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$B \to f_0(980)(\pi, \eta^{(\prime)})$ decays in the PQCD approach*

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Abstract Based on the assumption of a two-quark structure of the scalar meson $f_0(980)$, we calculate the branching ratios and CP-violating asymmetries for the four $B \to f_0(980)\pi$ and $B \to f_0(980)\eta^{(\prime)}$ decays by employing the perturbative QCD (pQCD) factorization approach. The leading order pQCD predictions for branching ratios are, $Br(B^- \to f_0(980)\pi^-) \sim 2.5\times10^{-6}$, $Br(\bar{B}^0 \to f_0(980)\pi^0) \sim 2.6\times10^{-7}$, $Br(\bar{B}^0 \to f_0(980)\eta) \sim 2.5\times10^{-7}$ and $Br(\bar{B}^0 \to f_0(980)\eta') \sim 6.7\times10^{-7}$, which are consistent with both the QCD factorization predictions and the experimental upper limits.

Key words B meson decay, scalar meson, pQCD factorization approach

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1 Introduction

Very recently, some $B \to SP$ decays have been studied, for example, by employing the QCD factorization (QCDF) approach or the perturbative QCD (pQCD) approach[1—3]. In the B factory, the first scalar meson $f_0(980)$ was observed in the decay mode $B \to f_0(980)K$ by Belle[4], and later confirmed by BaBar[5], then many $B \to SP$ channels have been measured[6, 7].

In this paper, we will calculate the branching ratios and CP asymmetries of $B^- \to f_0(980)\pi^-$, $\bar{B}^0 \to f_0(980)\pi^0$ and $\bar{B}^0 \to f_0(980)\eta^{(\prime)}$ decays in the pQCD approach at the leading order. This paper is organized as follows: In Sec. 2, we give a brief discussion about the physical properties of $f_0(980)$, and will calculate the decay amplitudes for the considered decays. Sec. 3 contains the numerical results and discussions.

2 Decay amplitudes of $B \to f_0(980)(\pi, \eta^{(\prime)})$ decays

At present we still do not have a clear understanding about the inner structure of the scalar mesons. There are many interpretations for the scalar mesons, such as the $qq\bar{q}\bar{q}$ four-quark state[8] or the $q\bar{q}$ state[9], the possibilities of the $K\bar{K}$ molecular state[10], and even an admixture with glueball states.

In the four-quark model, the flavor wave function of $f_0(980)$ is symbolically given by[8] $f_0 = s\bar{s}(u\bar{u}+d\bar{d})/\sqrt{2}$, which is supported by a lattice calculation. This scenario can explain some experiment phenomena, such as the mass degeneracy of $f_0(980)$ and $a_0(980)$, the large coupling of $f_0(980)$ and $a_0(980)$ to $K\bar{K}$. But we may wonder if the energetic $f_0(980)$ produced in B decays is dominated by the four-quark configuration as it needs to pick up two energetic quark-anti quark pairs to form a fast-moving light four-quark scalar meson[11].

In the naive 2-quark model, $f_0(980)$ is purely an $s\bar{s}$ state and this is supported by the data of $D_s^+ \to f_0\pi^+$ and $\phi \to f_0\gamma$. However, there also exists some experiment evidence, such as $\Gamma(J/\psi \to f_0\omega) \approx \frac{1}{2}\Gamma(J/\psi \to f_0\phi)$, $f_0(980) \to \pi\pi$ is not OZI suppressed relative to $a_0(980) \to \pi\eta$, indicating that $f_0(980)$ is not purely an $s\bar{s}$ state, but a mixture of $s\bar{s}$ and $n\bar{n} \equiv (u\bar{u}+d\bar{d})/\sqrt{2}$:

$$|f_0(980)\rangle = |s\bar{s}\rangle\cos\theta + |n\bar{n}\rangle\sin\theta\,, \tag{1}$$

where θ is the mixing angle. According to Ref. [12], θ lies in the ranges of $25^\circ < \theta < 40^\circ$ or $140^\circ < \theta <$

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165°. Because of our poor knowledge about the non-perturbative dynamics of QCD, we still can not distinguish between the four-quark and two-quark model assignment at present. Some authors, on the other hand, have shown that the scalar mesons with masses above 1 GeV can be identified as conventional $q\bar{q}$ states with a high probability[13, 14]. This conclusion was obtained by calculating the masses and the decay constants of these scalar mesons composed of quark-antiquark pairs based on the QCD sum rule. We here work in the two-quark model and identify $f_0(980)$ as the mixture of $s\bar{s}$ and $n\bar{n}$, in order to give quantitative predictions.

In the two-quark model, the decay constants for scalar meson $f_0(980)$ are defined by:

$$\langle f_0(p)|\bar{q}_2\gamma_\mu q_1|0\rangle = 0, \quad \langle f_0(p)|\bar{q}_2 q_1|0\rangle = m_{\mathrm{S}}\bar{f}_{\mathrm{S}}, \quad (2)$$

and

$$\langle f_0^{\mathrm{n}}|\bar{\mathrm{d}}\mathrm{d}|0\rangle = \langle f_0^{\mathrm{n}}|\bar{\mathrm{u}}\mathrm{u}|0\rangle = \frac{1}{\sqrt{2}} m_{\mathrm{f}_0}\tilde{f}^{\mathrm{n}}_{\mathrm{f}_0}, \ \langle f_0^{\mathrm{s}}|\bar{\mathrm{s}}\mathrm{s}|0\rangle = m_{\mathrm{f}_0}\tilde{f}^{\mathrm{s}}_{\mathrm{f}_0}, \quad (3)$$

where f_0^{n} and f_0^{s} represent the quark flavor states of $f_0(980)$. Using the QCD sum rules method, one can find that the scale-dependent scalar decay constants $f^{\mathrm{n}}_{\mathrm{f}_0}$ and $f^{\mathrm{s}}_{\mathrm{f}_0}$ are very close[1, 11]. So one usually assumes $\tilde{f}^{\mathrm{n}}_{\mathrm{f}_0} = \tilde{f}^{\mathrm{s}}_{\mathrm{f}_0}$ and denotes them as $\bar{f}_{\mathrm{f}_0}$ in the following.

The twist-2 and twist-3 light-cone distribution amplitudes (LCDAs) for different components of scalar meson $f_0(980)$ are defined by:

$$\langle f_0(p)|\bar{q}(z)_{\mathrm{l}} q(0)_{\mathrm{j}}|0\rangle = \frac{1}{\sqrt{2N_{\mathrm{c}}}}\int_0^1 \mathrm{d}x \mathrm{e}^{\mathrm{i}xp\cdot z} \times \left\{ \not{p}\Phi_{\mathrm{f}_0}(x) + m_{\mathrm{f}_0}\Phi^{\mathrm{S}}_{\mathrm{f}_0}(x) + m_{\mathrm{f}_0}(\not{n}_+\not{n}_- - 1)\Phi^{\mathrm{T}}_{\mathrm{f}_0}(x)\right\}_{\mathrm{jl}}. \quad (4)$$

Here we assume that $f_0^{\mathrm{n}}(p)$ and $f_0^{\mathrm{s}}(p)$ have the same form and are denoted as $f_0(p)$, and $n_+ = (1,0,0_{\mathrm{T}})$ and $n_- = (0,1,0_{\mathrm{T}})$ are the light-like vectors.

The twist-2 LCDA $\Phi_{\mathrm{f}}(x,\mu)$ can be expanded as the Gegenbauer polynomials:

$$\Phi_{\mathrm{f}}(x,\mu) = \frac{1}{2\sqrt{2N_{\mathrm{c}}}}\bar{f}_{\mathrm{f}}(\mu)6x(1-x) \times \sum_{m=1}^{\infty} B_m(\mu) C_m^{3/2}(2x-1), \quad (5)$$

where the values for Gegenbauer moments are taken at scale $\mu = 1$ GeV: $B_1 = -0.78\pm0.08$, $B_2 = 0$ and $B_3 = 0.02\pm0.07$.

As for the twist-3 distribution amplitudes $\Phi^{\mathrm{s}}_{\mathrm{f}}$ and $\Phi^{\mathrm{T}}_{\mathrm{f}}$, we adopt the asymptotic form:

$$\Phi^{\mathrm{S}}_{\mathrm{f}} = \frac{1}{2\sqrt{2N_{\mathrm{c}}}}\bar{f}_{\mathrm{f}}, \quad \Phi^{\mathrm{T}}_{\mathrm{f}} = \frac{1}{2\sqrt{2N_{\mathrm{c}}}}\bar{f}_{\mathrm{f}}(1-2x). \quad (6)$$

The B meson is treated as a heavy-light system. We here use the same B meson wave function as in Refs. [15, 16]. For the η-η' system, we use the quark-flavor basis with $\eta_{\mathrm{q}} = (\mathrm{u}\bar{\mathrm{u}} + \mathrm{d}\bar{\mathrm{d}})/\sqrt{2}$ and $\eta_{\mathrm{s}} = \mathrm{s}\bar{\mathrm{s}}$, employ the same wave function, the identical distribution amplitudes $\phi^{\mathrm{A,P,T}}_{\eta_{\mathrm{q,s}}}$, and use the same values for other relevant input parameters, such as $f_{\mathrm{q}} = (1.07\pm0.02)f_\pi$, $f_{\mathrm{s}} = (1.34\pm0.06)f_\pi$, $\phi = 39.3°\pm1.0°$, etc., as given in Ref. [17]. From these currently known studies[15, 16, 18] we believe that there is not much room left for the contribution due to the gluonic component of $\eta^{(\prime)}$, and therefore neglect the possible gluonic component in both the η and η' mesons.

The pQCD factorization approach has been used to study the $\mathrm{B}\to\mathrm{f}_0(980)\mathrm{K}$ decays[2, 3]. Following the same procedure of Ref. [3], we here would like to study $\mathrm{B}\to\mathrm{f}_0(980)\pi$ and $\mathrm{f}_0(980)\eta^{(\prime)}$ decays by employing the pQCD approach at the leading order.

Since the b quark is rather heavy we consider the B meson at rest for simplicity. By using the light-cone coordinates the B meson and the two final state meson's momenta can be written as

$$\begin{aligned} P_{\mathrm{B}} &= \frac{M_{\mathrm{B}}}{\sqrt{2}}(1,1,0_{\mathrm{T}}), \quad P_2 = \frac{M_{\mathrm{B}}}{\sqrt{2}}(1,0,0_{\mathrm{T}}), \\ P_3 &= \frac{M_{\mathrm{B}}}{\sqrt{2}}(0,1,0_{\mathrm{T}}), \end{aligned} \quad (7)$$

where the meson masses have been neglected. Putting the anti-quark momenta in B, P and S mesons as k_1, k_2, and k_3, respectively, we can choose

$$\begin{aligned} k_1 &= (x_1 P_1^+, 0, \boldsymbol{k}_{1\mathrm{T}}), \\ k_2 &= (x_2 P_2^+, 0, \boldsymbol{k}_{2\mathrm{T}}), \\ k_3 &= (0, x_3 P_3^-, \boldsymbol{k}_{3\mathrm{T}}). \end{aligned} \quad (8)$$

In the pQCD approach, the decay amplitude $\mathcal{A}(\mathrm{B}\to\mathrm{Pf}_0)$ can be written conceptually as

$$\begin{aligned} \mathcal{A}(\mathrm{B}\to\mathrm{Pf}_0) \sim & \int \mathrm{d}^4k_1\mathrm{d}^4k_2\mathrm{d}^4k_3 \ \mathrm{Tr}[C(t)\Phi_{\mathrm{B}}(k_1)\Phi_{\mathrm{P}}(k_2) \times \\ & \Phi_{\mathrm{f}_0}(k_3)H(k_1,k_2,k_3,t)], \sim \\ & \int \mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3 b_1\mathrm{d}b_1 b_2\mathrm{d}b_2 b_3\mathrm{d}b_3 \times \\ & \mathrm{Tr}[C(t)\Phi_{\mathrm{B}}(x_1,b_1)\Phi_P(x_2,b_2) \times \\ & \Phi_{\mathrm{f}_0}(x_3,b_3)H(x_i,b_i,t)S_{\mathrm{t}}(x_i)\,\mathrm{e}^{-S(t)}], \end{aligned} \quad (9)$$

where the term "Tr" denotes the trace over Dirac and color indices. $C(t)$ is the Wilson coefficient. The function $H(x_i,b_i,t)$ is the hard part and can be calculated perturbatively, while b_i is the conjugate space coordinate of $k_{i\mathrm{T}}$, and t is the largest energy scale in the hard function. The function Φ_{M} is the wave function which describes hadronization of the quark and anti-quark to the meson M. The threshold function

$S_t(x_i)$ smears the end-point singularities on x_i. The last term, $e^{-S(t)}$, is the Sudakov form factor which suppresses the soft dynamics effectively.

For our considered decays, the relevant weak effective Hamiltonian $\mathcal{H}_{\rm eff}$ can be written as

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\sum_{q=u,c} V_{qb}V_{qd}^*\Big\{[C_1(\mu)O_1^q(\mu)+C_2(\mu)O_2^q(\mu)]+ \sum_{i=3}^{10}C_i(\mu)\,O_i(\mu)\Big\}, \tag{10}$$

where the Fermi constant $G_F=1.16639\times10^{-5}\,\mathrm{GeV}^{-2}$, V_{ij} is the Cabbibo-Kobayashi-Maskawa (CKM) matrix elements, $C_i(\mu)$ are Wilson coefficients at the renormalization scale μ and O_i are the four-fermion operators for the case of $b\to d$ transition.

In the pQCD approach, the typical Feynman diagrams contributing to the $\bar{B}^0\to f_0(980)\pi^0$, $B^-\to f_0(980)\pi^-$ and $\bar{B}^0\to f_0(980)\eta^{(\prime)}$ decays at the leading order are illustrated in Fig. 1. By analytical calculations of the relevant Feynman diagrams, one can find the total decay amplitudes for the considered decays:

$$\begin{aligned}\mathcal{M}(f_0\pi^0) = {}& \frac{\xi_u}{\sqrt{2}}\big[(-M_{e\pi}+M_{a\pi}+M_{ef}+M_{af})C_2+(F_{a\pi}+F_{ef}+F_{af})a_2\big]F_1(\theta)+\frac{\xi_t}{\sqrt{2}}\Big\{\Big[F_{e\pi}^{P2}\Big(a_6-\frac{1}{2}a_8\Big)+\\ & M_{e\pi}\Big(C_3+2C_4-\frac{1}{2}C_9+\frac{1}{2}C_{10}\Big)+M_{e\pi}^{P2}\Big(2C_6+\frac{1}{2}C_8\Big)+(M_{e\pi}^{P1}+M_{a\pi}^{P1}+M_{ef}^{P1}+M_{af}^{P1})\Big(C_5-\frac{1}{2}C_7\Big)+\\ & (M_{a\pi}+M_{ef}+M_{af})\Big(C_3-\frac{3}{2}a_{10}\Big)-(M_{a\pi}^{P2}+M_{ef}^{P2}+M_{af}^{P2})\frac{3}{2}C_8-\\ & (F_{a\pi}+F_{ef}+F_{af})\Big(-a_4-\frac{3}{2}a_7+\frac{3}{2}a_9+\frac{1}{2}a_{10}\Big)+(F_{a\pi}^{P2}+F_{ef}^{P2}+F_{af}^{P2})\Big(a_6-\frac{1}{2}a_8\Big)\Big]F_1(\theta)+\\ & \Big[M_{e\pi}\Big(C_4-\frac{1}{2}C_{10}\Big)+M_{e\pi}^{P2}\Big(C_6-\frac{1}{2}C_8\Big)\Big]F_2(\theta)\Big\},\end{aligned}\tag{11}$$

$$\begin{aligned}\mathcal{M}(f_0\pi^-) = {}& \xi_u\big[M_{e\pi}C_2+(M_{a\pi}+M_{ef}+M_{af})C_1+(F_{a\pi}+F_{ef}+F_{af})a_1\big]F_1(\theta)-\xi_t\Big\{\Big[F_{e\pi}^{P2}\Big(a_6-\frac{1}{2}a_8\Big)+\\ & M_{e\pi}\Big(C_3+2C_4-\frac{1}{2}C_9+\frac{1}{2}C_{10}\Big)+M_{e\pi}^{P1}\Big(C_5-\frac{1}{2}C_7\Big)+(M_{a\pi}^{P1}+M_{ef}^{P1}+M_{af}^{P1})(C_5+C_7)-\\ & (M_{a\pi}+M_{ef}+M_{af})(C_3+C_9)+(F_{a\pi}+F_{ef}+F_{af})(a_4+a_{10})+(F_{a\pi}^{P2}+F_{ef}^{P2}+F_{af}^{P2})\Big(a_6-\frac{1}{2}a_8\Big)\Big]F_1(\theta)+\\ & \Big[M_{e\pi}\Big(C_4-\frac{1}{2}C_{10}\Big)+M_{e\pi}^{P2}\Big(C_6-\frac{1}{2}C_8\Big)\Big]F_2(\theta)\Big\},\end{aligned}\tag{12}$$

$$\begin{aligned}\mathcal{M}(f_0\eta) = {}& \xi_u\{[(M_{e\eta}+M_{a\eta}+M_{ef}+M_{af})C_2+(F_{a\eta}+F_{af})a_2]+F_{ef}a_2f_q\}F_1(\theta)F_1(\phi)-\xi_t\Big\{\Big[F_{e\eta}^{P2}\Big(a_6-\frac{1}{2}a_8\Big)+\\ & (M_{e\eta}+M_{a\eta}+M_{ef}+M_{af})\Big(C_3+2C_4-\frac{1}{2}C_9+\frac{1}{2}C_{10}\Big)+(M_{e\eta}^{P1}+M_{a\eta}^{P1}+M_{ef}^{P1}+M_{af}^{P1})\Big(C_5-\frac{1}{2}C_7\Big)+\\ & (M_{e\eta}^{P2}+M_{a\eta}^{P2}+M_{ef}^{P2}+M_{af}^{P2})\Big(2C_6+\frac{1}{2}C_8\Big)+(F_{a\eta}+F_{ef}f_q+F_{af})\Big(2a_3+a_4-2a_5-\frac{1}{2}a_7+\frac{1}{2}a_9-\frac{1}{2}a_{10}\Big)+\\ & (F_{a\eta}^{P2}+F_{ef}^{P2}+F_{af}^{P2})\Big(a_6-\frac{1}{2}a_8\Big)\Big]F_1(\theta)F_1(\phi)+\Big[(F_{a\eta}+F_{ef}f_s+F_{af})\Big(a_3-a_5+\frac{1}{2}a_7-\frac{1}{2}a_9\Big)+\\ & (M_{e\eta}+M_{a\eta}+M_{ef}+M_{af})\Big(C_4-\frac{1}{2}C_{10}\Big)+(M_{e\eta}^{P2}M_{a\eta}^{P2}+M_{ef}^{P2}+M_{af}^{P2})\Big(C_6-\frac{1}{2}C_8\Big)\Big]F_2(\theta)F_2(\phi)\},\end{aligned}\tag{13}$$

where $\xi_u=V_{ub}^*V_{ud}$, $\xi_t=V_{tb}^*V_{td}$, $F_1(\theta)=\sin\theta/\sqrt{2}$ and $F_2(\theta)=\cos\theta$ are the mixing factors for the $f_0(980)$ meson, while $F_1(\phi)=\cos\phi/\sqrt{2}$ and $F_2(\phi)=-\sin\phi$ are the mixing factors for the η-η' system. For the $B\to f_0(980)\eta'$ decay, the corresponding decay amplitude $\mathcal{M}(\bar{B}^0\to f_0\eta')$ can be obtained from $\mathcal{M}(\bar{B}^0\to f_0\eta)$ in Eq. (13) by replacements of $F_1(\phi)\to F_1'=\sin\phi/\sqrt{2}$ and $F_2(\phi)\to F_2'=\cos\phi$.

The Wilson coefficients a_i in Eqs. (11)—(13) are the combinations of the ordinary Wilson coefficients

$C_i(\mu)$,

$$
\begin{aligned}
a_1 &= C_2+\frac{C_1}{3}, \quad a_2=C_1+\frac{C_2}{3},\\
a_i &= C_i+\frac{C_{i+1}}{3}, \quad \text{for} \quad i=3,5,7,9, \\
a_i &= C_i+\frac{C_{i-1}}{3}, \quad \text{for} \quad i=4,6,8,10.
\end{aligned} \tag{14}
$$

The non-zero individual decay amplitudes in Eqs. (11)—(13), such as $F_{\mathrm{e}\pi}^{\mathrm{P2}}$, $M_{\mathrm{e}\pi}$, $M_{\mathrm{e}\pi}^{\mathrm{P1}}$, $M_{\mathrm{e}\pi}^{\mathrm{P2}}$, $\cdots$, are obtained by evaluating analytically the different Feynman diagrams in Fig. 1. For $\bar{\mathrm{B}}^0\to \mathrm{f}_0(980)\pi^0$ and $\mathrm{B}^-\to \mathrm{f}_0(980)\pi^-$ decays, we have

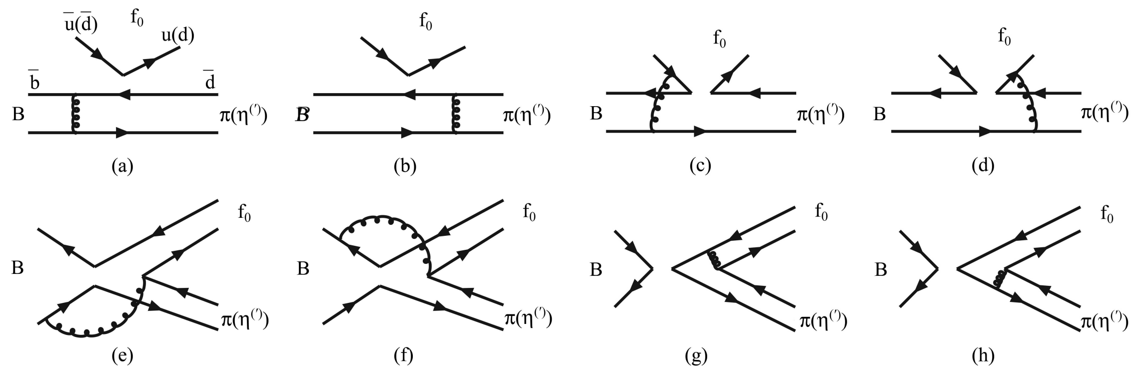

Fig. 1. Typical Feynman diagrams contributing to the $\mathrm{B}\to \mathrm{f}_0(980)\pi(\eta^{(\prime)})$ decays at the leading order.

$$
\begin{aligned}
F_{\mathrm{e}\pi}^{\mathrm{P2}} &= -16\pi C_{\mathrm{F}} m_{\mathrm{B}}^4 r_{\mathrm{f}} \bar{f}_{\mathrm{f}} \int_0^1 \mathrm{d}x_1 \mathrm{d}x_3 \int_0^\infty b_1 \mathrm{d}b_1\, b_3 \mathrm{d}b_3\, \Phi_{\mathrm{B}}(x_1,b_1) \Big\{ \big[\Phi_\pi^{\mathrm{A}}(x_3)+r_\pi x_3(\Phi_\pi^{\mathrm{P}}(x_3)-\Phi_\pi^{\mathrm{T}}(x_3))+ \\
&\quad 2r_\pi \Phi_\pi^{\mathrm{P}}(x_3)\big] E_{\mathrm{e}i}(t) h_{\mathrm{e}}(x_1,x_3,b_1,b_3) + 2r_\pi \Phi_\pi^{\mathrm{P}}(x_3) E_{\mathrm{e}i}(t') h_{\mathrm{e}}(x_3,x_1,b_3,b_1) \Big\},
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
\mathcal{M}_{\mathrm{e}\pi} &= 32\pi C_{\mathrm{F}} m_{\mathrm{B}}^4/\sqrt{2N_{\mathrm{C}}} \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 \int_0^\infty b_1 \mathrm{d}b_1\, b_2 \mathrm{d}b_2\, \Phi_{\mathrm{B}}(x_1,b_1) \Phi_{\mathrm{f}}(x_2) \times \\
&\quad \Big\{ [(1-x_2)\Phi_\pi(x_3) - r_\pi x_3(\Phi_\pi^{\mathrm{P}}(x_3)-\Phi_\pi^{\mathrm{T}}(x_3))] E'_{\mathrm{e}i}(t) h_{\mathrm{n}}(x_1,\bar{x}_2,x_3,b_1,b_2) - \\
&\quad [(x_2+x_3)\Phi_\pi(x_3) - r_\pi x_3(\Phi_\pi^{\mathrm{P}}(x_3)+\Phi_\pi^{\mathrm{T}}(x_3))] E'_{\mathrm{e}i}(t') h_{\mathrm{n}}(x_i,b_1,b_2) \Big\},
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
\mathcal{M}_{\mathrm{e}\pi}^{\mathrm{P1}} &= \frac{32}{\sqrt{6}} \pi C_{\mathrm{F}} m_{\mathrm{B}}^4 r_{\mathrm{f}} \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 \int_0^\infty b_1 \mathrm{d}b_1\, b_2 \mathrm{d}b_2\, \Phi_{\mathrm{B}}(x_1,b_1) \Big\{ E'_{\mathrm{e}i}(t) h_{\mathrm{n}}(x_1,\bar{x}_2,x_3,b_1,b_2) \times \\
&\quad \big[(x_2-1)\Phi_\pi^{\mathrm{A}}(x_3)(\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)+\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2)) + r_\pi(x_2-1)(\Phi_\pi^{\mathrm{P}}(x_3)-\Phi_\pi^{\mathrm{T}}(x_3))(\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)+\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2)) - \\
&\quad r_\pi x_3(\Phi_\pi^{\mathrm{P}}(x_3)+\Phi_\pi^{\mathrm{T}}(x_3))(\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)-\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2))\big] + E'_{\mathrm{e}i}(t') h_{\mathrm{n}}(x_i,b_1,b_2) \times \\
&\quad \big[x_2 \Phi_\pi^{\mathrm{A}}(x_3)(\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)-\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2)) + r_\pi x_2(\Phi_\pi^{\mathrm{P}}(x_3)-\Phi_\pi^{\mathrm{T}}(x_3))(\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)-\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2)) + \\
&\quad r_\pi x_3(\Phi_\pi^{\mathrm{P}}(x_3)+\Phi_\pi^{\mathrm{T}}(x_3))(\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)+\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2))\big] \Big\},
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
\mathcal{M}_{\mathrm{e}\pi}^{\mathrm{P2}} &= -\frac{32}{\sqrt{6}} \pi C_{\mathrm{F}} m_{\mathrm{B}}^4 \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 \int_0^\infty b_1 \mathrm{d}b_1\, b_2 \mathrm{d}b_2\, \Phi_{\mathrm{B}}(x_1,b_1) \Phi_{\mathrm{f}}(x_2) \Big\{ \big[(x_2-x_3-1)\Phi_\pi^{\mathrm{A}}(x_3) + \\
&\quad r_\pi x_3(\Phi_\pi^{\mathrm{P}}(x_3)+\Phi_\pi^{\mathrm{T}}(x_3))\big] E'_{\mathrm{e}i}(t) h_{\mathrm{n}}(x_1,\bar{x}_2,x_3,b_1,b_2) + \\
&\quad \big[x_2 \Phi_\pi^{\mathrm{A}}(x_3) - r_{\mathrm{K}} x_3(\Phi_{\mathrm{K}}^{\mathrm{P}}(x_3)-\Phi_{\mathrm{K}}^{\mathrm{T}}(x_3))\big] E'_{\mathrm{e}i}(t') h_{\mathrm{n}}(x_i,b_1,b_2) \Big\},
\end{aligned} \tag{18}
$$

$$\begin{aligned}\mathcal{M}_{\mathrm{a}\pi} = {} & \frac{32}{\sqrt{6}}\pi C_{\mathrm{F}} m_{\mathrm{B}}^4 \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 \int_0^\infty b_1 \mathrm{d}b_1\, b_2 \mathrm{d}b_2\, \Phi_{\mathrm{B}}(x_1,b_1) \Big\{ \big[-x_2 \Phi_\pi^{\mathrm{A}}(x_3) \Phi_{\mathrm{f}}(x_2) + \\ & r_\pi r_{\mathrm{f}} \Phi_{\mathrm{f}}^{\mathrm{T}}(x_2) \big((x_2+x_3-1)\Phi_\pi^{\mathrm{P}}(x_3) + (-x_2+x_3+1)\Phi_\pi^{\mathrm{T}}(x_3) \big) + r_\pi r_{\mathrm{f}} \Phi_{\mathrm{f}}^{\mathrm{S}}(x_2) \big((x_2-x_3+3)\Phi_\pi^{\mathrm{P}}(x_3) - \\ & (x_2+x_3-1)\Phi_\pi^{\mathrm{T}}(x_3) \big) \big] E'_{\mathrm{a}i}(t) h_{\mathrm{na}}(x_1,x_2,x_3,b_1,b_2) - E'_{\mathrm{a}i}(t') h'_{\mathrm{na}}(x_1,x_2,x_3,b_1,b_2) \times \\ & \big[(x_3-1)\Phi_\pi^{\mathrm{A}}(x_3)\Phi_{\mathrm{f}}(x_2) + r_\pi r_{\mathrm{f}} \Phi_{\mathrm{f}}^{\mathrm{S}}(x_2) \big((x_2-x_3+1)\Phi_\pi^{\mathrm{P}}(x_3) - (x_2+x_3-1)\Phi_\pi^{\mathrm{T}}(x_3) \big) + \\ & r_\pi r_{\mathrm{f}} \Phi_{\mathrm{f}}^{\mathrm{T}}(x_2) \big((x_2+x_3-1)\Phi_\pi^{\mathrm{P}}(x_3) - (1+x_2-x_3)\Phi_\pi^{\mathrm{T}}(x_2) \big) \big] \Big\}, \end{aligned} \tag{19}$$

$$\begin{aligned}\mathcal{M}_{\mathrm{a}\pi}^{\mathrm{P1}} = {} & \frac{32}{\sqrt{6}}\pi C_{\mathrm{F}} m_{\mathrm{B}}^4 \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 \int_0^\infty b_1 \mathrm{d}b_1\, b_2 \mathrm{d}b_2\, \Phi_{\mathrm{B}}(x_1,b_1) \Big\{ \big[r_\pi (1+x_3) \Phi_{\mathrm{f}}(x_2) (\Phi_\pi^{\mathrm{T}}(x_3) - \Phi_\pi^{\mathrm{P}}(x_3)) + \\ & r_{\mathrm{f}}(x_2-2)\Phi_\pi(x_3)(\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)+\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2)) \big] E'_{\mathrm{a}i}(t) h_{\mathrm{na}}(x_1,x_2,x_3,b_1,b_2) - \big[r_\pi (x_3-1)\Phi_{\mathrm{f}}(x_2)(\Phi_\pi^{\mathrm{T}}(x_3) - \\ & \Phi_\pi^{\mathrm{P}}(x_3)) + r_{\mathrm{f}} x_2 \Phi_\pi(x_3)(\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)+\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2)) \big] E'_{\mathrm{a}i}(t') h'_{\mathrm{na}}(x_1,x_2,x_3,b_1,b_2) \Big\}, \end{aligned} \tag{20}$$

$$\begin{aligned}\mathcal{M}_{\mathrm{a}\pi}^{\mathrm{P2}} = {} & -\frac{32}{\sqrt{6}}\pi C_{\mathrm{F}} m_{\mathrm{B}}^4 \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 \int_0^\infty b_1 \mathrm{d}b_1\, b_2 \mathrm{d}b_2\, \Phi_{\mathrm{B}}(x_1,b_1) \Big\{ \big[(x_3-1)\Phi_{\mathrm{f}}(x_2)\Phi_\pi^{\mathrm{A}}(x_3) + 4 r_\pi r_{\mathrm{f}} \Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)\Phi_\pi^{\mathrm{P}}(x_3) + \\ & r_\pi r_{\mathrm{f}} ((x_2-x_3-1)(\Phi_\pi^{\mathrm{P}}(x_3)\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2) - \Phi_\pi^{\mathrm{T}}(x_3)\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2)) - (x_2+x_3-1)(\Phi_\pi^{\mathrm{P}}(x_3)\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2) - \Phi_\pi^{\mathrm{T}}(x_3)\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)) \big] \times \\ & E'_{\mathrm{a}i}(t) h_{\mathrm{na}}(x_1,x_2,x_3,b_1,b_2) + [x_2 \Phi_{\mathrm{f}}(x_2)\Phi_\pi^{\mathrm{A}}(x_3) - x_2 r_\pi r_{\mathrm{f}} (\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)+\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2))(\Phi_\pi^{\mathrm{P}}(x_3)-\Phi_\pi^{\mathrm{T}}(x_3)) - \\ & r_\pi r_{\mathrm{f}} (1-x_3)(\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)-\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2))(\Phi_\pi^{\mathrm{P}}(x_3)+\Phi_\pi^{\mathrm{T}}(x_3))] E'_{\mathrm{a}i}(t') h'_{\mathrm{na}}(x_1,x_2,x_3,b_1,b_2) \Big\}, \end{aligned} \tag{21}$$

$$\begin{aligned}F_{\mathrm{a}\pi} = {} & -F_{\mathrm{a}\pi}^{\mathrm{P1}} = 8\pi C_{\mathrm{F}} m_{\mathrm{B}}^4 f_{\mathrm{B}} \int_0^1 \mathrm{d}x_2 \mathrm{d}x_3 \int_0^\infty b_2 \mathrm{d}b_2\, b_3 \mathrm{d}b_3 \Big\{ \big[(x_3-1)\Phi_\pi^{\mathrm{A}}(x_3)\Phi_{\mathrm{f}}(x_2) - 2 r_\pi r_{\mathrm{f}} (x_3-2)\Phi_\pi^{\mathrm{P}}(x_3)\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2) + \\ & 2 r_\pi r_{\mathrm{f}} x_3 \Phi_\pi^{\mathrm{T}}(x_3)\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2) \big] E_{\mathrm{a}i}(t) h_{\mathrm{a}}(x_2,1-x_3,b_2,b_3) + [x_2 \Phi_\pi^{\mathrm{A}}(x_3)\Phi_{\mathrm{f}}(x_2) - 2 r_\pi r_{\mathrm{f}} \Phi_\pi^{\mathrm{P}}(x_3)((x_2+1)\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2) + \\ & (x_2-1)\Phi_{\mathrm{f}}^{\mathrm{T}})] E_{\mathrm{a}i}(t') h_{\mathrm{a}}(1-x_3,x_2,b_3,b_2) \Big\}, \end{aligned} \tag{22}$$

$$\begin{aligned}F_{\mathrm{a}\pi}^{\mathrm{P2}} = {} & -16\pi C_{\mathrm{F}} m_{\mathrm{B}}^4 f_{\mathrm{B}} \int_0^1 \mathrm{d}x_2 \mathrm{d}x_3 \int_0^\infty b_2 \mathrm{d}b_2\, b_3 \mathrm{d}b_3 \Big\{ [r_\pi (x_3-1)\Phi_{\mathrm{f}}(x_2)(\Phi_\pi^{\mathrm{P}}(x_3)+\Phi_\pi^{\mathrm{T}}(x_3)) + 2 r_{\mathrm{f}} \Phi_\pi(x_3)\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)] \times \\ & E_{\mathrm{a}i}(t) h_a(x_2,\bar{x}_3,b_2,b_3) - [2 r_\pi \Phi_\pi^{\mathrm{P}}(x_3)\Phi_{\mathrm{f}}(x_2) + r_{\mathrm{f}} x_2 \Phi_{\mathrm{K}}^{\mathrm{A}}(x_3)(\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2) - \\ & \Phi_{\mathrm{f}}^{\mathrm{S}}(x_2))] E_{\mathrm{a}i}(t') h_{\mathrm{a}}(\bar{x}_3,x_2,b_3,b_2) \Big\}, \end{aligned} \tag{23}$$

$$\begin{aligned}F_{\mathrm{ef}} = {} & F_{\mathrm{ef}}^{\mathrm{P1}} = 8\pi C_{\mathrm{F}} m_{\mathrm{B}}^4 f_\pi \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \int_0^\infty b_1 \mathrm{d}b_1\, b_2 \mathrm{d}b_2\, \Phi_{\mathrm{B}}(x_1,b_1) \Big\{ [(1+x_2)\Phi_{\mathrm{f}}(x_2) - r_{\mathrm{f}}(1-2x_2)(\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)+\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2))] \times \\ & E_{\mathrm{e}i}(t) h_{\mathrm{e}}(x_1,x_2,b_1,b_2) - 2 r_{\mathrm{f}} \Phi_{\mathrm{f}}^{\mathrm{S}}(x_2) E_{\mathrm{e}i}(t') h_{\mathrm{e}}(x_2,x_1,b_2,b_1) \Big\}, \end{aligned} \tag{24}$$

$$\begin{aligned}F_{\mathrm{ef}}^{\mathrm{P2}} = {} & 16\pi C_{\mathrm{F}} m_{\mathrm{B}}^4 f_\pi r_\pi \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \int_0^\infty b_1 \mathrm{d}b_1\, b_2 \mathrm{d}b_2\, \Phi_{\mathrm{B}}(x_1,b_1) \Big\{ -[\Phi_{\mathrm{f}}(x_2) + r_{\mathrm{f}}(x_2 \Phi_{\mathrm{f}}^{\mathrm{T}}(x_2) - (x_2+2)\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2))] \times \\ & E_{\mathrm{e}i}(t) h_{\mathrm{e}}(x_1,x_2,b_1,b_2) + 2 r_{\mathrm{f}} \Phi_{\mathrm{f}}^{\mathrm{S}}(x_2) E_{\mathrm{e}i}(t') h_{\mathrm{e}}(x_2,x_1,b_2,b_1) \Big\}, \end{aligned} \tag{25}$$

$$\mathcal{M}_{\mathrm{ef}} = \frac{32}{\sqrt{6}}\pi C_{\mathrm{F}} m_{\mathrm{B}}^4 \int_0^1 \mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3 \int_0^{\infty} b_1\mathrm{d}b_1\, b_2\mathrm{d}b_2\, \Phi_{\mathrm{B}}(x_1,b_1)\Phi_{\pi}^{\mathrm{A}}(x_3)\Big\{-[(x_3-1)\Phi_{\mathrm{f}}(x_2)-r_{\mathrm{f}}x_2(\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)-\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2))]\times E_{\mathrm{ei}}'(t)h_{\mathrm{n}}(x_1,1-x_3,x_2,b_1,b_3)+[-(x_2+x_3)\Phi_{\mathrm{f}}(x_2)-r_{\mathrm{f}}x_2(\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)+\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2))]E_{\mathrm{ei}}'(t')h_{\mathrm{n}}(x_1,x_3,x_2,b_1,b_3)\Big\}, \tag{26}$$

$$\mathcal{M}_{\mathrm{ef}}^{\mathrm{P1}} = \frac{32}{\sqrt{6}}\pi C_{\mathrm{F}} m_{\mathrm{B}}^4 r_{\pi} \int_0^1 \mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3 \int_0^{\infty} b_1\mathrm{d}b_1\, b_3\mathrm{d}b_3\, \Phi_{\mathrm{B}}(x_1,b_1)\Big\{E_{\mathrm{ei}}'(t)h_{\mathrm{n}}(x_1,1-x_3,x_2,b_1,b_3)\times [(x_3-1)\Phi_{\mathrm{f}}(x_2)(\Phi_{\pi}^{\mathrm{P}}(x_3)+\Phi_{\pi}^{\mathrm{T}}(x_3))+r_{\mathrm{f}}\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2)((x_2+x_3-1)\Phi_{\pi}^{\mathrm{P}}(x_3)+(-x_2+x_3-1)\Phi_{\pi}^{\mathrm{T}}(x_3))+ r_{\mathrm{f}}\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)((x_2-x_3+1)\Phi_{\pi}^{\mathrm{P}}(x_3)-(x_2+x_3-1)\Phi_{\pi}^{\mathrm{T}}(x_3))]+[-x_3\Phi_{\mathrm{f}}(x_2)(\Phi_{\pi}^{\mathrm{T}}(x_3)-\Phi_{\pi}^{\mathrm{P}}(x_3))-r_{\mathrm{f}}x_3(\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)- \Phi_{\mathrm{f}}^{\mathrm{T}}(x_2))(\Phi_{\pi}^{\mathrm{P}}(x_3)-\Phi_{\pi}^{\mathrm{T}}(x_3))-r_{\mathrm{f}}x_2(\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)+\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2))(\Phi_{\pi}^{\mathrm{P}}(x_3)+\Phi_{\pi}^{\mathrm{T}}(x_3))]E_{\mathrm{ei}}'(t')h_{\mathrm{n}}(x_1,x_3,x_2,b_1,b_3)\Big\}, \tag{27}$$

$$\mathcal{M}_{\mathrm{ef}}^{\mathrm{P2}} = -\frac{32}{\sqrt{6}}\pi C_{\mathrm{F}} m_{\mathrm{B}}^4 \int_0^1 \mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3 \int_0^{\infty} b_1\mathrm{d}b_1\, b_2\mathrm{d}b_2\, \Phi_{\mathrm{B}}(x_1,b_1)\Phi_{\pi}^{\mathrm{A}}(x_3)\Big\{[(x_3-x_2-1)\Phi_{\mathrm{f}}(x_2)- r_{\mathrm{f}}x_2(\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)+\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2))]E_{\mathrm{ei}}'(t)h_{\mathrm{n}}(x_1,1-x_2,x_3,b_1,b_2)+[x_2\Phi_{\mathrm{f}}(x_2)+r_{\mathrm{f}}x_2(\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)-\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2))]\times E_{\mathrm{ei}}'(t')h_{\mathrm{n}}(x_1,x_3,x_2,b_1,b_2)\Big\}, \tag{28}$$

$$\mathcal{M}_{a} = -\frac{32}{\sqrt{6}}\pi C_{\mathrm{F}} m_{\mathrm{B}}^4 \int_0^1 \mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3 \int_0^{\infty} b_1\mathrm{d}b_1\, b_3\mathrm{d}b_3\, \Phi_{\mathrm{B}}(x_1,b_1)\Big\{[x_3\Phi_{\pi}^{\mathrm{A}}(x_3)\Phi_{\mathrm{f}}(x_2)+r_{\pi}r_{\mathrm{f}}\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2)\times ((x_2-x_3+1)\Phi_{\pi}^{\mathrm{T}}(x_3)-(x_2+x_3-1)\Phi_{\pi}^{\mathrm{P}}(x_3))+r_{\pi}r_{\mathrm{f}}\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)((-x_2+x_3+3)\Phi_{\pi}^{\mathrm{P}}(x_3)+(x_2+x_3-1)\Phi_{\pi}^{\mathrm{T}}(x_3))]\times E_{\mathrm{ai}}'(t)h_{\mathrm{na}}(x_1,x_3,x_2,b_1,b_3)+E_{\mathrm{ai}}'(t')h_{\mathrm{na}}'(x_1,x_3,x_2,b_1,b_3)[(x_2-1)\Phi_{\pi}^{\mathrm{A}}(x_3)\Phi_{\mathrm{f}}(x_2)+ r_{\pi}r_{\mathrm{f}}\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2)((-x_2+x_3+1)\Phi_{\pi}^{\mathrm{T}}(x_3)-(x_2+x_3-1)\Phi_{\pi}^{\mathrm{P}}(x_3))+ r_{\pi}r_{\mathrm{f}}\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)((x_2-x_3-1)\Phi_{\pi}^{\mathrm{P}}(x_3)+(x_2+x_3-1)\Phi_{\pi}^{\mathrm{T}}(x_3))]\Big\}, \tag{29}$$

$$\mathcal{M}_{\mathrm{af}}^{\mathrm{P1}} = \frac{32}{\sqrt{6}}\pi C_{\mathrm{F}} m_{\mathrm{B}}^4 \int_0^1 \mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3 \int_0^{\infty} b_1\mathrm{d}b_1\, b_3\mathrm{d}b_3\, \Phi_{\mathrm{B}}(x_1,b_1)\Big\{[r_{\mathrm{f}}(x_2+1)\Phi_{\pi}^{\mathrm{A}}(x_3)(\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)-\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2))+ r_{\pi}(x_3-2)\Phi_{\mathrm{f}}(x_2)(\Phi_{\pi}^{\mathrm{P}}(x_3)+\Phi_{\pi}^{\mathrm{T}}(x_3))]E_{\mathrm{ai}}'(t)h_{\mathrm{na}}(x_1,x_3,x_2,b_1,b_3)-[r_{\mathrm{f}}(x_2-1)\Phi_{\pi}^{\mathrm{A}}(x_3)(\Phi_{\mathrm{f}}^{\mathrm{S}}(x_3)- \Phi_{\mathrm{f}}^{\mathrm{T}}(x_3))+r_{\pi}x_3\Phi_{\mathrm{f}}(x_2)(\Phi_{\pi}^{\mathrm{P}}(x_3)+\Phi_{\pi}^{\mathrm{T}}(x_3))]E_{\mathrm{ai}}'(t')h_{\mathrm{na}}'(x_1,x_3,x_2,b_1,b_3)\Big\}. \tag{30}$$

$$F_{\mathrm{af}} = F_{\mathrm{af}}^{\mathrm{P1}}=8\pi C_{\mathrm{F}} m_{\mathrm{B}}^4 f_{\mathrm{B}} \int_0^1 \mathrm{d}x_2\mathrm{d}x_3 \int_0^{\infty} b_2\mathrm{d}b_2\, b_3\mathrm{d}b_3\Big\{[(x_2-1)\Phi_{\pi}^{\mathrm{A}}(x_3)\Phi_{\mathrm{f}}(x_2)+2r_{\pi}r_{\mathrm{f}}(x_2-2)\Phi_{\pi}^{\mathrm{P}}(x_3)\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)- 2r_{\pi}r_{\mathrm{f}}x_2\Phi_{\pi}^{\mathrm{P}}(x_3)\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2)]E_{\mathrm{ai}}(t)h_{\mathrm{a}}(x_3,1-x_2,b_3,b_2)+[x_3\Phi_{\pi}^{\mathrm{A}}(x_3)\Phi_{\mathrm{f}}(x_2)+2r_{\pi}r_{\mathrm{f}}\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)((x_3+1)\Phi_{\pi}^{\mathrm{P}}(x_3)+ (x_3-1)\Phi_{\pi}^{\mathrm{T}}(x_3))]E_{\mathrm{ai}}(t')h_{\mathrm{a}}(1-x_2,x_3,b_2,b_3)\}, \tag{31}$$

$$F_{\mathrm{af}}^{\mathrm{P2}} = 16\pi C_{\mathrm{F}} m_{\mathrm{B}}^4 f_{\mathrm{B}} \int_0^1 \mathrm{d}x_2\mathrm{d}x_3 \int_0^{\infty} b_2\mathrm{d}b_2\, b_3\mathrm{d}b_3\,\Big\{[r_{\mathrm{f}}(x_2-1)\Phi_{\pi}^{\mathrm{A}}(x_3)(\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)+\Phi_{\mathrm{f}}^{\mathrm{T}}(x_2))-2r_{\pi}\Phi_{\pi}^{\mathrm{P}}(x_3)\Phi_{\mathrm{f}}(x_2)]\times E_{\mathrm{ai}}(t)h_{\mathrm{a}}(x_3,\bar{x}_2,b_2,b_3)-[2r_{\mathrm{f}}\Phi_{\mathrm{K}}^{\mathrm{A}}(x_3)\Phi_{\mathrm{f}}^{\mathrm{S}}(x_2)+r_{\pi}x_3\Phi_{\mathrm{f}}(x_2)(\Phi_{\pi}^{\mathrm{P}}(x_3)-\Phi_{\pi}^{\mathrm{T}}(x_3))]\times E_{\mathrm{ai}}(t')h_{a}(1-x_2,x_3,b_2,b_3)\Big\}, \tag{32}$$

where $r_{\rm f}=m_{\rm f}/m_{\rm B}$ and $r_\pi=m_0^\pi/m_{\rm B}$. The explicit expressions of hard functions $E^{(\prime)}_{{\rm ei,a}i}(t)$ and $h_{\rm e,a}(x_i,b_j),\cdots$ can be found for example in Ref. [16]. For $\bar{\rm B}^0\to{\rm f}_0(980)\eta^{(\prime)}$ decays, one can find the corresponding decay amplitudes from those given in Eqs. (15)—(32) by simple replacements.

3 Numerical results and discussions

For numerical calculation, we will use the following input parameters:

$$\begin{aligned}
m({\rm f}_0(980)) &= 0.98\ {\rm GeV}, \quad m_\pi=0.14\ {\rm GeV},\\
m_\eta &= 547.5\ {\rm MeV}, \quad m_{\eta'}=957.8\ {\rm MeV},\\
M_{\rm B} &= 5.28\ {\rm GeV}, \quad m_0^\pi=1.4\ {\rm GeV},\\
M_{\rm W} &= 80.42\ {\rm GeV}, \quad \bar f_{{\rm f}_0}=(0.37\pm0.02)\ {\rm GeV}\\
f_{\rm B} &= 0.19\ {\rm GeV}, \quad f_\pi=0.13\ {\rm GeV},\\
\tau_{{\rm B}^\pm} &= 1.671\ {\rm ps}, \quad \tau_{{\rm B}^0}=1.536\ {\rm ps},\\
V_{\rm tb} &= 0.9997, \quad |V_{\rm td}|=0.0082,\\
V_{\rm ud} &= 0.974, \quad |V_{\rm ub}|=0.00367, \qquad (33)
\end{aligned}$$

with the CKM angle $\beta=21.6^\circ$ and $\gamma=60^\circ$.

It is straightforward to calculate the branching ratios of the considered decays. If ${\rm f}_0(980)$ is purely composed of $\bar{\rm n}{\rm n}$, the pQCD predictions for the branching ratios are

$$\begin{aligned}
\mathcal{B}(\bar{\rm B}^0\to{\rm f}_0(980)\pi^0) &= (0.89^{+0.10+0.16+0.05}_{-0.08-0.13-0.03})\times10^{-6},\\
\mathcal{B}({\rm B}^-\to{\rm f}_0(980)\pi^-) &= (16.4^{+1.7+1.1+0.8}_{-1.6-1.2-0.9})\times10^{-6},\\
\mathcal{B}(\bar{\rm B}^0\to{\rm f}_0(980)\eta) &= (2.0^{+0.2+0.4+0.1}_{-0.2-0.3-0.1})\times10^{-6},\\
\mathcal{B}(\bar{\rm B}^0\to{\rm f}_0(980)\eta') &= (1.3^{+0.2+0.3+0.0}_{-0.1-0.2-0.1})\times10^{-6}, \quad (34)
\end{aligned}$$

where the theoretical uncertainties are from the decay constant of $\bar f_{{\rm f}_0}=0.37\pm0.02$ GeV, the Gegenbauer moments $B_1=-0.78\pm0.08$ and $B_3=0.02\pm0.07$. If ${\rm f}_0(980)$ is purely composed of $\bar{\rm s}{\rm s}$, the branching ratios will be

$$\begin{aligned}
\mathcal{B}(\bar{\rm B}^0\to{\rm f}_0(980)\pi^0) &= (4.66^{+0.52+1.01+0.10}_{-0.49-0.90-0.06})\times10^{-8},\\
\mathcal{B}({\rm B}^-\to{\rm f}_0(980)\pi^-) &= (8.56^{+1.80+2.77+0.96}_{-0.21-1.04-0.00})\times10^{-8},\\
\mathcal{B}(\bar{\rm B}^0\to{\rm f}_0(980)\eta) &= (0.24^{+0.02+0.02+0.05}_{-0.03-0.03-0.03})\times10^{-6},\\
\mathcal{B}(\bar{\rm B}^0\to{\rm f}_0(980)\eta') &= (0.38^{+0.05+0.04+0.04}_{-0.04-0.03-0.03})\times10^{-6}, \quad (35)
\end{aligned}$$

where the theoretical uncertainties are from the same hadron parameters as above.

Table 1. The pQCD predictions (in unit of 10^{-6}) for the branching ratios of ${\rm B}\to{\rm f}_0(980)\pi$, ${\rm f}_0(980)\eta^{(\prime)}$ decays.

channel	$\theta_1=32.5^\circ\pm7.5^\circ$	$\theta_2=152.5^\circ\pm12.5^\circ$	data[19]	QCDF[11]
$Br({\rm B}^-\to{\rm f}_0(980)\pi^-)$	2.5 ± 1.0	$1.6^{+1.8}_{-0.6}$	<3.0	0.9
$Br(\bar{\rm B}^0\to{\rm f}_0(980)\pi^0)$	0.26 ± 0.06	$0.04^{+0.06}_{-0.02}$		0.03
$Br(\bar{\rm B}^0\to{\rm f}_0(980)\eta)$	0.25 ± 0.07	0.59 ± 0.20	<0.4	
$Br(\bar{\rm B}^0\to{\rm f}_0(980)\eta')$	0.67 ± 0.06	0.26 ± 0.03	<1.5	

Table 2. The pQCD predictions (in units of 10^{-2}) for the CP-violating asymmetries of ${\rm B}\to{\rm f}_0(980)\pi$, ${\rm f}_0(980)\eta^{(\prime)}$ decays.

channel	$A_{CP}^{\rm dir}$		$A_{CP}^{\rm mix}$	
	$\theta_1=[25^\circ,40^\circ]$	$\theta_2=[140^\circ,165^\circ]$	$\theta_1=[25^\circ,40^\circ]$	$\theta_2=[140^\circ,165^\circ]$
${\rm B}^-\to{\rm f}_0(980)\pi^-$	[50, 64]	[−39, 7.0]		
$\bar{\rm B}^0\to{\rm f}_0(980)\pi^0$	[−7.5, −2.3]	[−99, −56]	~-69	[−25, 7.1]
$\bar{\rm B}^0\to{\rm f}_0(980)\eta$	[−43, −5.0]	[−55, −30]	[−72, 12]	[−63, −23]
$\bar{\rm B}^0\to{\rm f}_0(980)\eta'$	[−42, −28]	[−29, 8.5]	[−57, −38]	[−75, −38]

When ${\rm f}_0(980)$ is treated as a mixing state of $\bar{\rm n}{\rm n}$ and $\bar{\rm s}{\rm s}$, the leading order pQCD predictions are listed in Table 1, where the two ranges of the mixing angle θ, $\theta_1=[25^\circ,40^\circ]$ and $\theta_2=[140^\circ,165^\circ]$, are taken into account. The QCDF predictions as given in Ref. [11] are also listed in Table 1 as a comparison. The remaining theoretical uncertainties induced by the errors of other input parameters and the wave functions are generally 30%—50%, and not shown here explicitly.

In Fig. 2, we show the θ-dependence of the central values of the pQCD predictions for the branching ratios of the four considered decays. One should note that the large theoretical uncertainties of the pQCD predictions are not shown here explicitly. The two vertical bands show the two ranges of the mixing angle θ preferred by the known experiments[12], while

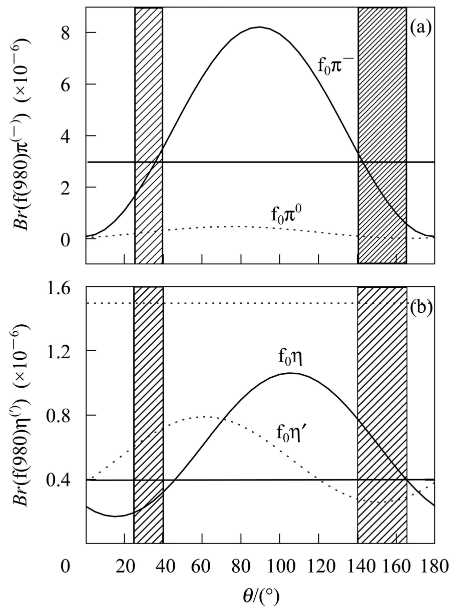

Fig. 2. The θ-dependence of the central values of the pQCD predictions for the branching ratios of (a) $B \to f_0(980)\pi$ decays, and (b) $\bar{B}^0 \to f_0\eta^{(\prime)}$ decays.

the three horizontal solid or dots lines show the corresponding experimental upper limits[19] as listed in Table 1. From the numerical results as shown in Table 1 and Fig. 2, one can not distinguish two regions of the mixing angle θ from currently available data, if the still large theoretical uncertainties are taken into account.

Now we turn to the evaluations of the CP-violating asymmetries of $B \to f_0(980)\pi$, $f_0(980)\eta^{(\prime)}$ decays in the pQCD approach. The pQCD predictions for the direct CP-violating asymmetries of the four considered decays are listed in Table 2. Although the CP-violating asymmetries are large in size, it is still difficult to measure them, since their branching ratios are generally very small, say around 10^{-6}—10^{-8}.

In this paper, based on the assumption of a two-quark structure of the scalar meson $f_0(980)$, we calculated the branching ratios and CP-violating asymmetries of the four $B \to f_0(980)\pi$ and $\bar{B}^0 \to f_0(980)\eta^{(\prime)}$ decays by employing the leading order pQCD factorization approach. The pQCD predictions are generally consistent with both the QCDF predictions and the currently available experimental upper limits.

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