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Decomposition of the equation of state of asymmetric nuclear matter into different spin-isospin channels*

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Abstract We investigate the equation of state of asymmetric nuclear matter and its isospin dependence in various spin-isospin ST channels within the framework of the Brueckner-Hartree-Fock approach extended to include a microscopic three-body force (TBF). It is shown that the potential energy per nucleon in the isospin-singlet $T=0$ channel is mainly determined by the contribution from the tensor SD coupled channel. At high densities, the TBF effect on the isospin-triplet $T=1$ channel contribution turns out to be much larger than that on the $T=0$ channel contribution. At low densities around and below the normal nuclear matter density, the isospin dependence is found to come essentially from the isospin-singlet SD channel and the isospin-triplet $T=1$ component is almost independent of isospin asymmetry. As the density increases, the $T=1$ channel contribution becomes sensitive to the isospin asymmetry and at high enough densities its isospin dependence may even become more pronounced than that of the $T=0$ contribution. The present results may provide some microscopic constraints for improving effective nucleon-nucleon interactions in a nuclear medium and for constructing new functionals of effective nucleon-nucleon interaction based on microscopic many-body theories.

Key words equation of state, asymmetric nuclear matter, decomposition into spin-isospin channels, three-body force, Brueckner-Hartree-Fock approach

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1 Introduction

Effective nucleon-nucleon (NN) interactions such as the Skyrme and Skyrme-like interactions play an important role in predicting the properties of finite nuclei[1—7], nuclear matter and neutron stars[8—11], the nucleus-nucleus interaction potential[12, 13] and fission barriers[14]. The parameters of the effective interactions are usually constrained by the ground state properties of stable nuclei and the saturation properties of nuclear matter, and thus they are shown to be quite successful for describing nuclear phenomena related to nuclear systems not far from the normal nuclear matter density ($\rho_0=0.17$ fm^{-3}) at small isospin-asymmetries. However, as soon as the density deviates from the normal nuclear matter density and the isospin asymmetry becomes large, the discrepancy among the predictions of the Skyrme-Hartree-Fock (SHF) approach by adopting different Skyrme parameters could be extremely large[15, 16]. As for the isospin dependence of single-particle properties, different Skyrme parameters may lead to an opposite isospin splitting of the neutron and proton effective masses in neutron-rich nuclear matter even at densities around ρ_0[17]. In order to improve the predictive power of the Skyrme interaction at high densities and large isospin asymmetries, some work was done

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in recent years to constrain the Skyrme parameters by fitting the bulk properties of asymmetric nuclear matter obtained by the SHF approach to those predicted by the microscopic many-body theories. For example, in Ref. [18], Chabanat et al. proposed a number of sets of Skyrme parameters by reproducing the equation of states (EOSs) of symmetric nuclear matter and pure neutron matter predicted by the microscopic variational approach[19]. In Ref. [20], the authors constructed the LNS parameters for the Skyrme interaction by fitting to the EOS of asymmetric nuclear matter and the neutron/proton effective mass splitting in neutron-rich matter around saturation density obtained within the Brueckner-Hartree-Fock (BHF) approach extended to include a microscopic three-body force (TBF)[21, 22]. Although these recent parametrizations of Skyrme interaction can reproduce fairly well the EOSs of symmetric nuclear matter and pure neutron matter predicted by microscopic approaches (variational method and BHF approach), the deviation from the microscopic results is shown to become significantly large even for symmetric nuclear matter as soon as the EOS is decomposed into different spin-isospin channels[17]. Therefore it is of interest to investigate the EOS of asymmetric nuclear matter and its isospin dependence in various spin-isospin ST channels within the framework of the microscopic BHF approach for a deeper understanding of the mechanism of the isospin dependence of the nuclear EOS and for providing more rigorous and elaborate microscopic constraints for effective NN interactions.

In the present paper, we shall decompose the EOS of asymmetric nuclear matter into various spin-isospin ST channels using the BHF approach extended to include a microscopic three-body force. We shall discuss particularly the isovector part and the isospin dependence of the EOS of asymmetric nuclear matter in different spin-isospin ST channels. The obtained results are expected to provide some useful information for constraining the spin-isospin properties of effective NN interactions.

2 Theoretical approaches

Our present calculation is based on the Brueckner theory[23]. The Brueckner approach for asymmetric nuclear matter and its extension to include a microscopic TBF can be found in Refs. [21, 24]. Here we simply give a brief review for completeness. The starting point of the BHF approach is the reaction $\boldsymbol{G}$-matrix, which satisfies the following isospin dependent Bethe-Goldstone (BG) equation,

$$\begin{aligned}\boldsymbol{G}(\rho,\beta,\omega) = {} & \upsilon_{\mathrm{NN}}+\upsilon_{\mathrm{NN}}\times \\ & \sum_{k_1k_2}\frac{|k_1k_2\rangle Q(k_1,k_2)\langle k_1k_2|}{\omega-\epsilon(k_1)-\epsilon(k_2)}\times \\ & \boldsymbol{G}(\rho,\beta,\omega),\end{aligned} \tag{1}$$

where $k_i \equiv (\boldsymbol{k}_i,\sigma_1,\tau_i)$, denotes the momentum, the z-component of spin and isospin of a nucleon, respectively. υ_{NN} is the realistic NN interaction and ω is the starting energy. The asymmetry parameter is defined as $\beta=(\rho_{\mathrm{n}}-\rho_{\mathrm{p}})/\rho$, where ρ,ρ_{n}, and ρ_{p} denote the total, neutron and proton number densities, respectively. In solving the BG equation for the $\boldsymbol{G}$-matrix, the continuous choice[25] for the auxiliary potential $U(k)$ is adopted since it provides a much faster convergence of the hole-line expansion than the gap choice[26]. Under the continuous choice, the auxiliary potential describes the BHF mean field felt by a nucleon during its propagation in the nuclear medium[27].

The BG equation has been solved in the total angular momentum representation[24]. Using the standard angular-averaging scheme for the Pauli operator and the energy denominator, the BG equation can be decoupled into different partial wave $\alpha=\{JST\}$ channels[28], where J denotes the total angular momentum, S the total spin and T the total isospin of a two-particle state.

For the NN interaction, we adopt the Argonne V_{18} (AV_{18}) two-body interaction[29] plus a microscopic TBF based on the meson-exchange current approach[30]. The parameters of the TBF model have been self-consistently determined so as to reproduce the AV_{18} two-body force by using the one-boson-exchange potential model[21]. The TBF contains the contributions from different intermediate virtual processes such as virtual nucleon-antinucleon pair excitations, and nucleon resonances (for details, see Ref. [30]). The TBF effects on the EOS of nuclear matter and its connection to the relativistic effects in the DBHF approach have been reported in Ref. [21].

The TBF contribution has been included by reducing the TBF to an equivalently effective two-body interaction via a suitable average with respect to the third-nucleon degrees of freedom according to the standard scheme[30]. The effective two-body interaction $\tilde{\upsilon}$ can be expressed in r-space as[21]

$$\begin{aligned}&\langle \boldsymbol{r}_1\boldsymbol{r}_2|\tilde{\upsilon}|\boldsymbol{r}_1'\boldsymbol{r}_2'\rangle=\frac{1}{4}\mathrm{Tr}\sum_n\int \mathrm{d}\boldsymbol{r}_3\mathrm{d}\boldsymbol{r}_3'\phi_n^*(\boldsymbol{r}_3{}')(1-\eta(r_{23}'))\times \\ &(1-\eta(r_{13}'))W_3(\boldsymbol{r}_1'\boldsymbol{r}_2'\boldsymbol{r}_3'|\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3)(1-\eta(r_{13}))\times \\ &(1-\eta(r_{23}))\phi_n(r_3),\end{aligned} \tag{2}$$

where the trace is taken with respect to the spin and isospin of the third nucleon. The function $\eta(r)$ is the defect function. Since the defect function is directly determined by the solution of the BG equation[30], it must be calculated self-consistently with the $\boldsymbol{G}$ matrix and the s.p. potential $U(k)$[21] at each density and isospin asymmetry. It is evident from Eq. (2) that the effective force $\tilde{v}$ rising from the TBF in the nuclear medium is density dependent. A detailed description and justification of the method can be found in Ref. [30].

3 Results and discussion

In Fig. 1 we display the potential part of the EOS of symmetric nuclear matter (i.e., the potential energy per nucleon as a function of density) in various spin-isospin channels of $ST = 00, 10, 01, 11$, and $T = 0, 1$. The solid curves are obtained by including the TBF, while the dashed curves are the results by adopting purely the AV_{18} two-body interaction. The empty and filled squares indicate the contributions of the $T = 0$ $^3SD_2$ tensor channel in the two cases by adopting the pure AV_{18} two-body interaction and the AV_{18} plus the TBF. We notice from the figure that the absolute values of the potential energy per nucleon in the two even partial wave channels ($ST = 10$ and $ST = 01$) are much larger than those in the two odd channels ($ST = 00$ and $ST = 11$). It is also seen that the potential energy in the isospin-singlet $T = 0$ channel is mainly determined by the contribution from the tensor $^3SD_2$ channel. At relatively low densities up to about 0.25 fm^{-3}, the attraction of the potential energy in both $T = 0$ and $T = 1$ channels increases monotonically as a function of density. In the high density region of 0.25 fm$^{-3} \leqslant \rho \leqslant 0.4$ fm^{-3}, the density dependence of the potential energy per nucleon in the $T = 0$ channel becomes very weak (slowly increasing with density) in both cases of including and not including the TBF.

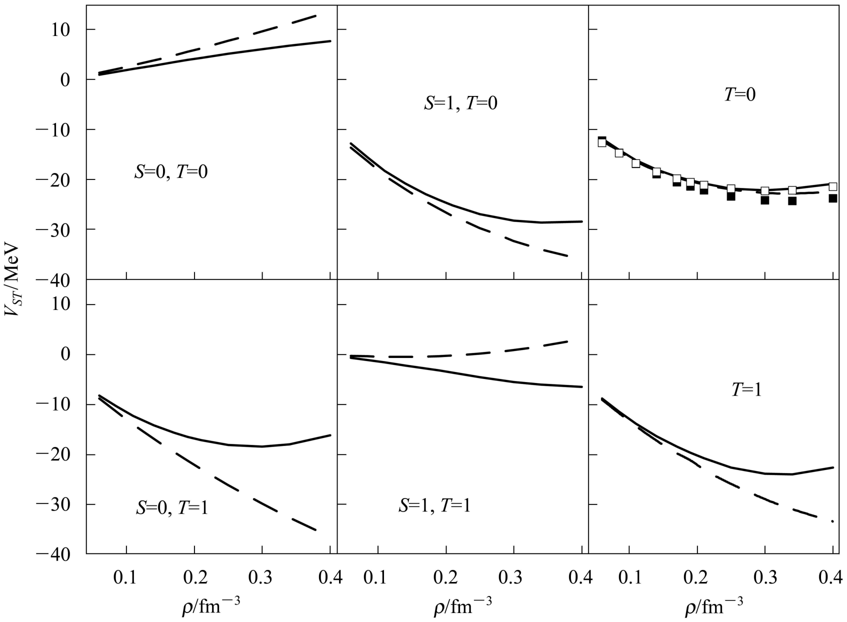

Fig. 1. Decomposition of the potential part of the EOS (i.e., potential energy per nucleon vs. density) of symmetric nuclear matter into various spin-isospin ST channels. Dashed curves: results obtained by adopting purely the AV_{18} two-body NN interaction; Solid curves: results obtained by adopting the AV_{18} plus the TBF; Full and empty squares: potential energy per nucleon in the tensor $^3SD_2$ channel calculated in the two cases of including and not including the TBF, respectively.

The TBF effect can be seen clearly by comparing the solid curves with the corresponding dashed curves. At small densities, the TBF effect turns out to be reasonably weak. As density increases, the TBF effect becomes significant. The TBF contributions are shown to be repulsive for the two even partial wave channels ($ST = 10$ and $ST = 01$), while they are attractive for the two odd channels ($ST = 00$ and $ST = 11$). When the TBF is not included, the two components in the $ST = 10$ and $ST = 01$ channels are strongly attractive and their attraction increases monotonically with density. The TBF makes these two components much less attractive at high densities and as a consequence these two components become slowly increasing functions of density at high enough densities. In the case of not including the TBF, the components in the two odd partial wave channels are repulsive. Inclusion of the TBF makes the potential energy per nucleon in the spin-triplet odd channel ($ST = 11$) become negative from positive. When the TBF is included, the potential energies per nucleon in both the isospin singlet $T = 0$ and isospin triplet $T = 1$ channels are shown to be almost independent of density (solid curves in the right two panels of Fig. 1) in the high-density region of $0.25\ \text{fm}^{-3} \leqslant \rho \leqslant 0.4\ \text{fm}^{-3}$ and thus the kinetic part, which increases monotonically with density as $\rho^{2/3}$, of the energy per nucleon is expected to determine the stiffness of the EOS at high enough densities ($\rho \geqslant 0.25\ \text{fm}^{-3}$).

It is worth mentioning that the two spin-triplet $ST = 10$ and $ST = 11$ components obtained in the present paper are quite different from those predicted by Baldo et al. (Crosses in Fig. 6 of Ref. [6]). The two components predicted by Baldo et al. are slowly increasing functions of density, while the $ST = 11$ component obtained in the present paper decreases slowly with density. The discrepancy between the two calculations is mainly caused by the two different TBFs adopted. In the calculation of Baldo et al. the phenomenological Urbana TBF[31] was adopted.

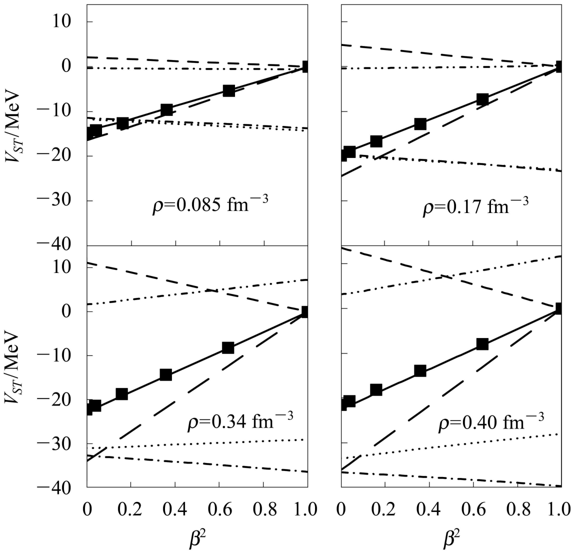

Fig. 2. Isospin-asymmetry dependence of the contributions to the EOS of asymmetric nuclear matter from various spin-isospin channels for several values of density $\rho = 0.085$, 0.17, 0.34 and 0.4 fm^{-3}, respectively. Solid curves: $T = 0$ channel; long-dashed curves: $ST = 00$ channel; short-dashed curve: $ST = 10$ channel; dotted curves: $T = 1$ channel; dot-dashed curves: $ST = 01$ channel; double-dot-dashed curves: $ST = 11$ channel; squares: SD tensor channel. The results are obtained in the case of not including the TBF.

Now we discuss the isospin dependence of the various spin-isospin ST channel contributions to the EOS of asymmetric nuclear matter. In Fig. 2 we show the potential energy per nucleon in the $ST = 00, 10, 01, 11$ and $T = 0, 1$ channels vs. β^2 for several values of density. The results of Fig. 2 have been obtained by adopting purely the AV_{18} two-body force. It is seen from the figure that the isospin dependence of the isospin-triplet ($ST = 01, 11$, and $T = 1$) components is extremely weak as compared with the isospin-singlet ($ST = 00, 10$, and $T = 0$) components, and thus the isospin dependence of the potential part of the EOS of asymmetric nuclear matter is determined essentially by the contribution of the isospin-singlet $T = 0$ channel. As for the $T = 0$ channel, the isospin dependence turns out to stem almost completely from the contribution of the tensor SD coupled channel (comparing the solid curves and the corresponding squares), while the contributions of the other isospin-singlet channels cancel almost completely.

In order to discuss the TBF effects, we give in Fig. 3 the β^2-dependence of the potential energy per nucleon in various spin-isospin channels obtained by including the TBF. It is seen that at relatively low densities around and below the normal nuclear matter density, the TBF effect is reasonably small. The components of the potential energy in the isospin-triplet channels ($ST = 01, 11$, and $T = 1$) are almost independent of β and thus the isospin dependence of the potential energy is essentially determined by that of its component in the isospin $T = 0$ channel. As the density increases, the isospin dependence of the com-

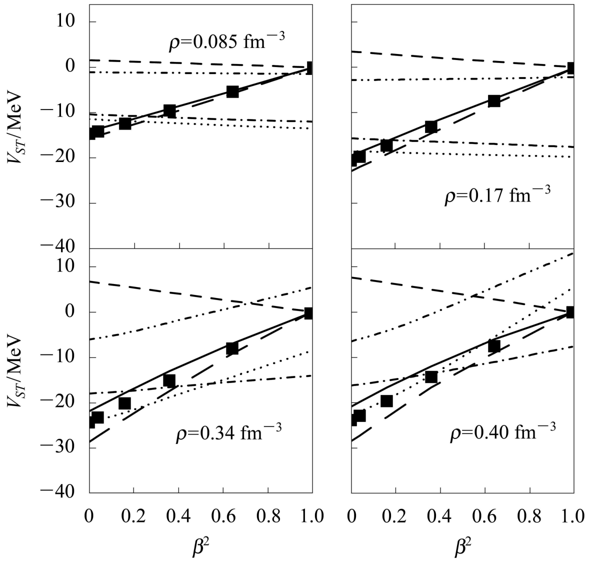

Fig. 3. The same as in Fig. 2 but the results are obtained by including the TBF.

ponents in the isospin-triplet channels becomes significant since the TBF effect increases continuously as the density increases. We may notice that the TBF affects the isospin dependence of the isospin-triplet channels much more significantly as compared to the isospin-singlet channels. At high densities, the TBF effect on the isospin-singlet $T=0$ channel is rather weak, but it affects strongly the isospin dependence of the potential energy per nucleon in the isospin-triplet $T=1$ channel. At high enough densities (for example, $\rho=0.4\ \mathrm{fm}^{-3}$) the sensitivity of the $T=1$ channel potential energy may even become more pronounced than that in the $T=0$ channel (comparing the solid and dotted curves in the lower-right panel of Fig. 3) due to the TBF effect. By comparing Fig. 3 with Fig. 2, we can see that the isospin dependence of the $T=1$ channel potential energy at high densities comes mainly from the TBF contribution.

4 Summary

In the present paper, we have investigated the EOS of asymmetric nuclear matter by decomposing its potential part into various spin-isospin ST channels using the BHF approach extended to include a microscopic three-body force. We have decomposed the isovector part of the EOS of asymmetric nuclear matter into different ST channels and discussed particularly the isospin dependence of the EOS of asymmetric nuclear matter in different spin-isospin ST channels. The potential energy per nucleon in the isospin-singlet $T=0$ channel and its isospin-dependence are shown to be determined to a large extent by the contribution of the tensor SD coupled channel. At low densities around and below the normal nuclear matter density, the TBF effect is fairly weak and the potential energy in the isospin-triplet $T=1$ channel is almost independent of isospin-asymmetry. Consequently, the isospin dependence is found to come essentially from the isospin-singlet SD channel. At high densities, the TBF effect on the isospin-triplet $T=1$ channel contribution turns out to be much larger than that on the $T=0$ channel contribution. As the density increases, the $T=1$ channel contribution becomes more and more sensitive to the isospin asymmetry and at high enough densities its isospin dependence may even become more pronounced than that of the $T=0$ contribution.

The present results may shed light on understanding the origin of the isospin dependence of the EOS of asymmetric nuclear matter and provide some useful information for constraining the effective NN interactions and their isospin dependence in an asymmetric nuclear medium.

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