

Dynamical study of the possible molecular state X(3872) with the s -channel one gluon exchange interaction^{*}

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Abstract The recently observed X(3872) resonance, which is difficult to assign a conventional $c\bar{c}$ charmonium state in the quark model, may be interpreted as a molecular state. Such a molecular state is a hidden flavor four quark state because of its charmonium-like quantum numbers. The s -channel one gluon exchange is an interaction which only acts in the hidden flavor multi-quark system. In this paper, we will study the X(3872) and other similar hidden flavor molecular states in a quark model by taking into account the s -channel one gluon exchange interaction.

Key words quark model, mesons, molecular states

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1 Introduction

Recently, many new resonances have been discovered experimentally. Many of them have the proper quantum numbers of the $q\bar{q}$ meson states. However, their mass values do not fit the conventional $q\bar{q}$ states in the quark model. Among them, X(3872) was first observed in the $J/\psi\pi^+\pi^-$ channel by the Belle collaboration in 2003 [1], and has been confirmed by the CDF [2], D0 [3] and BABAR collaborations [4]. Its quantum numbers are probably $J^{PC} = 1^{++}$. The corresponding charmonium candidates in the quark model are $2^3P_1(3990)$ and $3^3P_1(4290)$, which are 50–200 MeV above $M_X = 3872$ MeV.

Many people suggested that X(3872) is mainly a $D\bar{D}^*$ molecular state [5–9]. However, to bind the quarks and anti-quarks together in such a four quark state or other multi-quark states, we need to introduce new interaction into the quark model. Swanson proposed that the X(3872) is mainly a $D\bar{D}^*$ molecule bound by the meson-meson interaction derived from the one pion exchange and the quark exchange [7]. In Wong's work [8], the meson-meson interaction

is derived from a QED-type effective interaction in terms of effective charges for quarks and antiquarks. In Refs. [10–15], further investigations based on the molecule assumption were carried out.

Since the strength of the one pion exchange interaction seems insufficiently strong to bind the $D\bar{D}^*$ molecular state, other authors argued that X(3872) may be a dominant $c\bar{c}$ charmonium with some admixture of $D\bar{D}^*$ [16–18]. In Ref. [19], after taking into account the sigma meson exchange potential, the interpretation of X(3872) as a loosely bound molecular state was further disfavored.

We should notice that the color structure of a multi-quark state is much richer than that of a $q\bar{q}$ conventional meson state. Unlike the conventional mesons or baryons, the $q\bar{q}$ and qq pairs in a multi-quark state can be in the color 8_c and 6_c representations, respectively. Some color interactions which have no effects in the $q\bar{q}$ or q^3 colorless system may make significant contributions in a multi-quark system. So the complete interactions in the quark model can be quite different after we take into account these multi-quark states.

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The s -channel one gluon exchange interaction is an interaction between quark and anti-quark of the same flavor which annihilate into a virtual gluon. It has no effect on the conventional $q\bar{q}$ mesons but acts in the hidden flavor multi-quark system like the charmonium-like molecular states. In this work, we will investigate the hidden flavor molecular states by considering the s -channel gluon exchange interaction in the quark model. In the next section, we will model the potential of the s -channel one gluon exchange interaction starting from the non-relativistic reduction. Then the J^{PC} quantum numbers of the molecular states will be selected by an analysis of the spin dependence of the interaction strength. In Sec. 3, we will carry out the numerical calculation of X(3872) and some other charmonium-like states as the molecular states. Also, we will make a prediction about the similar bottomium-like molecular states. Finally, we will give a brief summary.

2 The potential of s -channel one gluon exchange interaction

In our work, we will use the Bhaduri quark model, which is a rather simple non-relativistic quark potential model. In the Bhaduri model, the Hamiltonian can be written as [20]

$$H = \sum_i \left(m_i + \frac{P_i^2}{2m_i} \right) - \frac{3}{4} \sum_{i < j} (F_i \cdot F_j V_{ij}^C + F_i \cdot F_j S_i \cdot S_j V_{ij}^{SS}), \quad (1)$$

where m_i are the constituent quark masses and

$$F_i^c = \frac{1}{2} \lambda_i^c \quad (c = 1, \dots, 8)$$

are the well-known $SU_c(3)$ Gell-Mann matrices. Apart from a constant, here the central potential is the usual one gluon exchange coulomb potential plus the linear confinement:

$$V_{ij}^C = -\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a_0^2} - M_0, \quad (2)$$

where $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ is the distance between quark i and j . The color-magnetic interaction reads

$$V_{ij}^{SS} = \frac{4\kappa}{m_i m_j} \frac{1}{r_0^2 r_{ij}} e^{-r_{ij}/r_0}, \quad (3)$$

where the δ -interaction has been smeared smoothly with the prescription

$$\delta^3(\mathbf{r}) \rightarrow \frac{1}{4\pi r_0^2 r_{ij}} e^{-r_{ij}/r_0}. \quad (4)$$

The model parameter values are

$$\begin{aligned} \kappa &= 102.67 \text{ MeV} \cdot \text{fm}, & a_0 &= 0.0326 \text{ (MeV}^{-1} \cdot \text{fm)}^{\frac{1}{2}}, \\ M_0 &= 913.5 \text{ MeV}, & r_0 &= 0.4545 \text{ fm} \\ m_u &= m_d = 337 \text{ MeV}, & m_s &= 600 \text{ MeV}, \\ m_c &= 1870 \text{ MeV}, & m_b &= 5259 \text{ MeV}. \end{aligned}$$

The s -channel one gluon exchange interaction (SOGE) takes place when a $q\bar{q}$ pair annihilates into a virtual gluon (Fig. 1). The non-relativistic reduction of the potential is:

$$V^{\text{SOGE}}(\mathbf{r}_{ij}) = -\frac{1}{2} \left(\frac{4}{3} + F_q \cdot F_{\bar{q}} \right) \left(1 + \frac{4}{3} S_q \cdot S_{\bar{q}} \right) \times G(4m^2) \delta^3(\mathbf{r}_{ij}) \delta(f_i, f_j), \quad (5)$$

where $G(4m^2)$ is the one gluon exchange amplitude. The last $\delta(f_i, f_j)$ indicates that the quark and the anti-quark must have the same flavor. Clearly, the two factors in the brackets mean that this interaction only occurs when the $q\bar{q}$ pair is in the color octet and with spin $S = 1$. In our model calculation, the δ -interaction should be smeared smoothly with the same prescription (4) used in the color-magnetic interaction in the same quark model. However, the QCD amplitude $G(4m^2)$ takes its argument value in the timelike region, where the QCD behavior is still not clear at the present time. In the spacelike region, this amplitude is well known from the gluon propagator in perturbative QCD and it reads

$$G_{\text{pert}}^t(q^2) = \frac{4\pi\alpha_s}{-q^2} \quad \text{for } q^2 < 0. \quad (6)$$

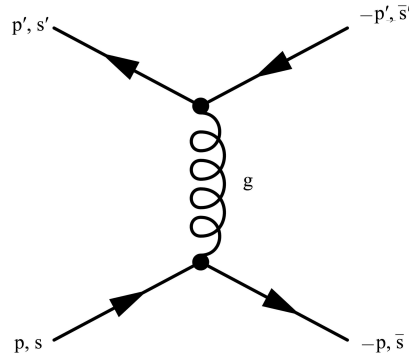


Fig. 1. The s -channel one gluon exchange.

However, in our study of the hidden flavor molecular states, in order to provide an attractive interaction to favor the formation of the bound states, we need that

$$G(4m^2) > 0,$$

which means that the above formula from perturbative QCD should not be directly used in the timelike region. This change of sign is first suggested in

Ref. [21] in the study of $\pi\pi$ and $K\pi$ scattering. Following Ref. [21], we assume that

$$G(4m^2) = -fG_{\text{pert}}^t(4m^2) = f\frac{\pi\alpha_s^2}{m^2}, \quad (7)$$

where f is an introduced strength factor. In our model, after the δ -function is smeared smoothly, the SOGE potential turns to be

$$V_{ij}^{\text{SOGE}} = -\frac{f}{4} \left(1 + \frac{3}{4} F_q \cdot F_{\bar{q}} \right) \left(\frac{3}{4} + S_q \cdot S_{\bar{q}} \right) \frac{4\kappa}{m_i^2 r_0^2 r_{ij}} e^{-r_{ij}/r_0} \delta(f_i, f_j). \quad (8)$$

First we observe that the potential V^{SOGE} is proportional to m_i^{-2} , so the interaction mainly takes place between the light $q\bar{q}$ pairs of $q = u, d, s$ in any multi-quark system.

Next, we will analyse the spin dependence of V^{SOGE} in each J^{PC} channel of molecular state. For the molecular states concerned, let us assume that their flavor structure is $Q\bar{q}q\bar{Q}$, where $Q = c, b$ and $q = u, d, s$. Since we can neglect the V^{SOGE} interaction between $Q\bar{Q}$, the interaction acts only when the $q\bar{q}$ pair is in color octet and $S = 1$, as we have mentioned before. More specifically, the interaction will be switched on if the spin coupling of the four quarks is $[(Q\bar{Q})_{J_1}(q\bar{q})_{J_2=1}]_J$. The color structure of $Q\bar{Q}$ and $q\bar{q}$ should be octet, obviously. However, the color (re)coupling is irrelevant to our analysis here and will not be explicitly presented. Furthermore, we will assume no spatial excitation of any quark. Hence all quarks have zero orbital angular momentum. Then the quantum numbers J^{PC} can be determined easily for $J_1 = 0, 1$. We have the following four V^{SOGE} interaction channels: $J_1 = 0 : 1^{+-}$; $J_1 = 1 : 0^{++}, 1^{++}, 2^{++}$. Finally, we can recover the molecular states by the angular momentum recoupling. We obtain

1) 0^{++}

$$[(Q\bar{Q})_1(q\bar{q})_1]_0 = -\frac{\sqrt{3}}{2}(Q\bar{q})_0(q\bar{Q})_0 - \frac{1}{2}[(Q\bar{q})_1(q\bar{Q})_1]_0,$$

2) 1^{++}

$$[(Q\bar{Q})_1(q\bar{q})_1]_1 = -\frac{1}{\sqrt{2}}(Q\bar{q})_1(q\bar{Q})_0 + \frac{1}{\sqrt{2}}(Q\bar{q})_0(q\bar{Q})_1,$$

3) 1^{+-}

$$(Q\bar{Q})_0(q\bar{q})_1 = \frac{1}{2}(Q\bar{q})_1(q\bar{Q})_0 + \frac{1}{2}(Q\bar{q})_0(q\bar{Q})_1 + \frac{1}{\sqrt{2}}[(Q\bar{q})_1(q\bar{Q})_1]_1,$$

4) 2^{++}

$$[(Q\bar{Q})_1(q\bar{q})_1]_2 = [(Q\bar{q})_1(q\bar{Q})_1]_2.$$

Then the factor of interaction strength can be read from the coefficients, which we present in Table 1. We see that the V^{SOGE} interaction favors forming the molecular states with $J^{PC} = 1^{++}, 2^{++}$.

Table 1. The list of spin factor of V^{SOGE} interaction strength in a molecular state J^{PC} made of two meson states $J_1^{P_1}$ and $J_2^{P_2}$. c.c. = charge conjugation.

$J_1^{P_1} J_2^{P_2}$	J^{PC}	spin factor
$0^- 0^-$	0^{++}	$\frac{3}{4}$
$1^- 1^-$	0^{++}	$\frac{1}{4}$
$1^- 0^- + \text{c.c.}$	1^{++}	1
$1^- 0^- - \text{c.c.}$	1^{+-}	$\frac{1}{2}$
$1^- 1^-$	1^{+-}	$\frac{1}{2}$
$1^- 1^-$	2^{++}	1

3 Numerical calculation

To calculate the molecular states, we use the Rayleigh-Ritz variation principle. The test wave function will be taken to be a series of Gaussian basis functions. The Gaussian basis functions are often utilized in variational calculations of atomic and molecular problems. Recently the method has also been used in the few body system in nuclear and particle physics [22–24].

In our case of the $Q\bar{q}q\bar{Q}$ molecular state, the test wave function of a molecular state between two clusters of $q\bar{q}$ meson states is a series:

$$\psi_{1234}(r_{12}, r_{34}, r_{1234}) = \sum_i \alpha_{1234}^i \psi_{12}(r_{12}) \psi_{34}(r_{34}) \exp(-\beta_{1234}^i r_{1234}^2), \quad (9)$$

where $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3$ and \mathbf{r}_4 are the coordinates of Q, \bar{q}, q and \bar{Q} , respectively. $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$. r_{1234} is the distance between the two meson clusters:

$$\mathbf{r}_{1234} = \frac{m_Q \mathbf{r}_1 + m_q \mathbf{r}_2}{m_Q + m_q} - \frac{m_q \mathbf{r}_3 + m_Q \mathbf{r}_4}{m_q + m_Q}, \quad (10)$$

$\psi_{ij}(r_{ij})$ is the meson wave function, which is also taken to be a Gaussian function series:

$$\psi_{ij}(r_{ij}) = \sum_k \alpha_{ij}^k \exp(-\beta_{ij}^k r_{ij}^2). \quad (11)$$

The wave function of a molecular state is determined by the variation principle in two steps. We first determine the wave function (11) of each meson cluster. Then the meson cluster functions ψ_{ij} are fixed in

(9) to obtain the wave function of the molecular state and their masses.

To reduce the amount of computation, the parameters β^i and α^i in a Gaussian function series are determined in two steps by one-dimensional minimization. We first determine an average β value using a single Gaussian function. Then a set $\{\beta^i\}$ of $2N+1$ elements is generated from scaling the β value up and down by a scale factor s [24]:

$$\beta^i = \beta s^{i-N}, \quad (12)$$

where $i = 0, 1, \dots, 2N$. The coefficients α^i are determined by diagonalization of the model Hamiltonian in the $2N+1$ -dimensional space spanned by these $2N+1$ different Gaussian functions. The final values of β^i and α^i and the mass of tetra quark states are determined by searching the scale factor s for a minimum of system energy.

In this way, we have calculated the possible 0^{++} (the combination of two 0^- mesons with the spin factor $\frac{3}{4}$), 1^{++} and 2^{++} molecular states in Table 1. We have calculated both the charmonium-like states $c\bar{q}q\bar{c}$, which will be compared with recent experimental results, and the bottomium-like states $b\bar{q}q\bar{b}$, which can be investigated in further experiments.

The results of relevant heavy quark $Q\bar{q}$ mesons are listed in Table 2. We see that the mass values calculated from the Bhaduri quark model deviate at most 30 MeV away from the experimental data. The difference between the calculation from the variation method with Gaussian function series (Cal. II) and the exact numerical calculation (Cal. I) is less than 0.5 MeV. So the variation method is an impressively good approximation for the numerical calculation of the $Q\bar{q}$ conventional mesons.

Table 2. Mass of $Q\bar{q}$ mesons. The experimental values are taken from PDG [25]. In Cal. I, the mass is obtained by solving the Schrödinger equation. In Cal. II, the mass is obtained by the variation method with Gaussian test functions using $N = 3$.

meson state	Exp./MeV	Cal. I/MeV	Cal. II/MeV
D^\pm	1869.62	1885.56	1886.14
D^0	1864.84		
$D^*(2007)^0$	2006.97	2019.96	2020.07
$D^*(2010)^\pm$	2010.27		
D_s^\pm	1968.49	1995.78	1996.36
$D_s^{*\pm}$	2112.3	2101.2	2101.4
B^\pm	5279.15	5300.84	5301.18
B^0	5279.53		
B^*	5325.1	5350.3	5350.5
B_s^0	5366.3	5371.9	5372.4
B_s^*	5412.8	5413.3	5413.6

The results of the molecular states are given in Table 3 and Table 4. Here the molecular state is presented by its binding energy:

$$E_b = M_1 + M_2 - M_X \quad (13)$$

and the rms radius $\langle r^2 \rangle^{1/2}$, where M_1, M_2 are the masses of the two compound mesons, and M_X is the mass of the molecular state.

We can see from Table 3 and Table 4 that indeed the 1^{++} and 2^{++} molecular states are bound preferable, since the V^{SOGE} are strong in these two channels. The binding of 1^{++} states is slightly deeper and tighter than that of 2^{++} s. So, from our model calculation, the 1^{++} molecular states should be easily observed in experiments.

Table 3. Binding energy E_b (in MeV) of molecular states vs paramter f .

f	-0.8	-1.0	-1.5	-2.0	-3.0
(DD) 0^{++}	-	-	11.8	28.1	71.7
(DD*) 1^{++}	1.8	6.2	24.2	48.7	109.3
(D*D*) 2^{++}	-	5.4	21.4	43.5	98.1
(D _s D _s) 0^{++}	-	-	-	-	5.3
(D _s D _s *) 1^{++}	-	-	-	-	15.4
(D _s *D _s *) 2^{++}	-	-	-	-	14.0
(BB) 0^{++}	8.8	15.3	35.2	58.5	110.9
(BB*) 1^{++}	16.8	26.8	55.8	88.5	160.4
(B*B*) 2^{++}	16.0	25.5	53.4	84.7	153.6
(B _s B _s) 0^{++}	-	-	3.4	9.3	25.7
(B _s B _s *) 1^{++}	-	1.9	8.9	18.9	44.4
(B _s *B _s *) 2^{++}	-	1.8	8.5	18.2	42.9

Table 4. rms $\langle r^2 \rangle^{1/2}$ (in fm) of molecular states vs paramter f .

f	-0.8	-1.0	-1.5	-2.0	-3.0
(DD) 0^{++}	-	-	1.33	1.00	0.76
(DD*) 1^{++}	2.67	1.68	1.07	0.86	0.69
(D*D*) 2^{++}	-	1.79	1.12	0.91	0.72
(D _s D _s) 0^{++}	-	-	-	-	1.63
(D _s D _s *) 1^{++}	-	-	-	-	1.12
(D _s *D _s *) 2^{++}	-	-	-	-	1.17
(BB) 0^{++}	1.11	0.94	0.74	0.64	0.54
(BB*) 1^{++}	0.92	0.80	0.66	0.58	0.50
(B*B*) 2^{++}	0.94	0.82	0.67	0.59	0.51
(B _s B _s) 0^{++}	-	-	1.42	1.01	0.74
(B _s B _s *) 1^{++}	-	1.77	1.03	0.81	0.63
(B _s *B _s *) 2^{++}	-	1.81	1.05	0.83	0.64

First, let us look at the $D^{(*)}D^{(*)}$ sector. If the X(3872) is a molecular state of D and D*, the binding energy can be estimated from the experimental

data [25].

$$E_b = M_D + M_{D^*} - M_X = 0.3 \text{ MeV},$$

which is very small. Accounting for the uncertainty of approximation in our model calculation, we think the reasonable range of the f value should be around -0.8 – -1.0 . If it is small $f \sim -0.8$, then it may be the only molecular state in the $D^{(*)}D^{(*)}$ sector. If $f \sim -1.0$, the 2^{++} state may also exist. From our numerical calculation, its mass should be about

$$M = M_{D^*} + M_{D^*} - E_b \approx 4008.5 \text{ MeV}.$$

The Belle collaboration has reported an $X(3930)$ state of 2^{++} at mass $M = 3929 \text{ MeV}$ [26], which is a candidate for the $c\bar{c}$ charmonium 2^3P_2 excited state χ'_{c2} . However, its mass is about 40 MeV below the quark model calculations (in the Bhaduri model, the χ'_{c2} mass is $M = 3963.53 \text{ MeV}$). Several authors have discussed the mass shifts of the coupled channel effect [27–30]. If the above 2^{++} molecular state exists, the final state interaction will be important.

Now we turn to the possible molecular states in other sectors. We observe that in the $D_s^{(*)}D_s^{(*)}$ sector there are no such molecular states due to the mass increase of m_s of the light quark pair. On the other hand, in the $B^{(*)}B^{(*)}$ sector, the binding energy increases with the mass increase of m_b of the heavy quark pair. So these bottomium-like molecular states with 1^{++} , 2^{++} and even 0^{++} may also exist. We can

also observe the $B_s^{(*)}B_s^{(*)}$ molecular states 1^{++} and 2^{++} if $f \sim -1.0$.

4 Summary

The quark model is extended by introducing the s -channel one gluon exchange interaction. The interaction has no effect on the inner quark structure of conventional $q\bar{q}$ mesons and qqq baryons. Since the interaction is short ranged – a smeared δ -interaction the effect on the long ranged hadron-hadron interaction is expected to be very small. So the significant effect of this interaction is only in the so-called hidden flavor states of multi-quark system.

We have calculated the heavy quark molecular states of $qQ\bar{Q}\bar{q}$ with $Q = c, b$ and $q = u, d, s$. We find that the interaction can be strong enough to bind the 1^{++} and 2^{++} states, and possibly the 0^{++} states. To compare with the recent experiments, $X(3872)$ is a candidate for the DD^* molecular state and the $X(3930)$ may be the χ'_{c2} state which couples to D^*D^* 2^{++} molecular state. The calculation shows that it is easier to bind the bottomium-like molecular states. Thus we expect the similar bottomium-like molecular states of 1^{++} and 2^{++} to exist too.

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References

- 1 Choi S K et al (Belle). Phys. Rev. Lett., 2003, **91**: 262001
- 2 Acosta D E et al (CDF II). Phys. Rev. Lett., 2004, **93**: 072001
- 3 Abazov V M et al (D0). Phys. Rev. Lett., 2004, **93**: 162002
- 4 Aubert B et al (BABAR). Phys. Rev. D, 2005, **71**: 071103
- 5 Close F E, Page P R. Phys. Lett. B, 2004, **578**: 119–123
- 6 Voloshin M B. Phys. Lett. B, 2004, **579**: 316–320
- 7 Swanson E S. Phys. Lett. B, 2004, **588**: 189–195
- 8 WONG C Y. Phys. Rev. C, 2004, **69**: 055202
- 9 Tornqvist N A. Phys. Lett. B, 2004, **590**: 209–215
- 10 AlFiky M T, Gabbiani F, Petrov A A. Phys. Lett. B, 2006, **640**: 238–245
- 11 Fleming S, Kusunoki M, Mehen T, van Kolck U. Phys. Rev. D, 2007, **76**: 034006
- 12 Braaten E, LU M, Lee J. Phys. Rev. D, 2007, **76**: 054010
- 13 Hanhart C, Kalashnikova Y S, Kudryavtsev A E, Nefediev A V. Phys. Rev. D, 2007, **76**: 034007
- 14 Voloshin M B. Phys. Rev. D, 2007, **76**: 014007
- 15 Colangelo P, De Fazio F, Nicotri S. Phys. Lett. B, 2007, **650**: 166–171
- 16 Suzuki M. Phys. Rev. D, 2005, **72**: 114013
- 17 MENG C, GAO Y J, CHAO K T. 2005, arXiv:hep-ph/0506222
- 18 ZHU S L. Int. J. Mod. Phys. E, 2008, **17**: 283–322
- 19 LIU Y R, LIU X, DENG W Z, ZHU S L. Eur. Phys. J. C, 2008, **56**: 63–73
- 20 Bhaduri R K, Cohler L E, Nogami Y. Nuovo Cim. A, 1981, **65**: 376–390
- 21 LI Z, Guidry M, Barnes T, Swanson E S. 1994, arXiv:hep-ph/9401326
- 22 Kameyama H, Kamimura M, Fukushima Y. Phys. Rev. C, 1989, **40**: 974–987
- 23 Varga K, Suzuki Y. Phys. Rev. A, 1996, **53**: 1907–1910
- 24 Brink D M, Stancu F. Phys. Rev. D, 1998, **57**: 6778–6787
- 25 Amsler C et al (Particle Data Group). Phys. Lett. B, 2008, **667**: 1
- 26 Uehara S et al (Belle). Phys. Rev. Lett., 2006, **96**: 082003
- 27 Kalashnikova Y S. Phys. Rev. D, 2005, **72**: 034010
- 28 Pennington M R, Wilson D J. Phys. Rev. D, 2007, **76**: 077502
- 29 LI B Q, MENG C, CHAO K T. Phys. Rev. D, 2009, **80**: 014012
- 30 Ortega P G, Segovia J, Entem D R, Fernandez F. 2009, arXiv:hep-ph/0907.3997