in the understanding of the quark-antiquark interaction at their excited states.

*Part of this work is done with a financial support from DST, Government of India, under a Major Research Project SR/S2/HEP-20/2006.*

Table 1. Di-gluon decay width (in MeV) of  $S$ -wave chormonium system.

$c\bar{c}$ State		Potential index								
		0.3	0.5	0.7	0.8	0.9	1.0	1.1	1.3	
$\eta_c \rightarrow gg$	$\Gamma_{gg}(0)$	13.54	18.75	23.06	24.93	26.64	28.19	29.62	32.13	15.70 [11]
	$\Gamma_{ggR}(0)$	20.28	28.08	34.53	37.34	39.89	42.22	44.36	48.11	10.57 [11]
	$\Gamma_{gg}^{\text{eff}}(0)$	9.70	13.58	16.86	18.31	19.64	20.86	21.99	23.99	19.60 [12]
	$\Gamma_{ggR}^{\text{eff}}(0)$	14.52	20.33	25.25	27.42	29.41	31.24	32.93	35.93	23.03 [13]
$\eta_c' \rightarrow gg$	$\Gamma_{gg}(0)$	5.02	8.69	12.72	14.84	17.00	19.20	21.44	25.96	8.10 [11]
	$\Gamma_{ggR}(0)$	7.52	13.01	19.05	22.22	25.46	28.76	32.11	38.88	5.94 [11]
	$\Gamma_{gg}^{\text{eff}}(0)$	3.24	5.26	7.25	8.22	9.16	10.08	10.97	12.66	12.10 [12]
	$\Gamma_{ggR}^{\text{eff}}(0)$	4.85	7.88	10.86	12.31	13.72	15.10	16.43	18.95	14.00±0.7 [14]
$\eta_c'' \rightarrow gg$	$\Gamma_{gg}(0)$	3.05	5.91	9.48	11.50	13.68	16.00	18.44	23.70	
	$\Gamma_{ggR}(0)$	4.57	8.85	14.19	17.23	20.48	23.95	27.62	35.48	
	$\Gamma_{gg}^{\text{eff}}(0)$	1.90	3.31	4.77	5.50	6.21	6.91	7.59	8.87	
	$\Gamma_{ggR}^{\text{eff}}(0)$	2.85	4.96	7.15	8.24	9.31	10.35	11.36	13.29	

Table 2. Di-gluon decay width (in MeV) of  $P$ -wave chormonium system.

State		potential index								
		0.3	0.5	0.7	0.8	0.9	1.0	1.1	1.3	
$\chi_{c0} \rightarrow gg$	$\Gamma_{gg}(0)$	1.80	4.65	8.69	11.08	13.64	16.42	19.28	25.34	10.30 [8]
	$\Gamma_{ggR}(0)$	3.57	9.22	17.25	21.98	27.07	32.58	38.26	50.29	13.44 [13]
	$\Gamma_{gg}^{\text{eff}}(0)$	0.69	1.64	2.84	3.50	4.18	4.89	5.59	6.99	12.5±3.2 [7]
	$\Gamma_{ggR}^{\text{eff}}(0)$	1.37	3.25	5.64	6.95	8.30	9.70	11.09	13.88	10.4±0.7 [14]
$\chi_{c0}' \rightarrow gg$	$\Gamma_{gg}(0)$	1.32	4.01	8.59	11.71	15.31	19.50	24.15	35.34	9.61 [8]
	$\Gamma_{ggR}(0)$	2.62	7.95	17.05	23.24	30.38	38.70	47.93	70.14	
	$\Gamma_{gg}^{\text{eff}}(0)$	0.49	1.23	2.20	2.75	3.30	3.87	4.43	5.58	
	$\Gamma_{ggR}^{\text{eff}}(0)$	0.97	2.44	4.36	5.45	6.55	7.69	8.79	11.07	
$\chi_{c2} \rightarrow gg$	$\Gamma_{gg}(0)$	0.48	1.24	2.32	2.95	3.64	4.38	5.14	6.76	1.20 [13]
	$\Gamma_{ggR}(0)$	0.37	0.96	1.79	2.28	2.81	3.38	3.97	5.22	1.72 [11]
	$\Gamma_{gg}^{\text{eff}}(0)$	0.18	0.42	0.72	0.88	1.05	1.22	1.39	1.72	2.03±0.12 [14]
	$\Gamma_{ggR}^{\text{eff}}(0)$	0.14	0.32	0.56	0.68	0.81	0.94	1.07	1.33	
$\chi_{c2}' \rightarrow gg$	$\Gamma_{gg}(0)$	0.35	1.07	2.29	3.12	4.08	5.20	6.44	9.42	
	$\Gamma_{ggR}(0)$	0.27	0.83	1.77	2.41	3.15	4.01	4.97	7.27	
	$\Gamma_{gg}^{\text{eff}}(0)$	0.13	0.32	0.56	0.70	0.84	0.98	1.11	1.39	
	$\Gamma_{ggR}^{\text{eff}}(0)$	0.10	0.25	0.44	0.54	0.65	0.76	0.86	1.07	

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