# HOM study and parameter calculation of the TESLA cavity model

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**Abstract**   The Superconducting Proton Linac (SPL) is the project for a superconducting, high current H-accelerator at CERN. To find dangerous higher order modes (HOMs) in the SPL superconducting cavities, simulation and analysis for the cavity model using simulation tools are necessary. The existing TESLA 9-cell cavity geometry data have been used for the initial construction of the models in HFSS. Monopole, dipole and quadrupole modes have been obtained by applying different symmetry boundaries on various cavity models. In calculation, scripting language in HFSS was used to create scripts to automatically calculate the parameters of modes in these cavity models (these scripts are also available in other cavities with different cell numbers and geometric structures). The results calculated automatically are then compared with the values given in the TESLA paper. The optimized cavity model with the minimum error will be taken as the base for further simulation of the SPL cavities.

**Key words**   TESLA 9-cell cavity, superconducting proton linac, higher order modes, HFSS scripts

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## 1    Introduction

The Superconducting Proton Linac (SPL) is the project for a superconducting, high current H- accelerator at CERN [1]. It is part of the injector chain upgrade for the Large Hadron Collider (LHC). High gradient superconducting cavities [2] operating with high beam current will be used in SPL. In cavities, there exists not only the fundamental mode which is used to accelerate the beam but also high order modes (HOMs) which are considered as the source of beam instability and HOM heating. The HOMs are one of the major problems with operating a high current linac. It is very important to correctly find HOMs and get the values of parameters for possible dangerous HOMs. To do this, the existing TESLA cavity geometry data [3] are used firstly to construct models in HFSS and then the results obtained are compared with the values given in the TESLA paper (Ref. [4]). The cavity model with the minimum error will be taken as the base for further simulation of the SPL cavities.

Both cavity simulation and parameter calculations have been performed in computer code AN-SOFT HFSS v11. HFSS is a powerful and flexible tool for 3D full EM simulation with high accuracy. Different symmetry boundaries on various cavity models are applied to obtain the correct modes. By using script language in HFSS, the scripts have been written to automatically calculate the parameters of the modes. These scripts can also be available in cavities with different cell numbers and geometric structures. It is very useful to calculate the parameters of rescaled cavities and other cavities in the next step of the SPL project.

## 2    Cavity modeling in HFSS

The basic TESLA cavity geometry data are used for constructing the models in HFSS calculation. Different boundary conditions and different shapes are applied to obtain monopole, dipole and quadrupole modes separately. A 10-degree 9-cell cavity model is taken to get monopole modes according to the method of Ref. [5]. The 10-degree 9-cell cavity model

is shown in Fig. 1. Monopole modes are obtained by applying magnetic (H) symmetry boundaries on both sides of the model. For dipole modes, a 90-degree cavity model is taken with H symmetry on one side and electric (E) symmetry on the other side. For quadrupole modes, a 90-degree cavity model with E symmetry boundaries on both sides is adopted.
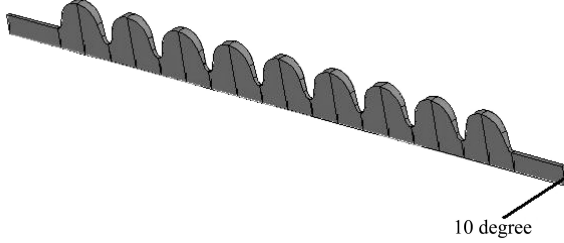


Fig. 1. 10-degree 9-cell cavity model for monopole modes.

Monopole modes can be also obtained by using a 90-degree cavity with perfect H symmetries on both sides; however, quadrupole modes also exist in this model, and the lowest frequencies of these quadrupole modes are comparable with the second band of monopole modes. It is not convenient to obtain only monopole modes in lower bands using the 90-degree cavity model. For the same reason, there exist sextupole modes when using the 90-degree cavity to get dipole modes and octopole modes when using the 90-degree cavity to get quadrupole, but these modes often exist in higher frequency bands and play a non-significant role. Thus, we usually pay less attention to these modes when considering the dipole and quadrupole modes. If necessary in some cases, one can obtain sextupole and octopole modes by using 60-degree and the 45-degree cavity models respectively to make further comparison.

In order to make the simulation fast and simple, a 4.5-cell cavity model has been used. It can be estimated that the calculation for the 4.5-cell cavity model will take much less time than for the 9-cell cavity model. It will be a fast way to use scripts for the 4.5-cell cavity model to calculate the parameters of all modes. The 4.5-cell cavity model for dipole and quadrupole modes is given in Fig. 2. For monopole modes, we take the first half of the 10-degree 9-cell cavity. The boundary conditions applied on two sides are the same as those used on the corresponding 9-cell cavity sides. On the end plane, both perfect H symmetry and perfect E symmetry need to be applied to obtain complete monopole, dipole or quadrupole modes.
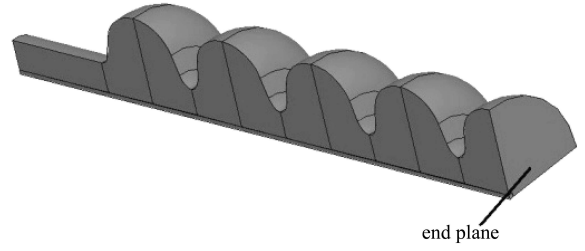


Fig. 2. 4.5-cell cavity model for dipole and quadrupole modes.

## 3   Calculation of the parameters

### 3.1   Definitions of the parameters

There exists no unified standard for the definitions of some parameters of the modes. To make a comparison with the TESLA paper, the definitions of parameters $G_1$, $R/Q$, and phase advance used in this section take those used in the TESLA paper.

The geometry parameter $G_1$ is related to the power dissipation on the cavity surface. It is defined as:

$$G_1 = \frac{\frac{1}{2}\omega\mu\iiint |H|^2 \mathrm{d}V}{\frac{1}{2}\iint |n \times H|^2 \mathrm{d}S} = \frac{\omega U}{\frac{1}{2}\iint |n \times H|^2 \mathrm{d}S}, \quad (1)$$

where $U$ is the total field energy in the cavity. $G_1$ is the geometry constant which is independent of the cavity material.

The ratio $R/Q$ is a very important quantity related to the interaction of the beam and cavity. It is defined as:

$$\frac{R^{(m)}}{Q} = \frac{1}{r^{2m}}\frac{2k^{(m)}(r)}{\omega} = \frac{1}{r^{2m}}\frac{|V_c|^2}{4\omega U^{(m)}}, \quad (2)$$

where $m = 0$, for monopole modes; $m = 1$, for dipole modes; $m = 2$, for quadrupole modes. $V_c$ is the accelerating voltage of a cavity which is defined as:

$$V_c = \int_{-z_1}^{z_2} E_z(r = a, z)\exp(-\mathrm{j}\omega z/c)\mathrm{d}z, \quad (3)$$

where for monopole modes, $a = 0$; for dipole, quadrupole modes, $a \neq 0$. $z_2 - z_1 = L$, $L$ is the length of cavity. The ratio of $R^{(m)}/Q$ is independent of radius $r$ and surface resistance.

The modes can be also described as $\pi/9$, $2\pi/9$, $\cdots$, $9\pi/9$ modes for each band according to their phase advance per cell. For a cavity with periodic structure, the phase advance per cell at position $z$ is defined as:

$$\varphi(z) = \arccos\left(\frac{E_z(r, z + L_{\mathrm{cell}}) + E_z(r, z - L_{\mathrm{cell}})}{2E_z(r, z)}\right), \quad (4)$$

where $E_z(r,z)$ is the longitudinal electric field. The equation to calculate the phase advance per cell can be only applied in the middle cells of the cavity and is not accurate for the end cells.

### 3.2 Calculating parameters with HFSS script

An HFSS field calculator is a powerful tool which can perform a lot of mathematical operations for any field quantities. It is sufficient to use the calculator to complete the parameter calculation of the modes[1]. All the calculator operations have direct equivalents in HFSS script codes described by scripting language. Writing the calculator operations code into one script is useful for the calculations needed to repeat. Calculations of the parameters of modes can also be written into scripts. Running these scripts, we can directly get the results of the parameters, which have been used to make a comparison with the values given in the TESLA paper. A block gram of these scripts is shown in Fig. 3. The scripts can be applied in both the 9-cell cavity and the 4.5-cell cavity models. For other cavities with different cell numbers and geometric structures, the scripts are also applied by changing a few codes.
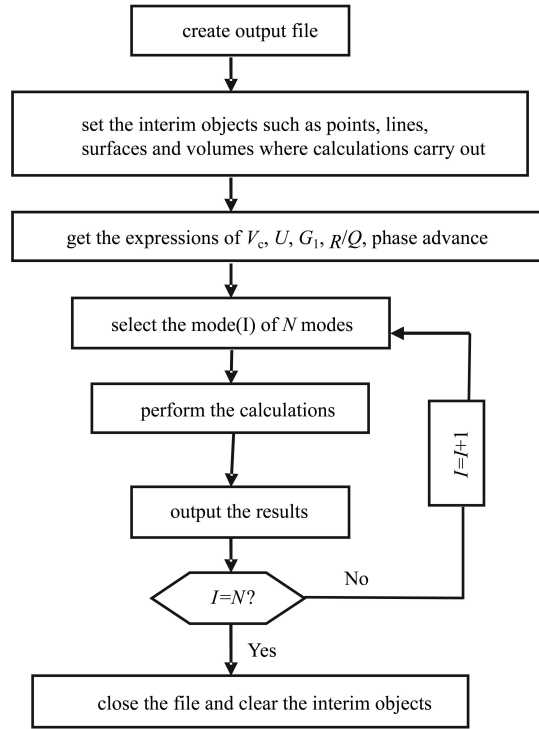


Fig. 3. Block gram of the scripts to calculate the parameters.

Table 1. A comparison of parameters calculated and the values given in the TESLA paper.

| mode | frequency/GHz | | | $G_1/\Omega$ | | | $(R/Q)/\Omega$ | | | phase/($^\circ$) | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| | Calc. | paper | Diff. | Calc. | paper | Diff. | Calc. | paper | Diff. | Calc. | paper | Diff. |
| M1 | 1.2815 | 1.2756 | 0.0059 | 267.9 | 252.7 | 15.2 | 0.0004 | 0.0002 | 0.0002 | 20 | 20 | 0 |
| M2 | 1.2836 | 1.2776 | 0.0060 | 268.1 | 252.9 | 15.2 | 0.0007 | 0.0001 | 0.0006 | 40 | 39.9 | 0.1 |
| M3 | 1.2868 | 1.2807 | 0.0061 | 268.6 | 253.2 | 15.4 | 0.0041 | 0.0013 | 0.0028 | 60.1 | 59.9 | 0.2 |
| M4 | 1.2908 | 1.2845 | 0.0063 | 269.2 | 253.5 | 15.7 | 0.0052 | 0.0005 | 0.0047 | 80 | 79.8 | 0.2 |
| M5 | 1.2951 | 1.2885 | 0.0066 | 269.8 | 253.9 | 15.9 | 0.0091 | 0.0005 | 0.0086 | 100 | 99.8 | 0.2 |
| M6 | 1.2991 | 1.2924 | 0.0067 | 270.3 | 254.2 | 16.1 | 0.0473 | 0.0019 | 0.0454 | 120.1 | 119.7 | 0.4 |
| M7 | 1.3024 | 1.2955 | 0.0069 | 270.8 | 254.5 | 16.3 | 0.0155 | 0.0339 | −0.0184 | 139.8 | 139.6 | 0.2 |
| M8 | 1.3046 | 1.2976 | 0.0070 | 271.1 | 254.7 | 16.4 | 4.9905 | 0.0163 | 4.9742 | 160.1 | 159.2 | 0.9 |
| M9 | 1.3053 | 1.2983 | 0.0070 | 270.6 | 254.8 | 15.8 | 498.1035 | 511.0652 | −12.9617 | 180* | 176.1 | 3.9 |
| M10 | 2.3863 | 2.38 | 0.0063 | 390.1 | 370.6 | 19.5 | 0.0763 | 0.001 | 0.0753 | 161.5 | 159.9 | 1.6 |
| M11 | 2.3912 | 2.3856 | 0.0056 | 390.8 | 370.7 | 20.1 | 0.173 | 0.0196 | 0.1534 | 143.3 | 139.9 | 3.4 |
| M12 | 2.3991 | 2.3943 | 0.0048 | 391.5 | 370.9 | 20.6 | 0.6606 | 0.0329 | 0.6277 | 124.8 | 119.9 | 4.9 |
| M13 | 2.4095 | 2.4055 | 0.0040 | 391.6 | 371.2 | 20.4 | 0.6736 | 0.0547 | 0.6189 | 106.1 | 100.1 | 6 |
| M14 | 2.4218 | 2.4181 | 0.0037 | 391.5 | 371.3 | 20.2 | 1.9867 | 0.4943 | 1.4924 | 87.4 | 80.6 | 6.8 |
| M15 | 2.4349 | 2.4308 | 0.0041 | 390.6 | 371.2 | 19.4 | 3.0853 | 0.0075 | 3.0778 | 68.6 | 61.4 | 7.2 |
| M16 | 2.4473 | 2.4419 | 0.0054 | 389.2 | 370.6 | 18.6 | 5.455 | 10.2352 | −4.7802 | 50.1 | 43 | 7.1 |
| M17 | 2.4573 | 2.4499 | 0.0074 | 387.1 | 369.0 | 18.1 | 64.6458 | 77.6533 | −13.0075 | 32 | 25.9 | 6.1 |
| M18 | 2.4635 | 2.4539 | 0.0096 | 384.3 | 365.9 | 18.4 | 83.7533 | 73.8717 | 9.8816 | 14.8 | 11.5 | 3.3 |

∗ The phase advance of M9 cannot be calculated using equation (4) because good field flatness cannot be achieved by just taking cavity geometry data from the TESLA documents. The ideal phase is 180°.

1) "ANSOFT HFSS FIELD CALCULATOR COOKBOOK", a document of Ansoft Corporation.

# 4    Results and comparisons with TESLA paper

The parameters of monopole, dipole and quadrupole modes of both the 4.5-cell cavity and the 9-cell cavity models are calculated automatically by running the scripts. There is little difference between the results obtained by the 9-cell and the 4.5-cell cavities. The first two bands of monopole mode parameters are listed in Table 1 and compared with the values of the TESLA paper. From M1 to M9 are the first band (TM010) modes, and M9 ($\pi$-mode) is the fundamental mode used to accelerate the beam. From M10 to M18 are the second band (TM011) modes which belong to HOMs.
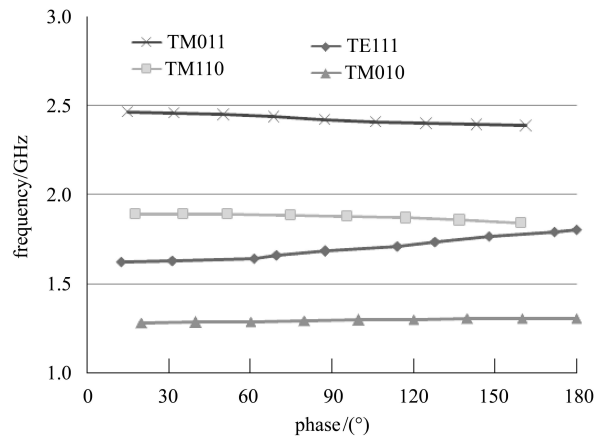


Fig. 4.   Dispersion curves for monopole and dipole modes in the lowest two bands.

From Table 1, it is found that the difference between the results calculated and the values from the paper is small. It is also in agreement with the TESLA paper that the fundamental mode M9 has the largest $R/Q$ and M17 and M18 are the dangerous HOMs with larger $R/Q$. More accurate values might be obtained if one tunes the size of the end cells carefully to get good electric field flatness. Dipole modes and quadrupole modes are all HOMs and the results are also in agreement with the paper for most modes. We should pay more attention to the first two bands of dipole modes (TE111 and TM110) for they are the most dangerous HOMs. From the results，we can also get the dispersion curves for specified bands. Dispersion curves for monopole and dipole modes in the lowest two bands are given in Fig. 4.

# 5    Summary

Monopole, dipole and quadrupole modes are gotten by the HFSS code using both the 9-cell cavity and the 4.5-cell cavity models. A 10-dgeree cavity model is adopted to correctly get the monopole modes. These cavity models are a very important reference for further simulation of the SPL cavities after slight tuning to get good field flatness. The HFSS scripts used for the calculation of the mode parameters for both the 9-cell cavity and the 4.5-cell cavity models have been written and can be run successfully to get the results. The results are consistent with the values listed in the TESLA paper and dangerous HOMs with larger $R/Q$ values have been found. These scripts which are available in cavities with different cell numbers and geometric structures will be useful in the process of tuning and even in further calculation of the SPL cavities and other cavities.

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