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Superfluid nuclear matter in BCS theory and beyond*

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Abstract Medium polarization effects are studied for $^1S_0$ pairing in nuclear matter within BHF approach. The screening potential is calculated in the RPA limit, suitably renormalized to cure the low density mechanical instability of nuclear matter. The self-energy corrections are consistently included resulting in a strong depletion of the Fermi surface. The self-energy effects always lead to a quenching of the gap, whereas it is almost completely compensated by the anti-screening effect in nuclear matter.

Key words pairing in nuclear matter, self-energy effect, medium effect

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1 Introduction

The attractive components of the bare nuclear interaction have led to the investigation of pairing in neutron stars[1] disregarding possible repulsive effect exerted by screening of the force via the medium. A pairing suppression has been found by most calculations in neutron matter[2]. On the contrary, other pairing configurations have not yet been explored since the repulsive components of the direct nuclear interaction cannot support the formation of Cooper pairs. But there are strong indications that, in a nuclear matter environment, the medium polarization of the interaction can favor the formation of Cooper pairs similar to the lattice vibrations in ordinary superconductors. In nuclear matter the medium enhancement of neutron-neutron $^1S_0$ pairing is to be traced back to the proton particle-hole excitations[3], and in finite nuclei to the surface vibrations[4].

A complete microscopic treatment of pairing problem requires vertex and self-energy corrections to be treated on the same footing. In the present work we use a realistic two body force V18[5] in the Born term, and use as vertices in the induced interaction a force which is based on a more modern G-matrix calculation[6] as this was the case for the Gogny force[3, 7]. We now get reasonable renormalization effects of the pairing force and we calculate the corresponding gaps as a function of density in symmetric nuclear matter.

Firstly the screening interaction is discussed in the RPA limit. Then the results are presented. Last part is devoted to the discussion and to the conclusions.

2 Screening interaction

In the present calculation we adopt the G-matrix itself and we try to reduce its complexity with reasonable approximations. For the sake of application to the pairing in the $^1S_0$ channel we select the two particle state with total spin S=0 and isospin T=1. Then the one-bubble interaction can be written as

$$<1\bar{1}|\mathcal{V}_1|1'\bar{1}'>=\frac{1}{4}\sum_{22'\mathrm{ST}}(-)^S(2S+1)(2T+1) <12|G^{\mathrm{ph}}_{\mathrm{ST}}|1'2'>_{\mathrm{A}}<2'\bar{1}|G^{\mathrm{ph}}_{\mathrm{ST}}|2\bar{1}'>_{\mathrm{A}}\Lambda^0(22'), \quad (1)$$

where $1\equiv(\boldsymbol{k}_1,\sigma_1,\tau_1)$ $(1'\equiv(\boldsymbol{k}_{1'},\sigma_{1'},\tau_{1'}))$ and $\bar{1}\equiv(-\boldsymbol{k}_1,\sigma_1,\tau_1)$ $(\bar{1}'\equiv(-\boldsymbol{k}_{1'},\sigma_{1'},\tau_{1'}))$ are the momenta of the pair in the entrance (exit) channel. Λ is the static polarization part. The G-matrix is converted into the ph sector, as it is required to solve the Bethe-Salpeter equation and to sum up the bubble series. We use

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the standard recoupling procedure from pp sector to ph sector[8]. Since the G-matrix incorporates short range pp correlations, its momentum range is shrunk remarkably in comparison with the bare interaction. At variance with the bare interaction, the G-matrix cannot sustain large momentum transfers $\boldsymbol{q}=\boldsymbol{k}-\boldsymbol{k}'$, that justifies the approximation to average it around the Fermi surface, in the limit q=0. As a consequence the q dependence is only located in the integral of the polarization part, giving the Lindhard function.

The problems with the calculation of the ph multi bubble contribution are the following. First, the bubble series with G-matrix insertions have to be previously summed up. But, since the interaction vertices in the ph channel involve particle excitations around the Fermi surface, they can be approximated by the Landau parameters. Second, even replacing the bare interaction vertices by G-matrices, there appears the low density singularity in the RPA in nuclear matter ($F_0=-1$). This problem, discussed in Ref.[2] is remedied by dressing the vertex insertions according to the Babu-Brown induced interaction theory[9].

One can determine the Landau parameters from the microscopic Brueckner theory, performing the double variational derivative of the energy per particle. So doing, a number of contributions are generated that can be calculated one by one[10]. A simple and powerful way to calculate the Landau parameters is to suitably fit the BHF equation of state (EoS) with a functional of the occupation numbers and then to perform the double derivative. A Skyrme-like functional has proved to reproduce accurately the EoS of symmetric as well as spin and isospin asymmetric nuclear matter[11]. Therefore we determine the Landau parameters in that way. The latter are plotted in Fig.1 as a function of the Fermi momentum. As expected F_0 exhibits the well known instability below the saturation point, which makes the RPA series difficult to handle. As in previous papers[3, 12] this drawback can be overcome by the induced interaction theory of Babu and Brown[9]. The numerical results are depicted in Fig.1. The salient feature of the induced interaction is that the renormalization of F_0 prevents any singular behavior to occur below the saturation density.

Therefore we dressed the residual interaction first with the short range correlations (G-matrix instead of bare interaction) and then by the renormalized long range correlations $V_{\rm ph}$ replacing the G-matrix in the RPA series. Since the calculation of the induced interaction with the G-matrix is a quite complex job, we have simplified the problem replacing the G-matrix with the Landau parameters. The way we determine the Landau parameters, the approximation turns out to be better than starting from the G-matrix itself.

The vertex insertions dressing the bubbles must be treated on different footing than the external ones. Using the Landau parameters corresponds to the Landau limit, which is a quite reasonable approximation. In this case the RPA summation of the ph interaction turns out to be algebraic and, expressed in term of the dressed bubble, it is written as

$$\Lambda(q)_{\rm ST}=\frac{\Lambda^0(q)}{1+\Lambda^0(q)\mathcal{L}_{\rm ST}}, \tag{2}$$

where $\mathcal{L}_{\rm ST}$ are the Landau parameters, whose components are commonly denoted by: $\mathcal{L}_{00}=F$, $\mathcal{L}_{01}=F'$, $\mathcal{L}_{10}=G$, $\mathcal{L}_{11}=G'$. In this expression we clearly see

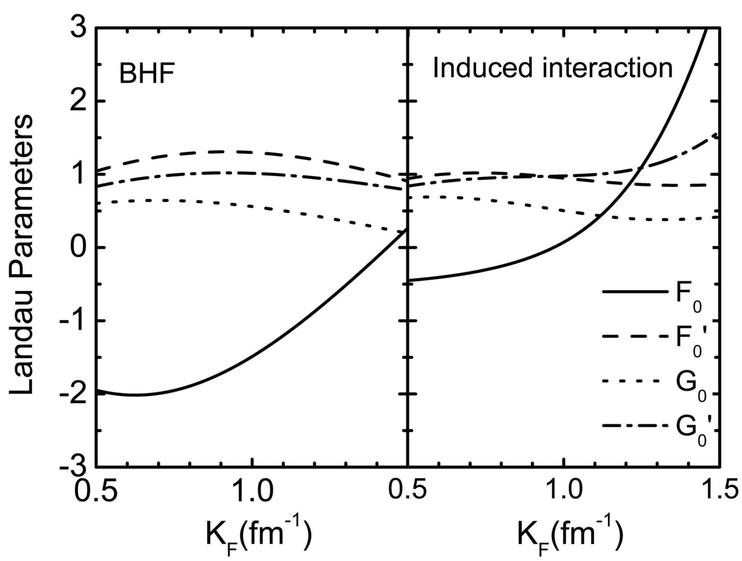

Fig. 1. Landau parameters of nuclear matter.

how the induced interaction prevents any divergence to occur since $|\Lambda\mathcal{L}| \leqslant |\mathcal{L}| \leqslant 1$. Replacing in Eq. (1) the bare bubble Λ^0 with the dressed bubble Λ we get the full screening interaction used in the calculation.

3 Results

In this paper we only focus on the $^1S_0$ pairing interaction in symmetric nuclear matter. In nuclear matter the screening interaction is split as follows

$$\mathcal{V}_1 = \frac{1}{4}(\Lambda(q)_{00}G_{00}^{\rm ph}G_{00}^{\rm ph}+\Lambda(q)_{01}G_{01}^{\rm ph}G_{01}^{\rm ph})- \frac{3}{4}(\Lambda(q)_{10}G_{10}^{\rm ph}G_{10}^{\rm ph}+\Lambda(q)_{11}G_{11}^{\rm ph}G_{11}^{\rm ph}). \quad (3)$$

The various contributions are plotted in left part of Fig.2 in terms of pp states. In nuclear matter the pp G-matrix elements are dominated by the deuteron channel ($^3SD_1$ coupled pp channel), which is very attractive and therefore it reinforces the density mode and weakens the spin mode. In other words, the main isospin effect is to reverse the role of the medium, i.e. antiscreening instead of screening. In previous papers this effect has been discussed in terms of proton-proton ph screening against neutron-neutron ph screening in the neutron-neutron $^1S_0$ channel[13]. The latter gives repulsion the former attraction. At variance with Ref.[14] the proton-proton ph screening is stronger than neutron-neutron ph screening. This effect is to be traced back to stronger in medium renormalization of the force in the $T=0$ channel than in the $T=1$ one. Antiscreening is the overall effect. As we discussed before, the screening effects in fact reinforce the attractive strength of the bare interaction, which can be seen in right part of Fig. 2.

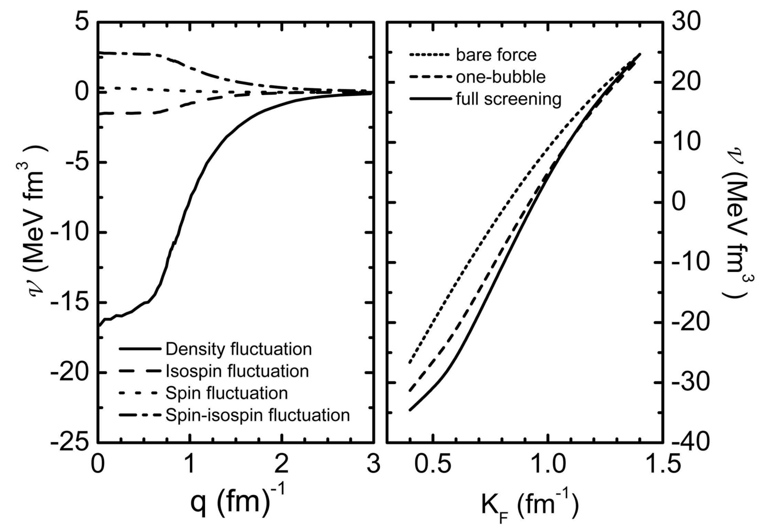

Fig. 2. (Left) Individual components of the ph residual interaction; (Right) Pairing interaction.

One can distinguish the bare interaction which is responsible for the pairing between the two particles in the $^1S_0$ state, from the screening interaction induced by the surrounding particles. Therefore the interaction can be cast as follows

$$< k|\mathcal{V}|k' >= \int\frac{{\rm d}\Omega}{4\pi}[\mathcal{V}_0(\boldsymbol{k},\boldsymbol{k}')+\mathcal{V}_1(|\boldsymbol{k}-\boldsymbol{k}'|)]. \quad (4)$$

We solved the general gap equation[15]

$$\Delta_{\boldsymbol{k}}(\omega) = \sum_{k'}\int\frac{{\rm d}\omega'}{2\pi{\rm i}}\mathcal{V}_{\boldsymbol{k},\boldsymbol{k}'}(\omega,\omega')F_{\boldsymbol{k}'}(\omega'). \quad (5)$$

The results are plotted in Fig. 3. Due to the antiscreening effect, the magnitude of the gap variation is much more sizeable: the gap rises up from 3 MeV to 5 MeV for Fermi momentum $k_{\rm F}=0.8\ {\rm fm}^{-1}$.

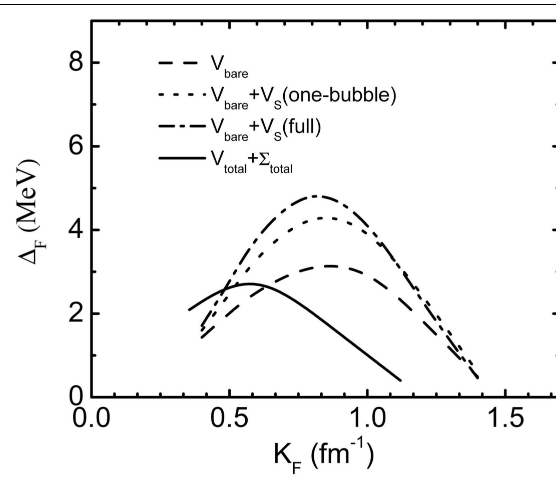

Fig. 3. Pairing gap in the $^1S_0$ channel for symmetric nuclear matter.

There are two kinds of selfenergy effects: dispersive effect and Fermi surface depletion. The first one is a correction to the sp spectrum in the energy denominator. Additional strong reduction is due to the depletion of the Fermi surface which hinders transitions around the Fermi surface. In nuclear matter the self-energy effects are much stronger already at moderately low density, as it has to be expected, and the peak value shifts down to very low density $k_{\mathrm{F}} \approx 0.5-0.6\ \mathrm{fm}^{-1}$.

4 Discussion and conclusions

In this paper an exhaustive treatment of the $^1S_0$ pairing in nuclear matter has been reported. The medium effects on the interaction and the self-energy corrections to the mean field, both developed in the framework of the Brueckner theory, have been included in the solution of the gap equation.

Within the pure mean field approximation[16] the $^1S_0$ gap is not affected by the medium. In nuclear matter the most remarkable result is the antiscreening effect of the medium polarization. In fact in nuclear matter isospin modes arise that reverses the competition between the attractive density modes and the repulsive spin-density modes due to the presence of isospin modes. However the enhancement of the gap to almost 5 MeV is almost completely suppressed by the strong correlation effects on the self-energy. But, even a small variation of the force strength implies a large variation of the gap. These effects also push to lower density the peak value of the gap.

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