Towards Lambda-nucleon coupling constants in relativistic mean field theory *

LÜ Hong-Feng(吕洪凤)[1;1)] YAO Jiang-Ming(尧江明)[2;2)]

1 (College of Science, China Agriculture University, Beijing 100083, China)

2 (School of Physics, Peking University, Beijing 100871, China)

Abstract New parameter sets for Λ-nucleon coupling in relativistic mean field theory are proposed based on nucleon-nucleon effective interaction PK1. Hypernuclear properties are described well through a systematical study. Effects of hyperon tensor coupling term on spin-orbit splitting are also investigated self-consistently.

Key words Lambda-nucleon coupling constants, Lambda-hypernuclei, relativistic mean field theory

PACS 21.30.Fe, 21.80.+a,21.10.Dr

1 Introduction

For the world of matter made of u, d, s quarks, Λ hypernuclei are the optimal objects that allow one to study the behaviors of bound hyperons. Such an object extend hyperon-nucleon, hyperon-hyperon interactions into a unified picture of baryon-baryon interactions for understanding short-range nuclear forces like Λ-N spin-dependent forces[1, 2] and high density nuclear matter like neutron-star[3].

Short-range parts of nuclear force such like repulsive core and spin-orbit force causes elementary properties of nucleus, like saturation, magic numbers etc. Such characters naturally appear in relativistic methods, e.g., relativistic mean field (RMF) theory [4].

Based on effective nucleon-nucleon interaction PK1, which provides good description for the properties of both nuclear matter and nuclei in and far from the valley of β stability[5], new effective lambda-nucleon interaction are proposed in RMF theory. It's noted that a tensor coupling term of Λ to the vector fields are included self-consistently.

This paper is organized as follows: Section 2 contains an outline of the RMF model. Effective Λ-nucleon interaction determining procedure, application of adjusted interaction on hyperon splitting energy, baryon spin-orbit potential, central potential and single particle energy of the spherical ground states, are analyzed in Section 3. The results are briefly summarized in Section 4.

2 Theoretical framework

In RMF theory, one describes the baryons (B=n, p, Λ) in a nucleus as Dirac spinors ($\psi_{\rm B}$, $m_{\rm B}$) moving in the fields of mesons: isoscalar-scalar meson (σ, m_σ, g_σ), isoscalar-vector meson (ω, m_ω, g_ω), isovector-vector meson ($\boldsymbol{\rho}$, m_ρ, g_ρ) and the photo (A). The field tensor for the ω-meson is given as $\Omega_{\mu\nu}=\partial_\mu\omega_\nu-\partial_\nu\omega_\mu$ and by similar expressions for the ρ-meson, and the photon. The Lagrangian density including the non-linear self-coupling of the σ field (coupling constants g_2, g_3), the ω field (coupling constant c_3) and the tensor coupling term (coupling constant $f_{\omega\rm BB}$) for the vector meson is constructed as:

$$\mathcal{L} = \bar\psi_{\rm B}(\not{p}_{\rm B}-m_{\rm B}-g_{\sigma\rm BB}\sigma-g_{\omega\rm BB}\not{\omega}-g_{\rho\rm BB}\not{\boldsymbol{\rho}}\boldsymbol{\tau})\psi_{\rm B}- \bar\psi_{\rm B}Q_{\rm B}\not{A}\psi_{\rm B}+\frac{f_{\omega\rm BB}}{2m_{\rm B}}\bar\psi_{\rm B}\sigma_{\mu\nu}\partial^\mu\omega^\nu\psi_{\rm B}+ \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma-\frac{1}{2}m_\sigma^2\sigma^2-\frac{1}{3}g_2\sigma^3-\frac{1}{4}g_3\sigma^4-$$

Received 3 September 2008

* Supported by Scientific Research Foundation of China Agriculture University (2007005)

1) E-mail: hongfeng@cau.edu.cn

2) E-mail: jmyao@pku.edu.cn

$$\frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu}+\frac{1}{2}m_{\omega}^{2}\omega_{\mu}\omega^{\mu}+\frac{1}{4}c_3(\omega_{\mu}\omega^{\mu})^2-$$
$$\frac{1}{4}\boldsymbol{R}_{\mu\nu}\boldsymbol{R}^{\mu\nu}+\frac{1}{2}m_{\rho}^{2}\boldsymbol{\rho}_{\mu}\boldsymbol{\rho}^{\mu}-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad (1)$$

where $\sigma^{\mu\nu} = \frac{1}{2i}[\gamma^{\mu},\gamma^{\nu}]$, the charge Q_B is either $e(1-\tau_3)/2$ for nucleon or 0 for hyperon Λ. As the ωBB tensor coupling terms is negligible for nucleons[6], only the tensor coupling term for Λ will be considered in this work.

The equations of motion for baryons can be derived from the Lagrangian density in Eq.(1) with variational principle as:

$$[\gamma_{\mu}(i\partial^{\mu}-V_{B}^{\mu})-(m_B+S_B)+\frac{f_{\omega BB}}{4m_B}\sigma_{\mu\nu}\Omega^{\mu\nu}]\psi_B=0, \quad (2)$$

where the scalar potential S_B and vector potential V_B^{μ} are given by

$$\begin{cases} S_B = g_{\sigma BB}\sigma, \\ V_B^{\mu} = g_{\omega BB}\omega^{\mu}+g_{\rho BB}\tau_3\cdot\rho_3^{\mu}+Q_B A^{\mu}. \end{cases} \quad (3)$$

Potentials are determined in the mean-field approximation from solutions of Klein-Gordon equations for mesons and Coulomb field. The system of equations being restricted to the spherical symmetry is solved self-consistently in coordinate space within the box of 20 fm and a step size of 0.1 fm. PK1 set for nucleon-nucleon interaction is adopted. For the detailed formalism and numerical techniques, see Ref.[7] and the references therein.

3 Results

To get the consistent estimation of hypernuclei, the scalar and vector vertex factors $R_{\sigma}=g_{\sigma\Lambda\Lambda}/g_{\sigma NN}$, $R_{\omega}=g_{\omega\Lambda\Lambda}/g_{\omega NN}$ for Λ-nucleon interaction are optimized to the available binding energy of Λ in ground state nuclei ${}^{12-14}_{\Lambda}$C, ${}^{14,15}_{\Lambda}$N, ${}^{16}_{\Lambda}$O, ${}^{28}_{\Lambda}$Si, ${}^{32}_{\Lambda}$S, ${}^{40}_{\Lambda}$Ca, ${}^{51}_{\Lambda}$V, ${}^{89}_{\Lambda}$Y, ${}^{139}_{\Lambda}$La and ${}^{208}_{\Lambda}$Pb. Ratios (R_{σ}, R_{ω}) are varied with a step size 0.02. By systematically minimizing the accumulated squared deviation

$$\chi^2 \equiv \sum_i \frac{(E_i^{exp.}-E_i^{cal.})^2}{(\Delta E_i^{exp.}/E_i^{exp.})^2}, \quad (4)$$

proper parameter sets are found in least-square fit. $\Delta E_i^{exp.}$ is experimental error bar and the relative error of the data $\frac{\Delta E_i^{exp.}}{E_i^{exp.}}$ plays a role as sensitivity coefficient. Since there is no experimental uncertainty of ${}^{14}_{\Lambda}$N, its relative weight is fixed with the largest percentage 3% of those light hypernuclei.

Table 1. Parameter sets for hyperon-meson interaction based on PK1 set, where $R_x=g_{x\Lambda\Lambda}/g_{xNN}|_{x=\sigma,\omega}$. The total square deviation from the experimental data $\Delta^2=\sum_i(E_i^{exp.}-E_i^{cal.})^2$, the relative square deviation $\delta^2=\sum_i(E_i^{exp.}-E_i^{cal.})^2/(E_i^{exp.})^2$, $\chi^2=\sum_i(E_i^{exp.}-E_i^{cal.})^2\times(E_i^{exp.})^2/(\Delta E_i^{exp.})^2$.

sets	$f_{\omega\Lambda\Lambda}/g_{\omega\Lambda\Lambda}=0$		$f_{\omega\Lambda\Lambda}/g_{\omega\Lambda\Lambda}=1$		
	S01	S02	ST1	ST2	FT1
R_{σ}	0.615	0.377	0.618	0.386	0.649
R_{ω}	0.667	0.377	0.667	0.386	0.704
$\chi^2(10^4)$	11.143	22.133	3.523	12.399	3.233
$\delta^2(10^{-4})$	373	577	576	437	346
Δ^2	11.06	20.32	8.28	14.11	8.77

Besides χ^2, mean-square error δ and average error Δ are respectively introduced for comparison. χ^2 distribution for the deviation of Λ-single particle energies either without $(f_{\omega\Lambda\Lambda}/g_{\omega\Lambda\Lambda}=0)$ and with $(f_{\omega\Lambda\Lambda}/g_{\omega\Lambda\Lambda}=1)$ Λ tensor coupling term are shown in Table 1. Sets S01 and ST1 are obtained by keeping $R_{\omega}=2/3$, which comes from naive quark model. Sets S02 and ST2 are get by simply assuming $R_{\omega}=R_{\sigma}$. Set FT1 is adjusted keeping both R_{ω} and R_{σ} free.According to the value of χ^2, FT1 is the best one among these five sets.

To know the systematical application, comparison of experimental data[8—13] (filled squares) and the calculated Λ binding energies versus $A^{-2/3}$ (A is the hypernuclear mass, from 9 to 208) with Λ-nucleon interactions S01, S02, ST1, ST2, and FT1 are presented in Fig. 1. Obviously, the parameter sets with tensor coupling term show a better agreement than those without tensor coupling term.

As p state spin-orbit splitting 0.152±0.09 MeV of ${}^{13}_{\Lambda}$C[1] is well known to show the novel character of hypernuclear structure introduced by hyperon, the hyperon spin-orbit splitting of p states calculated with S01, S02, ST1, ST2 and FT1 sets are presented in Fig. 2. It demonstrates the importance of the tensor coupling term even there is no sufficient spin-orbit splitting data.

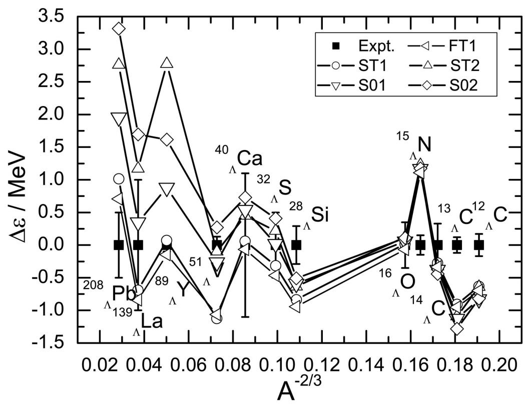

Fig. 1. The divergence of hyperon binding energy from experimental data, where nucleon-nucleon interaction is PK1.

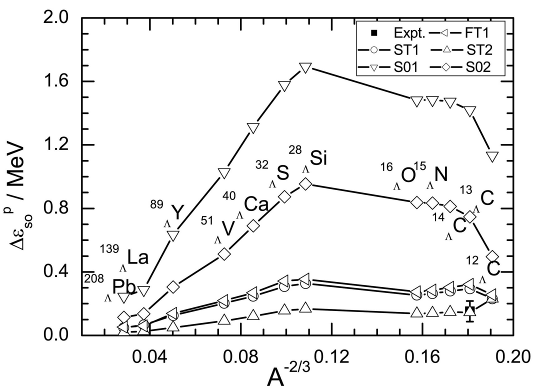

Fig. 2. The spin-orbit splitting of Λ 1p states with parameter sets listed in Table 1.

Microscopic reason for the phenomena shown in Fig. 2 can be found by converting Dirac equations into a Schrödinger-like equation, where the spin-orbit potential is:

$$V_{\rm so}\boldsymbol{l}\cdot\boldsymbol{s} = \left[\frac{1}{2\overline{m}^2}\frac{1}{r}\frac{\rm d}{{\rm d}r}(V-S)-\frac{2V_{\rm T}}{\overline{m}}\frac{1}{r}\right]\boldsymbol{l}\cdot\boldsymbol{s}, \quad (5a)$$

$$\overline{m} = m-\frac{1}{2}(V-S);\ V_{\rm T}=\frac{f_{\omega\Lambda\Lambda}}{2m_{\Lambda}}\partial_r\omega_0. \quad (5b)$$

As $f_{\omega\Lambda\Lambda}>0$, there are two negative contribution to the spin-orbit potential. This is also one of the great advantages of the relativistic treatment that the spin-orbit interaction is automatically included in baryons motion equations.

Taken ${}^{16}_{\Lambda}$O as an example, the spin-orbit potential $V_{\rm so}$ calculated with FT1 set shows that with tensor part, the contributions to hyperon spin orbit potential from nuclear core and tensor coupling term almost cancel each other, which will result in a highly suppressed spin-orbit potential with the magnitude less than 1 MeV. Spin-orbit splitting of Λ 1p states without and with tensor coupling term are reduced obviously from 1.68 to 0.26 MeV.

4 Summary

Preliminary studies on parameter sets for the Λ-nucleon interaction in relativistic mean field theory are proposed based on nucleon-nucleon effective interaction PK1. Two hyperon parameters–the scalar and vector vertex factors (R_σ, R_ω)– related to nucleon-nucleon coupling strength are determined by a fit to hyperon binding energy of ground state Λ-hypernuclei. Sets FT1 provides an excellent systematical description of hypernuclei. Spin properties are uniquely fixed for the relativistic character of the model. The effects of tensor coupling term of Λ hyperon to the vector fields on the spin-orbit splitting are studied self-consistently. Further delicate investigation is in progress.

References

1 Ajimura S et al. Phys. Rev. Lett., 2001, **86**: 4255
2 Ukai M et al. Phys. Rev. Lett., 2004, **93**: 232501
3 Hashimoto O, Tamura H. Prog. Part. Nucl. Phys., 2006, **57**: 564
4 Vretenar D, Afanasjev A V, Lalazissis G A et al. Phys. Rep., 2005, **409**: 101
5 LONG W H, MENG J, Van Giai N et al. Phys. Rev. C, 2004, **69**: 034319
6 Cohen J, Weber H J. Phys. Rev. C, 1991, **44**: 1181
7 Sharma M M, Nagarajan M A, Ring P. Phys. Lett. B, 1993, **312**: 377
8 Davis D H, Pniewski J. Contemp. Phys., 1986, **27**: 91
9 Hotchi H et al. Phys. Rev. C, 2001, **64**: 044302
10 Pile P H et al. Phys. Rev. Lett., 1991, **66**: 2585
11 Nagae T. Nucl. Phys. A, 2000, **670**: 269c
12 Hasegawa T et al. Phys. Rev. Lett., 1995, **74**: 224; Phys. Rev. C, 1996, **53**: 1210
13 Tamura H. Eur. Phys. J. A, 2002, **13**: 181