1D theory of laser plasma wake*

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Abstract In this paper, we get the 1D approximate analytical solution of the plasma electrostatic wake driven by the laser, and get the modified oscillating frequency of this wake. Finally we analyze the longitudinal beam dynamics in this electrostatic wake, and find that the high order terms don't change the topology of the longitudinal phase space.

Key words LWFA, electrostatic, beam dynamics

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Laser plasma wakefield accelerators (LWFA), in which the plasma wakefield is excited by an intense laser to accelerate the particles, have demonstrated accelerating gradient of hundreds of GV/m, and have become the highlight of advanced accelerator concept. In this paper, we get the 1D analytical solution of the plasma electrostatic wake driven by the laser, and get the modified oscillating frequency of this wake. Finally we analyze the longitudinal beam dynamics in this electrostatic wake, and find that the high order terms don't change the topology of the longitudinal phase space.

We consider the homogeneous unmagnetized plasma. Laser travels in the z direction with the initial frequency $\omega_0 \gg \omega_p$, ω_p is the electron plasma frequency, and the group and the phase speed $v_g \approx v_p \approx c$ (c is the light velocity). We adopt one dimensional theory, the LWFA system is governed by the following set of equations[1]:

$$\frac{\partial n_e}{\partial t} + \frac{\partial n_e v_{||}}{\partial z} = 0\,, \tag{1}$$

$$\widehat{z} \times \frac{\partial \vec{E}}{\partial z} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}\,, \tag{2}$$

$$\widehat{z} \times \frac{\partial \vec{B}}{\partial z} = -\frac{4\pi e}{c} n_e \vec{v} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}\,, \tag{3}$$

$$\frac{\partial E_{||}}{\partial z} = -4\pi e(n_e - n_0)\,. \tag{4}$$

Here, $\vec{E},\vec{B}$ are the electromagnetic fields, and $\vec{v}$ is the electron velocity. We introduce the coordinates $\zeta = k_p(z-ct)$ ($k_p = \omega_p/c$), and normalized scalar and vector potentials, $\phi(\zeta)$ and $a(\zeta)$, such that $E_{||} = -\frac{mc^2}{e}\frac{\partial \phi}{\partial \zeta}$, $E_{\perp} = \frac{mc^2}{e}\frac{\partial a}{\partial \zeta}$, where $\perp$ and $||$ refer to the components perpendicular and parallel to $\widehat{z}$. From the above equations, we can get the equations of $\phi(\zeta)$ and $a(\zeta)$,

$$\frac{\partial^2 a}{\partial^2 \zeta} = \frac{n_e}{n_0}\frac{a}{\gamma}\,, \tag{5}$$

$$\frac{\partial^2 \phi}{\partial^2 \zeta} = \left(\frac{n_e}{n_0} - 1\right)\,, \tag{6}$$

where $k_p = 2\pi/\lambda_p$ is the electron plasma wave number, and γ is the relativistic factor of electron. From the Hamiltonian of the electron and the continuity equation, one can get the following relation

$$\frac{n_e}{n_0} = \frac{\gamma}{1+\phi} = \frac{1+a^2+(1+\phi)^2}{2(1+\phi)^2}\,. \tag{7}$$

Finally, we get the 1D self-consistent equations for LWFA system

$$\frac{\mathrm{d}^2 a}{\mathrm{d}^2 \zeta} = \frac{a}{1+\phi}\,, \tag{8}$$

$$\frac{\mathrm{d}^2 \phi}{\mathrm{d}^2 \zeta} = \frac{1}{2}\left(\frac{1+a^2}{(1+\phi)^2} - 1\right). \tag{9}$$

In this paper, we assume that a is unchanged and ϕ is in the vicinity of zero (small variable) $\phi \propto \varepsilon$ to

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get the analytical solution of the electrostatic plasma wake. We expand Eq. (9) into the form

$$\phi''+\phi-\frac{3}{2}\phi^2+2\phi^3-\frac{5}{2}\phi^4+\cdots=$$
$$\varepsilon^2a^2(1-2\phi+3\phi^2-4\phi^3+5\phi^4+\cdots). \qquad (10)$$

Using KBM method[2], one expands ϕ as the following

$$\phi=u\cos\theta+\sum_{1}^{N}\varepsilon^n\phi_n(u,\theta)+o(\varepsilon^n)\,. \qquad (11)$$

The equations of u and θ are

$$\frac{\mathrm{d}u}{\mathrm{d}t}=\sum_{1}^{N}\varepsilon^nA_n(u)\,, \qquad (12)$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}t}=\sum_{1}^{N}\varepsilon^n\theta_n(u)\,. \qquad (13)$$

From Eq. (10), we remove the secular terms to obtain the first order solution

$$A_1=0, \qquad \theta_1=0, \qquad (14)$$

$$\phi_1=\frac{3u^2}{4}-\frac{u^2}{4}\cos2\theta\,, \qquad (15)$$

and also the second order solution

$$A_2=0, \quad \theta_2=a^2-\frac{9u^2}{32}\,, \qquad (16)$$

$$\phi_2=a^2+\frac{11u^3}{128}\cos3\theta\,. \qquad (17)$$

So the solution of the electrostatic plasma wake potential to second order is

$$\phi=u\cos\theta+\varepsilon\left(\frac{3u^2}{4}-\frac{u^2}{4}\cos2\theta\right)+\varepsilon^2\left(a^2+\frac{11u^3}{128}\cos3\theta\right), \qquad (18)$$

where $\theta=k_p\left(1+a^2-\frac{9u^2}{32}\right)(z-ct)$. In (18), if we reserve the zero order term only, then $\theta=k_p(z-ct)$, the wake oscillating frequency is the usual electron plasma frequency ω_p. But the wake oscillating frequency is modified as (19), if we include the high order terms, ω_p' is related to the oscillating amplitude

$$\omega_p'=\omega_p\left(1+a^2-\frac{9u^2}{32}\right). \qquad (19)$$

Now, we begin to analyze the longitudinal beam dynamics in the electrostatic potential ϕ. In doing so, the expression of θ should be expressed as $\theta=k_p\left(1+a^2-\frac{9u^2}{32}\right)(z-v_pt)$, $v_p<c$. We incorporate the new variable $s=k_pz$, the longitudinal equations of motion are

$$\frac{\mathrm{d}\gamma}{\mathrm{d}s}=\frac{\partial\phi}{\partial\theta}\,, \qquad (20)$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}s}=1-\frac{\beta_p}{\beta}\,, \qquad (21)$$

where $\beta_p=v_p/c$, $\beta=(1-\gamma^{-2})^{\frac{1}{2}}$. The function ϕ is shown in the Fig. 1.

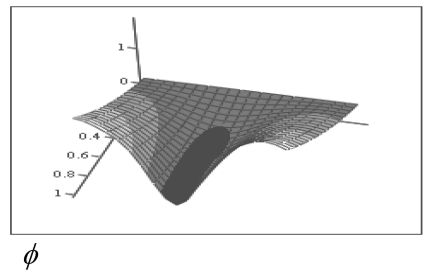

Fig. 1. The function ϕ.

The fixed points of the longitudinal equations of motion are $\gamma=\gamma_p$, $\theta=n\pi$ (n is integer). The Jacobian matrix of (20), (21) is

$$J=\begin{bmatrix}0 & \frac{\partial^2\phi}{\partial^2\theta}\\ \frac{1}{\gamma_p\beta_p^2} & 0\end{bmatrix}.$$

So the characteristic value λ satisfies $\lambda^2=\frac{1}{\gamma_p\beta_p^2}\frac{\partial^2\phi}{\partial^2\theta}$. It is obvious that $\theta=0$ is the stable point and $\theta=\pi,-\pi$ are the saddle points. If we remove the second and third terms in (18), we will get the same conclusion. So the high order terms in the electrostatic potential don't change the topology of the longitudinal phase space. Fig. 2 gives the longitudinal phase space near the zero phase, when $u=0.5$, $\beta_p=0.98$.

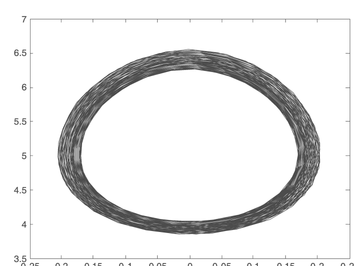

Fig. 2. Longitudinal phase space near $\theta=0$.

References

1 Chen Pisin, Spitkovsky A. Proceeding of AAC, 1998
2 Kalmykov S et al. Phil. Trans. R.Soc. A, 2006, **364**: 725