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Isospin structure in ^{68}Ge*

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Abstract The interacting boson model-3(IBM-3) has been used to study the low-energy level structure and electromagnetic transitions of ^{68}Ge nucleus. The main components of the wave function for some states are also analyzed respectively. The theoretical calculations are in agreement with experimental data, and the ^{68}Ge is in transition from $U(5)$ to $SU(3)$.

Key words IBM-3, energy level, isospin, electromagnetic transitions

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1 Introduction

Nuclei with $N \approx Z$ have been a subject of intense interest during the last few years.[1—5]. The main reason is that the structure of these nuclei provides a sensitive test for the isospin symmetry of nuclear force. The interacting boson model (IBM)[6—8] is an algebraic model used to study the nuclear collective motions. For lighter nuclei, the valence protons and neutrons are filling the same major shell and the isospin should be taken into account, so the IBM has been extended to the interacting boson model with isospin(IBM-3)[9]. In the IBM-3,three types of bosons including proton-proton(π),neutron-neutron(ν) and proton-neutron(δ) forms the isospin T=1 triplet. The dynamical symmetry group for IBM-3 is $U(18)$, which starts with $U_{\rm sd}(6)\times U_{\rm c}(3)$ and must contain $SU_{\rm T}(2)$ and $O(3)$ as subgroups because the isospin and the angular momentum are good quantum numbers. The natural chains of IBM-3 group $U(18)$ are the following)[10]

$$U(18)\supset(U_{\rm c}(3)\supset SU_{\rm T}(2))\times (U_{\rm sd}(6)\supset U_{\rm d}(5)\supset O_{\rm d}(5)\supset O_{\rm d}(3)),$$
$$U(18)\supset(U_{\rm c}(3)\supset SU_{\rm T}(2))\times (U_{\rm sd}(6)\supset O_{\rm sd}(6)\supset O_{\rm d}(5)\supset O_{\rm d}(3)),$$
$$U(18)\supset(U_{\rm c}(3)\supset SU_{\rm T}(2))\times (U_{\rm sd}(6)\supset SU_{\rm sd}(3)\supset O_{\rm d}(3)).$$

The subgroups $U_{\rm d}(5), O_{\rm sd}(6)$ and $SU_{\rm sd}(3)$ describe vibrational, γ-unstable and rotational nuclei respectively.

2 The IBM-3 Hamiltonian

The IBM-3 Hamiltonian can be written as[9]

$$H=\varepsilon_{\rm s}\hat{n}_{\rm s}+\varepsilon_{\rm d}\hat{n}_{\rm d}+H_2, \quad (1)$$

where

$$\begin{aligned}H_2 = {} & \frac{1}{2}\sum_{L_2T_2}C_{L_2T_2}[(\rm d^\dagger d^\dagger)^{L_2T_2}\cdot(\tilde{d}\tilde{d})^{L_2T_2}]+\\ & \frac{1}{2}\sum_{T_2}B_{0T_2}[(\rm s^\dagger s^\dagger)^{0T_2}\cdot(\tilde{s}\tilde{s})^{0T_2}]+\\ & \sum_{T_2}A_{2T_2}[(\rm s^\dagger d^\dagger)^{2T_2}\cdot(\tilde{d}\tilde{s})^{2T_2}]+\\ & \frac{1}{\sqrt{2}}\sum_{T_2}D_{2T_2}[(\rm s^\dagger d^\dagger)^{2T_2}\cdot(\tilde{d}\tilde{d})^{2T_2}]+\\ & \frac{1}{2}\sum_{T_2}G_{0T_2}[(\rm s^\dagger s^\dagger)^{0T_2}\cdot(\tilde{d}\tilde{d})^{0T_2}],\end{aligned} \quad (2)$$

and

$$(\rm b_1^\dagger b_2^\dagger)^{L_2T_2}\cdot(\tilde{b_3}\tilde{b_4})^{L_2T_2}=(-1)^{(L_2+T_2)}$$
$$\sqrt{(2L_2+1)(2T_2+1)}\times[(\rm b_1^\dagger b_2^\dagger)^{L_2T_2}\cdot(\tilde{b_3}\tilde{b_4})^{L_2T_2}]^{00}$$
$$(\tilde{\rm b}_{(lm,m_z)}=(-1)^{(l+m+1+m_z)}b_{(l-m-m_z)}$$

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where the symbols T_2 and L_2 represent the two-boson isospin and angular momentum, respectively. The parameters A,B,C,D, and G are the two-body matrix elements

$A_{T_2} = \langle sd20 \mid H_2 \mid sd20\rangle, T_2 = 0,1,2; B_{T_2} = \langle s^2 0T_2 \mid H_2 \mid s^2 0T_2\rangle, G_{T_2} = \langle s^2 0T_2 \mid H_2 \mid d^2 0T_2\rangle, D_{T_2} = \langle sd2T_2 \mid H_2 \mid d^2 2T_2\rangle$ and $C_{L_2T_2} = \langle d^2 L_2T_2 \mid H_2 \mid d^2 L_2T_2\rangle$, with $T_2 = 0,2, L_2 = 0,2,4; C_{L_21} = \langle d^2L_21 \mid H_2 \mid d^2L_21\rangle$ with $L_2 = 1,3$.

IBM-3 Hamiltonian can be expressed in Casimir operator form, i.e.,

$$H_{\text{Casimir}} = \lambda C_{2U_{\text{sd}}(6)} + a_{\text{T}}T(T+1) + a_1C_{1U_{\text{d}}(5)} + a_3C_{2SU_{\text{sd}}(3)} + a_2C_{2U_{\text{d}}(5)} + a_4C_{2O_{\text{d}}(5)} + a_5C_{O_{\text{d}}(3)}. \quad (3)$$

The low-lying levels of[6—8] nucleus can be described by the following Hamiltonians,

$$H_{\text{Casimir}} = -0.019C_{2U_{\text{sd}}(6)} + 1.46T(T+1) + 0.1556C_{1U_{\text{d}}(5)} + 0.14C_{2SU_{\text{sd}}(3)} + 0.155C_{2U_{\text{d}}(5)} + 0.115C_{2O_{\text{d}}(5)} + -0.127C_{O_{\text{d}}(3)}. \quad (4)$$

3 Energy levels

The energy levels and wave function are given by the computation program written by Van Isacker[11]. The parameters of the calculation are listed in Table 1.

Table 1. The parameters of the IBM-3 Hamiltonian of the ^{68}Ge nucleus.

$\varepsilon_{\text{d}\rho}(\rho=\pi,\nu,\delta)$	4.836		
$\varepsilon_{\text{s}\rho}(\rho=\pi,\nu,\delta)$	4.207		
$A_i(i=0,1,2)$	-5.34	3.442	3.442
$C_{i0}(i=0,2,4)$	-5.246	-5.461	-5.234
$C_{i2}(i=0,2,4)$	-3.541	3.334	3.526
$C_{i1}(i=1,3)$	-2.712	-3.982	
$B_i(i=0,2)$	-5.878	2.882	
$D_i(i=0,2)$	-1.048	-1.048	
$G_i(i=0,2)$	1.253	1.253	

The calculated and experimental energy levels[12] are exhibited in Fig.1. When the spin value is below 8^+, the theoretical calculations are in agreement with experimental data.

We have analyzed the wave function of the 0_1^+,2_1^+ and 4_1^+, they are:

$$|0_1^+\rangle = -0.64444 \mid s_\nu^4 s_\pi^2\rangle + 0.3676 \mid s_\nu^3 s_\pi d_\nu d_\pi\rangle + 0.3184 \mid s_\pi^1 s_\nu^2 d_\nu^2\rangle - 0.1592 \mid s_\nu^2 s_\pi d_\nu s_\delta d_\delta\rangle + \cdots,$$

$$|2_1^+\rangle = -0.4581 \mid s_\nu^4 s_\pi d_\pi\rangle + 0.3239 \mid s_\nu^3 s_\pi^2 d_\nu + 0.2290 \text{mid} s_\nu s_\delta^4 d_\nu\rangle + \cdots,$$

$$|4_1^+\rangle = 0.4062 \mid d_\nu^2 s_\delta^4\rangle - 0.2872 \mid s_\nu d_\nu s_\delta^3 d_\delta\rangle + 0.2623 \mid s_\nu^2 s_\pi d_\nu d_\delta s_\delta\rangle - 0.2364 \mid s_\nu^3 s_\pi d_\delta^2\rangle + \cdots.$$

We found that the main components of the wave function for the states above are s^N, $s^{N-1}d$, $s^{N-2}d^2$ and so on configurations. The wave function of these states contain a significant amount of δ boson component, which shows that it is necessary to consider the isospin effect for the light nuclei.

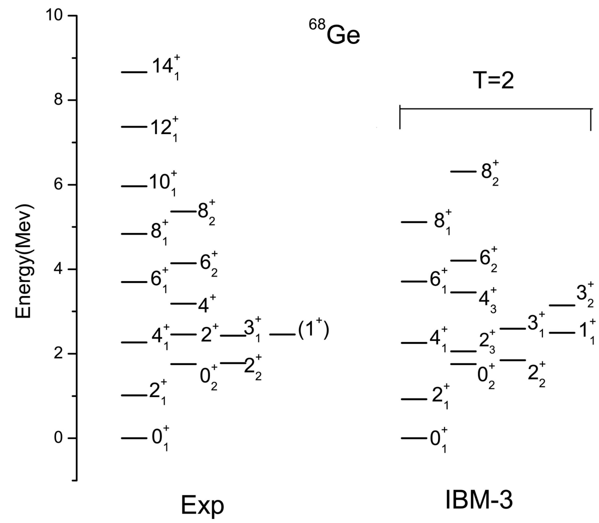

Fig. 1. Comparison between lowest excitation energy bands of the IBM-3 calculation and experimental excitation energies of ^{68}Ge.

4 Electromagnetic transition

In the IBM-3 model, the quadrupole operator was expressed as[13]:

$$Q = Q^0 + Q^1, \quad (5)$$

Where

$$Q^0 = \alpha_0\sqrt{3}[(\text{s}^\dagger\tilde{\text{d}})^{20} + (\text{d}^\dagger\tilde{\text{s}})^{20}] + \beta_0\sqrt{3}(\text{d}^\dagger\tilde{\text{d}})^{20}, \quad (6)$$

$$Q^1 = \alpha_1\sqrt{2}[(\text{s}^\dagger\tilde{\text{d}})^{21} + (\text{d}^\dagger\tilde{\text{s}})^{21}] + \beta_1\sqrt{2}(\text{d}^\dagger\tilde{\text{d}})^{21}. \quad (7)$$

The M1 transition is also a one-boson operator with an isoscalar part and an isovector part

$$M = M^0 + M^1, \quad (8)$$

Where

$$M^0 = g_0\sqrt{3}(\text{d}^\dagger\tilde{\text{d}})^{10} = g_0L/\sqrt{10}, \quad (9)$$

$$M^1 = g_1\sqrt{2}(\text{d}^\dagger\tilde{\text{d}})^{11}. \quad (10)$$

For the ^{68}Ge, $\alpha_0 = \beta_0 = 0.12$, $\alpha_1 = \beta_1 = 0.102$eb, $g_0 = 0{,}05$, $g_1 = 2.5$ respectively. Table 2 gives the electromagnetic transition rate calculated by IBM-3.

Table 2 shows that the calculated B(E2) values

Table 2. Experimental and calculated $B(\mathrm{E2})$ ($e^2\mathrm{fm}^4$) and $B(\mathrm{M1})(\mu_\mathrm{N}^2)$ for ^{68}Ge.

$J_j^+\mathrm{i} \rightarrow J_\mathrm{f}^+$	B(E2)		B(M1)	
	Exp.	Cal.	Exp.	Cal.
$2_1^+ \rightarrow 0_1^+$	290.22	299.06		
$2_2^+ \rightarrow 0_1^+$	2.97	0		
$2_2^+ \rightarrow 2_1^+$	7.09	0	0.007	0
$1_1^+ \rightarrow 2_2^+$		345.74		0.2541
$4_1^+ \rightarrow 2_1^+$	229.21	184.87		
$4_2^+ \rightarrow 2_1^+$	8.25	170.99		

are quite close to the experimental ones[12]. The calculated quadrupole moments of the 2_1^+ state is $Q(2_1^+)$=0.3256eb. From the IBM-3 Hamiltonian expressed in Casimir operator form, we know that the ^{68}Ge is in transition from $U(5)$ to $SU(3)$.

5 Conclusion

The interacting boson model-3(IBM-3) has been used to study the isospin states and electromagnetic transitions for ^{68}Ge nucleus. The main components of the wave function for some states are also analyzed respectively. The calculated quadrupole moments of the 2_1^+ state is 0.3256eb. According to this study, the ^{68}Ge is in transition from $U(5)$ to $SU(3)$. The calculated results are compared with available experimental data, and they are in general good agreement.

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