Microscopic calculation of the magnetic field of neutron stars*

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Abstract Based on the Dirac equation describing an electron moving in a uniform and cylindrically symmetric magnetic field which may be the result of the self-consistent mean field of the electrons themselves in a neutron star, we have obtained the eigen solutions and the orbital magnetic moments of electrons in which each eigen orbital can be calculated. From the eigen energy spectrum we find that the lowest energy level is the highly degenerate orbitals with the quantum numbers $p_z = 0$, $n = 0$, and $m \geqslant 0$. At the ground state, the electrons fill the lowest eigen states to form many Landau magnetic cells and each cell is a circular disk with the radius $\lambda_{\rm free}$ and the thickness $\lambda_{\rm e}$, where $\lambda_{\rm free}$ is the electron mean free path determined by Coulomb cross section and electron density and $\lambda_{\rm e}$ is the electron Compton wavelength. The magnetic moment of each cell and the number of cells in the neutron star are calculated, from which the total magnetic moment and magnetic field of the neutron star can be calculated. The results are compared with the observational data and the agreement is reasonable.

Key words microscopic calculation of magnetic field of neutron stars, Dirac equation of electrons in a uniform magnetic mean field, orbital magnetic moment and Landau magnetic cell

PACS 03.65.Ca, 41.20.Gz, 26.65.+t

1 Introduction

Large magnetic fields have been observed on neutron stars. The neutron star's surface magnetic field is very difficult to observe directly, however, it can be calculated by measuring some other significant quantities[1]. For example, the magnetic field from the pulse cycles of neutron stars is about 3×10^7—3×10^9 T[2]. We can also calculate the magnetic field by virtue of the age of the pulsar and its radiation frequency. In addition, the magnetic field of neutron stars from the observation of X-ray astronomy is about 10^8—2×10^9 T[3], and a direct measurement of an isolated neutron star's magnetic field is about 8×10^6 T[4]. At present, people generally think that the neutron star's surface magnetic field is about 10^8—10^{10} T[5]. In recent years, a class of super-high energy pulsars called magnetic stars have been found. The magnetic stars are also neutron stars, born in the center of the massive stars or supernovas. But we don't know exactly what mechanism makes every magnetic star have a powerful magnetic field 10^9—10^{14} T[6—8]. The strong magnetic field of neutron stars is sure to impact the state of neutron stars[5, 9]. For instance, research shows that in the mean-field approximation, the equation of state becomes stiffer in some degree[3].

Theoretical studies of the large magnetic fields on the surfaces of neutron stars are relevant to the ferromagnetic model, which gives the extreme magnetic field of neutron stars of about 10^8 T[10, 11], but it is about 2—3 orders of magnitude smaller than the observations of the magnetic stars.

In this letter, we shall conduct a novel microscopic calculation of the magnetic field of neutron stars. Based on the Dirac equation describing an electron moving in a uniform and cylindrically symmetric magnetic field which may be the result of the self-consistent mean field of electrons themselves in a neutron star, we have obtained the eigen solutions

Received 30 December 2008

* Supported by National Natural Science Foundation of China (90503008, 10775100), and Fund of Theoretical Nuclear Center of HIRFL of China

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and the orbital magnetic moments of electrons in which each eigen orbital can be calculated. From the eigen energy spectrum we find that the lowest energy level corresponds to the highly degenerate orbitals with the quantum numbers $p_z=0$, $n=0$, and $m \geqslant 0$. At the ground state, the electrons fill the lowest eigen states to form many Landau magnetic cells and each cell is a circular disk with the radius λ_{free} and the thickness λ_{e}, where λ_{free} is the electron mean free path determined by Coulomb cross section and electron density and λ_{e} is the electron Compton wavelength. The magnetic moment of each cell and the number of cells in the neutron star are calculated, from which the total magnetic moment and magnetic field of the neutron star can be calculated.

In the outer crust of neutron stars the electrons form electron gas. In the gas and within the circle with the radius of the mean free path λ_{free}, the electrons fill the lowest Landau levels and form magnetic cells; the sum of all the orbital magnetic moments of the electrons in the cell constitutes the magnetic moment of the cell, and the sum of all the cell's magnetic moments gives the total magnetic moment of the neutron star. According to this microscopic model, the calculated magnetic field of neutron stars is about 10^5—10^{11} T for a range of electron densities, which is in between the magnetic field values of magnetic stars and neutron stars. The results are compared with the observational data and the agreement is reasonable.

2 Dirac equation and orbital magnetic moment of electrons in a uniform magnetic field

The mean field in a neutron star is usually assumed to be uniform. To describe the motion of an electron in a neutron star, we consider the Dirac equation for an electron moving in a uniform magnetic field $B=(0,0,B)$

$$H\Psi_{\tau,p_z,\rho,n,m}=\tau E_{p_z,n,m}\Psi_{\tau,p_z,\rho,n,m}\,, \tag{1}$$

the Hamiltonian is

$$H=\alpha\cdot(P-qA)+\beta\mu\,, \tag{2}$$

which has cylindrical symmetry and conserves the z-component of momentum, thus P_z can be replaced by its eigen value p_z. Based on the observation that the γ-matrices have structure and are decomposable, we decompose the γ-matrices into the direct product of the operators in the spin space and the particle-antiparticle space. The α-matrices and β-matrices read[12, 13]

$$\alpha=\begin{pmatrix}\sigma & 0\\ 0 & -\sigma\end{pmatrix}\,, \tag{3}$$

$$\beta=\begin{pmatrix}0 & -I\\ -I & 0\end{pmatrix}\,, \tag{4}$$

thus

$$H=\tau_3\otimes\sigma\cdot(P-qA)-\tau_1\otimes I\mu\,, \tag{5}$$

where

$$A=\left(-\frac{1}{2}By,\ \frac{1}{2}Bx,0\right)=\left(-\frac{1}{2}Br\sin\phi,\ \frac{1}{2}Br\cos\phi,0\right).$$

In cylindrical coordinates, the Hamiltonian can be rewritten as,

$$H=\begin{pmatrix}p_z & \eta_+ & -\mu & 0\\ \eta_- & -p_z & 0 & -\mu\\ -\mu & 0 & -p_z & -\eta_+\\ 0 & -\mu & -\eta_- & p_z\end{pmatrix}\,, \tag{6}$$

the operators are defined as follows

$$\eta_+=-\mathrm{i}\mathrm{e}^{\mathrm{i}\phi}\left(\frac{\partial}{\partial r}-\mathrm{i}\frac{1}{r}\frac{\partial}{\partial\phi}-\frac{1}{2}qBr\right), \tag{7}$$

$$\eta_-=-\mathrm{i}\mathrm{e}^{-\mathrm{i}\phi}\left(\frac{\partial}{\partial r}+\mathrm{i}\frac{1}{r}\frac{\partial}{\partial\phi}+\frac{1}{2}qBr\right). \tag{8}$$

In the spin-coordinate space, the not-normalized local helicity operator can be defined as

$$\varSigma'=\begin{pmatrix}p_z & \eta_+\\ \eta_- & -p_z\end{pmatrix}\,. \tag{9}$$

Since $[\varSigma',H]=0$, $\varSigma'$ is a conserved quantity in spin and coordinate space and has common eigen solutions with H. The eigen equation of $\varSigma'$ is

$$\varSigma'\chi_{\rho,n,m}(r,\phi)=\rho Z_{p_z,n,m}\chi_{\rho,n,m}(r,\phi)\,. \tag{10}$$

The normalized local helicity operator is $\varSigma=\dfrac{1}{Z_{p_z,n,m}}\varSigma'$, so

$$\varSigma\chi_{\rho,n,m}(r,\phi)=\rho\chi_{\rho,n,m}(r,\phi)\,. \tag{11}$$

In the cylindrical-coordinates, we choose the following basis to solve the Dirac equation in spin-coordinate space, $u_{n,m}(r,\phi)=N_{n,m}\mathrm{e}^{\mathrm{i}m\phi}(\alpha r)^{|m|}\mathrm{e}^{-\alpha^2r^2/2}F(-n,|m|+1,\alpha^2r^2)$, with $\alpha=\sqrt{qB/2}$, $n\geqslant 0$ and m are integers, $F(-n,|m|+1,\alpha^2r^2)$ is the confluent hyper geometric function.

The normalized constant is

$$N_{n,m}=\frac{1}{\sqrt{\pi\frac{1}{\alpha}\sum_{k=0}^{n}\left(\frac{(-n)_k}{k!(|m|+1)_k}\right)^2\Gamma\left(2k+|m|+\frac{1}{2}\right)}},$$

the complete eigen solutions of (11) are:

(1) Right handed (R) solution, $\rho=1$:

$$\chi_{+,n,m}(r,\phi)=\begin{pmatrix} u_{n,m}(r,\phi) \\ \frac{1}{p_z+Z_{p_z,n,m}}\eta_- u_{n,m}(r,\phi)\end{pmatrix}. \quad (12)$$

(2) Left handed (L) solution, $\rho=-1$:

$$\chi_{-,n,m}(r,\phi)=\begin{pmatrix} u_{n,m}(r,\phi) \\ \frac{1}{p_z-Z_{p_z,n,m}}\eta_- u_{n,m}(r,\phi)\end{pmatrix}, \quad (13)$$

where $Z_{p_z,n,m}=\sqrt{(2n+|m|-m)qB+p_z^2}$.

In the above situation, the eigen wave functions in spin-coordinate space have been obtained analytically. In the following, we should solve the eigen wave functions in particle-anti-particle space. The Hamiltonian in the particle-antiparticle space can be written as

$$H_\tau=\rho Z_{p_z,n,m}\tau_3-\mu\tau_1\,. \quad (14)$$

The eigen equation in particle-antiparticle space is

$$H_\tau \upsilon_\tau=\tau E_{p_z,n,m}\upsilon_\tau\,, \quad (15)$$

where $E_{p_z,n,m}=\sqrt{(2n+|m|-m)qB+p_z^2+\mu^2}$ is the famous Landau levels in relativistic case.

For particle, $\tau=+1$

$$\upsilon_+=\sqrt{\frac{E_{p_z,n,m}+\rho Z_{p_z,n,m}}{2E_{p_z,n,m}}}\begin{pmatrix} 1 \\ -\frac{1}{E_{p_z,n,m}+Z_{p_z,n,m}}\end{pmatrix} \quad (16)$$

and for anti-particle, $\tau=-1$

$$\upsilon_-=\sqrt{\frac{E_{p_z,n,m}+\rho Z_{p_z,n,m}}{2E_{p_z,n,m}}}\begin{pmatrix} \frac{1}{E_{p_z,n,m}+Z_{p_z,n,m}} \\ 1\end{pmatrix}. \quad (17)$$

The total wave function can be factorized into three types of wave functions in coordinate z-space, spin-coordinate (r,ϕ) space, and particle-anti-particle space, it contains five quantum numbers: $\Psi_{\tau,p_z,\rho,n,m}\sim e^{ip_zz/\hbar}\chi_{n,m}(r,\phi)\otimes\upsilon_\tau$. specifically, for particle solutions, $\tau=+1$, $E'=\tau E_{p_z,n,m}=+E_{p_z,n,m}$, one has: R-particle solution $\rho=+1$,

$$\Psi_1=D\begin{pmatrix} u_{n,m} \\ \frac{1}{p_z+Z_{p_z,n,m}}\eta_- u_{n,m} \\ -\frac{\mu}{E_{p_z,n,m}+Z_{p_z,n,m}}u_{n,m} \\ -\frac{\mu}{E_{p_z,n,m}+Z_{p_z,n,m}}\frac{1}{p_z+Z_{p_z,n,m}}\eta_- u_{n,m}\end{pmatrix}e^{\frac{ip_zz}{\hbar}}. \quad (18)$$

And L-particle solution $\rho=-1$,

$$\Psi_2=D\begin{pmatrix} u_{n,m} \\ \frac{1}{p_z-Z_{p_z,n,m}}\eta_- u_{n,m} \\ -\frac{\mu}{E_{p_z,n,m}+Z_{p_z,n,m}}u_{n,m} \\ -\frac{\mu}{E_{p_z,n,m}+Z_{p_z,n,m}}\frac{1}{p_z-Z_{p_z,n,m}}\eta_- u_{n,m}\end{pmatrix}e^{\frac{ip_zz}{\hbar}}. \quad (19)$$

For the anti-particle solution, $\tau=-1$, $E'=\tau E_{p_z,n,m}=-E_{p_z,n,m}$, one has R-anti-particle solution $\rho=+1$

$$\Psi_3=D\begin{pmatrix} \frac{\mu}{E_{p_z,n,m}+Z_{p_z,n,m}}u_{n,m} \\ \frac{\mu}{E_{p_z,n,m}+Z_{p_z,n,m}}\frac{1}{p_z+Z_{p_z,n,m}}\eta_- u_{n,m} \\ u_{n,m} \\ \frac{1}{p_z+Z_{p_z,n,m}}\eta_- u_{n,m}\end{pmatrix}e^{\frac{ip_zz}{\hbar}}. \quad (20)$$

And L-anti-particle solution, $\rho=-1$,

$$\Psi_4=D\begin{pmatrix} \frac{\mu}{E_{p_z,n,m}+Z_{p_z,n,m}}u_{n,m} \\ \frac{\mu}{E_{p_z,n,m}+Z_{p_z,n,m}}\frac{1}{p_z-Z_{p_z,n,m}}\eta_- u_{n,m} \\ u_{n,m} \\ \frac{1}{p_z-Z_{p_z,n,m}}\eta_- u_{n,m}\end{pmatrix}e^{\frac{ip_zz}{\hbar}}, \quad (21)$$

where

$$D=\frac{1}{\sqrt{2}}\sqrt{\frac{E_{p_z,n,m}+\rho Z_{p_z,n,m}}{E_{p_z,n,m}}}.$$

For the ground state of electrons, $\tau=+1$, $\rho=\pm1$, $p_z=0$, $n=0$, $m\geqslant 0$, $Z_{0,0,m}=0$, $E_{0,0,m}=\mu$,

$\Psi_{+,p_z,\rho,n,m} = \Psi_{+,0,\rho,0,m}$.

The electron (particle) solution,

$$\Psi_{+,0,\rho,0,m} = \frac{1}{\sqrt{2}} \begin{pmatrix} u_{0,m} \\ 0 \\ -u_{0,m} \\ 0 \end{pmatrix} \equiv \Psi_0 \,. \tag{22}$$

The magnetic moment of the particles in the lowest levels is

$$\overline{\mu} = \langle \Psi_0 | \mu_l \hat{J}_z + (\mu_s - \mu_l) S_z | \Psi_0 \rangle. \tag{23}$$

The total angular momentum $\hat{J}_z$

$$\hat{J}_z = \begin{pmatrix} -\mathrm{i}\dfrac{\partial}{\partial \phi} + \dfrac{1}{2} & 0 \\ 0 & -\mathrm{i}\dfrac{\partial}{\partial \phi} - \dfrac{1}{2} \end{pmatrix} \otimes I \,, \tag{24}$$

$$\hat{J}_z |\Psi_0\rangle = \left(m + \frac{1}{2} \right) |\Psi_0\rangle. \tag{25}$$

Therefore, Ψ_0 is the normalized eigenstate of $\hat{J}_z$, $\langle \Psi_0 | \Psi_0 \rangle = 1$. So

$$\langle \Psi_0 | \hat{J}_z | \Psi_0 \rangle = \left(m + \frac{1}{2} \right) \mu_{\mathrm{p(e)}} \,. \tag{26}$$

Consider the expectation value of the intrinsic spin

$$S_z = \begin{pmatrix} \dfrac{1}{2} & 0 \\ 0 & -\dfrac{1}{2} \end{pmatrix} \otimes I \,, \tag{27}$$

$$\langle \Psi_0 | S_z | \Psi_0 \rangle = \frac{1}{2} \mu_{\mathrm{p(e)}} \,, \tag{28}$$

so

$$\overline{\mu}_m = 2(m+1) \mu_{\mathrm{p(e)}} \,. \tag{29}$$

The electron magnetic moment $\mu_{\mathrm{e}} = 9.274 \times 10^{-24}$ J·T^{-1} and the proton magnetic moment $\mu_{\mathrm{p}} = 5.059 \times 10^{-27}$ J·T^{-1}, from which one can see $\mu_{\mathrm{e}} \gg \mu_{\mathrm{p}}$, so we can neglect the contribution of protons.

3 Magnetic field of neutron stars from electrons in the outer crust

The outer crust of neutron stars is mainly composed of electrons and protons, its mass density is about 10^4—10^6 g/cm^3, and so the electron and proton number density is 10^{28}—10^{30}/cm^3. The thickness of the outer crust is about 1 km. In the outer crust and at the ground state, the electrons fill the lowest levels and form the Landau magnetic cells. The Landau magnetic cell is a circular disk with the radius λ_{free} and the thickness λ_{e}. The sum of the orbital magnetic moments of the electrons in the cell constitutes the magnetic moment of the Landau cell. The key point is to calculate the number of the magnetic cells in the outer crust and the magnetic moment of each Landau magnetic cell. In doing so, we can obtain the total magnetic moment and magnetic field of the neutron star. The thickness of the magnetic cell is the electron Compton wavelength $\lambda_{\mathrm{e}} = \dfrac{\hbar}{\mu c} = 3.86 \times 10^{-11}$ cm, and the radius of the magnetic cell is the mean free path of the electron

$$\lambda_{\mathrm{free}} = \frac{1}{\rho_{\mathrm{e}} \sigma} \,, \tag{30}$$

here, ρ_{e} is the number density of the electron, σ is the e-e and e-neuclus Coulomb cross section. In the e-e and e-neuclus collision, the Rutherford scattering differential cross section is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \sigma_c = \left(\frac{1}{4\pi\varepsilon_0} \frac{Z_1 Z_2 e^2}{4E} \right)^2 \frac{1}{\sin^4 \dfrac{\theta}{2}} \,, \tag{31}$$

here Z_1 and Z_2 are the charge numbers of the incident and the target particles, E is the energy of the electron, and θ is the scattering angle. To obtain a finite total cross section, one should consider the screening effect of the mean field of the crust or electron finite size effect, and the total cross section in the mean field is

$$\sigma = \int \sigma_c \mathrm{d}\Omega = \int_0^{\pi} \left(\frac{1}{4\pi\varepsilon_0} \frac{Z_1 Z_2 e^2}{4E} \right)^2 \frac{\sin\theta \mathrm{d}\theta}{\sin^4 \dfrac{\theta}{2} + \alpha^2} =$$

$$\left(\frac{1}{4\pi\varepsilon_0} \frac{Z_1 Z_2 e^2}{4E} \right)^2 4\pi \frac{1}{\alpha} \arctan \frac{1}{\alpha} \,, \tag{32}$$

where α represents the Coulomb screening factor of the electron in the mean field of the crust of the neutron star or the electron finite size effect. Due to the lack of information about the screening effect of the mean field, we take the information from the electron itself and assume $\alpha = \dfrac{r_{\mathrm{e}}}{\lambda_{\mathrm{e}}}$ representing both the finite size effect and the Compton wavelength effect of an electron. Here r_{e} is the classic radius of the electron, and λ_{e} is the Compton wavelength of the electron. At zero temperature approximation, the average energy of the electron can be assumed to be the Fermi energy

$$E = \frac{\hbar^2}{2m_{\mathrm{e}}} (3\pi^2 \rho_{\mathrm{e}})^{2/3} \,. \tag{33}$$

The number density of electrons in the outer crust is $\rho_{\mathrm{e}} = 10^{28}$—$10^{30}$ cm^{-3} and the corresponding energy is E=0.078—0.36 MeV. So the electron total Coulomb crossing section is $\sigma = 1.2 \times 10^{-20} - 2.6 \times 10^{-23}$ cm^2, and the mean free path of the electron with respect to Coulomb collisions is $\lambda_{\rm free}=\dfrac{1}{\rho_{\rm e}\sigma}=8.3\times10^{-9}$—$3.8\times10^{-8}$ cm, which is the radius of the Landau magnetic cell since beyond the mean free path, the Landau quantum orbitals of electrons will be destroyed by Coulomb collisions. Within the magnetic cell, the electrons fill the lowest Landau levels up to the maximum magnetic quantum number $m_{\rm f}$ (the angular momentum on the Fermi surface) determined by the relationship between the electron surface number density $\sigma_{\rm e}$ and the maximum magnetic quantum number $m_{\rm f}$

$$\sigma_{\rm e}=\frac{N}{\pi R_0^2}=2\frac{\sum\limits_{m=0}^{m_{\rm f}}1}{\pi R_0^2}=\frac{2(m_{\rm f}+1)}{\pi R_0^2}\,, \tag{34}$$

here the radius R_0 of the magnetic cell is assumed to be $R_0=\lambda_{\rm free}$ as mentioned above, and the electron surface number density $\sigma_{\rm e}$ can be obtained from the number density $\rho_{\rm e}$ times the Compton wavelength $\lambda_{\rm e}$ describing the position quantum uncertainty of electron in z-direction (the cylindrical symmetric axis along which p_z is conserved, while the plane of magnetic cells is perpendicular to it),

$$\sigma_{\rm e}=\rho_{\rm e}\lambda_{\rm e}=\frac{\rho_{\rm e}\hbar}{\mu c}\,, \tag{35}$$

in this way, the maximum magnetic quantum number is obtained as

$$m_{\rm f}=\frac{\rho_{\rm e}\hbar}{2\mu c}\times\pi\lambda_{\rm free}^2-1\,, \tag{36}$$

the total magnetic moment of the magnetic cells is the sum of all the orbitals of the cell,

$$M_0=\sum_{m=0}^{m_{\rm f}}2(m+1)\mu_{\rm e}=1.9\times10^3-7.6\times10^9\mu_{\rm e}\,. \tag{37}$$

The volume of the magnetic cell is equal to the area of the circular disk times its thickness $v_0=\pi R_0^2\times\lambda_{\rm e}=8.5\times10^{-27}$—$1.7\times10^{-25}$ cm^3. The radius of the neutron star is $R=10^6$ cm, and the thickness of the outer crust is $\Delta R=1$ km $=10^5$ cm. So, the volume of the outer crust is $V=\Delta R\times4\pi R^2=1.3\times10^{18}$ cm^3. And the number of the Landau magnetic cells in the outer crust of the neutron star is $N_{\rm cell}=V/v_0=1.5\times10^{44}-7.2\times10^{42}$, so the total magnetic moment of a neutron star is

$$M=N_{\rm cell}M_0\approx2.9\times10^{47}-5.5\times10^{52}\mu_{\rm e}\,. \tag{38}$$

So, the magnetic field of a neutron star near the surface can be derived from its magnetic moment and reads

$$B=\frac{\mu_0 M}{2\pi R^3}\approx5.4\times10^5-1.0\times10^{11}\ {\rm T}\,, \tag{39}$$

the magnetic constant $\mu_0=4\pi\times10^{-7}{\rm N\cdot A^{-2}}$, N= newton, A=Amper. The calculated magnetic field of the neutron star is about 10^5—10^{11} T which is reasonable in comparison with the observational data.

4 Conclusions and discussions

Based on the Dirac equation of electrons moving in a uniform and cylindrically symmetric magnetic field in a neutron star, we have obtained the eigen solutions and calculated the orbital magnetic moments of electrons in each lowest eigen orbital. From the eigen energy spectrum we find that the lowest energy level is the highly degenerate orbitals. At the ground state, the electrons fill the lowest eigen states to form many Landau magnetic cells and each cell is a circular disk with the radius $\lambda_{\rm free}$ of the electron mean free path and the thickness $\lambda_{\rm e}$ of the electron Compton wavelength. The magnetic moment of each cell and the number of cells in the neutron star are calculated, from which the total magnetic moment and magnetic field of the neutron star are calculated. The calculated magnetic field of the neutron star is about 10^5—10^{11} T which is reasonable in comparison with the observational data.

It should be noted that the responses of an electron to an external magnetic field in the relativistic-quantum case are different from those in the non-relativistic-classical case where the spin magnetic moment and the orbital magnetic moment are treated separately, and the responses of an electron to an external magnetic field are para-magnetically for its spin magnetic moment and anti-magnetically for its orbital magnetic moment. However in our case, the spin magnetic moment and the orbital magnetic moment cannot be treated separately and classically. The eigen energies of an electron (Eq. (15) and below), according to the Dirac equation with a cylindrically symmetric magnetic field, are functions of the orbital magnetic quantum number m. The lowest energy levels of the electron are the eigen energy states with the quantum numbers $n=0$, $p_z=0$, and $m\geqslant0$. The ground state of the electron system is thus the state with all its electrons occupying the highly degenerate lowest eigen energy levels, which results in a constructive enhancement of the total orbital magnetic quantum number and the corresponding magnetic moment of the system.

In this letter, the model is tentative and the calculation is primarily due to the uncertainty of the observational data. In the calculation, only the orbital magnetic moments of the electrons in the outer crust of neutron stars are considered. The contributions from protons and neutrons are neglected because their orbital magnetic moments are much smaller compare with those of electrons. Besides, the system is assumed to be in ground state and the finite temperature effect is not included. In the next study, the model should be improved, the finite temperature effect on the magnetic field should be considered and yet the contributions from protons and neutrons should be estimated.

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