New physical effects on the decay $B_s \to \gamma\gamma$ in the technicolor model

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Abstract In this paper we calculate the contributions to the branching ratio of $B_s \to \gamma\gamma$ from charged pseudo-Goldstone bosons appearing in the one generation Technicolor model. We find that the theoretical value of the branching ratio, $BR(B_s \to \gamma\gamma)$, including the contributions of PGBs, $P^{\pm}$ and $P_8^{\pm}$, is very different from the standard model (SM) prediction. The new physical effects can provide a one to two order of magnitude enhancement of the SM results. It is shown that the decay $B_s \to \gamma\gamma$ can test new physical signals from the technicolor model.

Key words TCM, charged PGBs, B_s

PACS 12.60.Nz, 13.20.Jf

1 Introduction

As is well known, the rare radiative decays of B mesons are particularly sensitive to contributions from new physics beyond the standard model. Both inclusive and exclusive processes, such as the decays $B_s \to X\gamma$, $B_s \to \gamma\gamma$ and $B \to X_s\gamma$ have received some attention in the literature[1—14]. In this paper, we will present our results in technicolor theories.

The one generation technicolor model (OGTM)[15, 16] is the simplest and most frequently studied model. The same as other models, the OGTM has defects such as the S parameter being large and positive[17]. But one can relax the constraint on the OGTM from the S parameter by introducing three additional parameters (V,W,X)[18]. The basic idea of the OGTM is: to introduce a new set of asymptotical free gauge interactions and let the technicolor force act on the technifermions. The technicolor interaction at 1 TeV becomes strong and causes a spontaneous breaking of the global flavor symmetry $SU(8)_L \times SU(8)_R \times U(1)_Y$. The result is $8^2-1=63$ massless Goldstone bosons. Three of the these objects replace the Higgs field and induce a mass for the $W^{\pm}$ and Z^0 gauge bosons. Due to the new strong interaction other Goldstone bosons acquire masses. As for the $B_s \to \gamma\gamma$, only the charged color singlet and color octets have contributions. The gauge couplings of the PGBs are determined by their quantum numbers. In Table 1 we listed the relevant couplings[19] needed in our calculation, where the V_{ud} is the corresponding element of the Kobayashi-Maskawa matrix. The Goldstone boson decay constant F_π[20] should be $F_\pi = v/2 = 123$ GeV, which corresponds to the vacuum expectation value of an elementary Higgs field.

Table 1. The relevant gauge couplings and effective Yukawa couplings for the OGTM.

$P^+P^-\gamma_\mu$	$-ie(p_+-p_-)_\mu$
$P^+_{8a}P^-_{8b}\gamma_\mu$	$-ie(p_+-p_-)_\mu\delta_{ab}$
$P^+\,u\,d$	$i\dfrac{V_{ud}}{2F_\pi}\sqrt{\dfrac{2}{3}}[M_u(1-\gamma_5)-M_d(1+\gamma_5)]$
$P^+_{8a}\,u\,d$	$i\dfrac{V_{ud}}{2F_\pi}\lambda_a[M_u(1-\gamma_5)-M_d(1+\gamma_5)]$
$P^+_{8a}P^-_{8b}g_{c\mu}$	$-gf_{abc}(p_a-p_b)_\mu$

2 Numerical calculations and results

At the lowest order (LO) in QCD the effective

Received 23 August 2008, Revised 19 November 2008

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Hamiltonian is

$$\mathcal{H}_{\rm eff}=\frac{-4G_{\rm F}}{\sqrt{2}}V_{\rm tb}V_{\rm ts}^{*}\sum_{i=1}^{8}C_i(M_{\rm W}^{-})O_i(M_{\rm W}^{-}). \quad (1)$$

Here, as usual, $G_{\rm F}$ denotes the Fermi coupling constant and $V_{\rm tb}V_{\rm ts}^{*}$ indicates the Cabibbo-Kobayashi-Maskawa matrix elements. The current-current, QCD penguin, electromagnetic and chromomagnetic dipole operators are of the form

$$O_1 = (\overline{c}_{{\rm L}\beta}\gamma^{\mu}b_{{\rm L}\alpha})(\overline{s}_{{\rm L}\alpha}\gamma_{\mu}c_{{\rm L}\beta})\,, \quad (2)$$

$$O_2 = (\overline{c}_{{\rm L}\alpha}\gamma^{\mu}b_{{\rm L}\alpha})(\overline{s}_{{\rm L}\beta}\gamma_{\mu}c_{{\rm L}\beta})\,, \quad (3)$$

$$O_3 = (\overline{s}_{{\rm L}\alpha}\gamma^{\mu}b_{{\rm L}\alpha})\sum_{\rm q=u,d,s,c,b}(\overline{q}_{{\rm L}\beta}\gamma_{\mu}q_{{\rm L}\beta})\,, \quad (4)$$

$$O_4 = (\overline{s}_{{\rm L}\alpha}\gamma^{\mu}b_{{\rm L}\beta})\sum_{\rm q=u,d,s,c,b}(\overline{q}_{{\rm L}\beta}\gamma_{\mu}q_{{\rm L}\alpha})\,, \quad (5)$$

$$O_5 = (\overline{s}_{{\rm L}\alpha}\gamma^{\mu}b_{{\rm L}\alpha})\sum_{\rm q=u,d,s,c,b}(\overline{q}_{{\rm R}\beta}\gamma_{\mu}q_{{\rm R}\beta})\,, \quad (6)$$

$$O_6 = (\overline{s}_{{\rm L}\alpha}\gamma^{\mu}b_{{\rm L}\beta})\sum_{\rm q=u,d,s,c,b}(\overline{q}_{{\rm R}\beta}\gamma_{\mu}q_{{\rm R}\alpha})\,, \quad (7)$$

$$O_7 = (e/16\pi^2)m_{\rm b}\overline{s}_{\rm L}\sigma^{\mu\nu}b_{\rm R}F_{\mu\nu}\,, \quad (8)$$

$$O_8 = (g/16\pi^2)m_{\rm b}\overline{s}_{\rm L}\sigma^{\mu\nu}T^{\rm a}b_{\rm R}G^{\rm a}_{\mu\nu}\,, \quad (9)$$

where α and β are color indices, $\alpha=1,\cdots,8$ label the $SU(3)$c generators, e and g refer to the electromagnetic and strong coupling constants, while $F_{\mu\nu}$ and $G^{\rm a}_{\mu\nu}$ denote the QED and QCD field strength tensors, respectively.

The Feynman diagrams that contribute to the matrix element are the following (Fig. 1).

In Fig. 2 the dashed lines represent the charged PGBs $\rm P^{\pm}$ and $\rm P_8^{\pm}$ of the OGTM. Now we first integrate out the top quark and the weak W bosons at the $\mu=M_{\rm W}$ scale, generating an effective five-quark theory and run the effective field theory down to the b-quark scale to obtain the leading logarithmic QCD corrections by using the renormalization group equation. The Wilson coefficients are processed independently and the coefficients $C_i(\mu)$ of the 8 operators are calculated from Fig. 2. The Wilson coefficients read[21] as

$$C_i(M_{\rm W})=0,\quad i=1,3,4,5,6,\quad C_2(M_{\rm W})=1, \quad (10)$$

$$C_7(M_{\rm W})=-A(\delta)+\frac{B(x)}{3\sqrt{2}G_{\rm F}F_{\pi}^2}+\frac{8B(y)}{3\sqrt{2}G_{\rm F}F_{\pi}^2}, \quad (11)$$

$$C_8(M_{\rm W})=-C(\delta)+\frac{D(x)}{3\sqrt{2}G_{\rm F}F_{\pi}^2}+\frac{8D(y)+E(y)}{3\sqrt{2}G_{\rm F}F_{\pi}^2}, \quad (12)$$

with $\delta = M_{\rm W}^2/m_{\rm t}^2$, $x = (m({\rm P}^{\pm})/m_{\rm t})^2$ and $y = (m({\rm P}_8^{\pm})/m_{\rm t})^2$. From Eqs. (11) and (12), we see that the situation of the color-octet charged PGBs is more complicated than that of the color-singlet charged PGBs because of the involvement of the color interactions. We have used the following abbreviations

$$A(\delta) = \frac{\frac{1}{3}+\frac{5}{24}\delta-\frac{7}{24}\delta^2}{(1-\delta)^3}+\frac{\frac{3}{4}\delta-\frac{1}{2}\delta^2}{(1-\delta)^4}\log\delta, \quad (13)$$

$$B(y) = \frac{-\frac{11}{36}+\frac{53}{72}y-\frac{25}{72}y^2}{(1-y)^3}+\frac{-\frac{1}{4}y+\frac{2}{3}y^2-\frac{1}{3}y^3}{(1-y)^4}\log y, \quad (14)$$

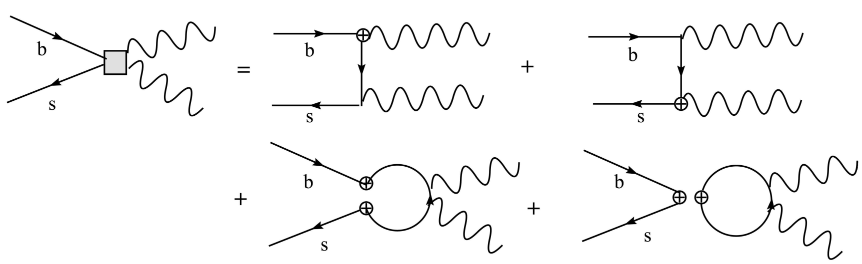

Fig. 1. Examples of Feynman diagrams that contribute to the matrix element.

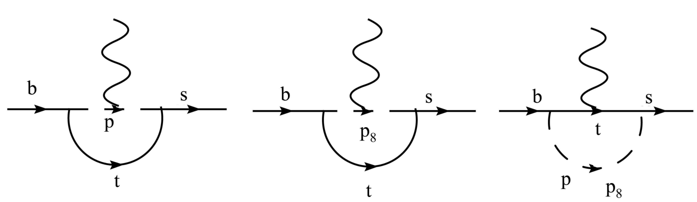

Fig. 2. The Feynman diagrams that contribute to the Wilson coefficients $C7$, $C8$.

$$C(\delta) = \frac{\frac{1}{8}-\frac{5}{8}\delta-\frac{1}{4}\delta^2}{(1-\delta)^3}-\frac{\frac{3}{4}\delta^2}{(1-\delta)^4}\log\delta, \tag{15}$$

$$D(y) = \frac{-\frac{5}{24}+\frac{19}{24}y-\frac{5}{6}y^2}{(1-y)^3}+\frac{\frac{1}{4}y^2-\frac{1}{2}y^3}{(1-y)^4}\log y, \tag{16}$$

$$E(y) = \frac{\frac{3}{2}-\frac{15}{8}y-\frac{15}{8}y^2}{(1-y)^3}+\frac{\frac{9}{4}y-\frac{9}{2}y^2}{(1-y)^4}\log y. \tag{17}$$

By caculating the graphs for the exchanged W boson in the SM one finds the functions A and C. Evaluating the graphs involving the color-singlet and color-octet charged PGBs in the OGTM one obtains the functions B, D and E. If $\delta<1$, $x,y\gg 1$, the OGTM contributions B, D and E have always a relative minus sign with respect to the SM contributions A and C. As a result, the OGTM contributions always interfere destructively with the SM contributions.

The leading-order results for the Wilson coefficients of all operators entering the effective Hamiltonian in Eq. (1) can be written in analytic form. They are

$$C_7^{\text{eff}}(m_b) = \eta^{16/23}C_7(M_W)+\frac{8}{3}(\eta^{14/23}-\eta^{16/23})\times C_8(M_W)+C_2(M_W)\sum_{i=1}^{8}h_i\eta^{a_i}\ , \tag{18}$$

with $\eta=\alpha_s(M_W)/\alpha_s(m_b)$,

$$h_i = \left(\frac{626126}{272277},-\frac{56281}{51730},-\frac{3}{7},-\frac{1}{14},-0.6494,-0.0380,-0.0186,-0.0057\right), \tag{19}$$

$$a_i = \left(\frac{14}{23},\frac{16}{23},\frac{6}{23},-\frac{12}{23},0.4086,-0.4230,-0.8994,0.1456\right). \tag{20}$$

To calculate $B_s\to\gamma\gamma$, one may follow a perturbative QCD approach which divides the evaluation of the soft gluon effects into the calculation of the B_s meson wave function and a systematic way of resumming large logarithms due to hard gluons with energies between 1 GeV and m_b. In order to calculate the matrix element of Eq. (1) for $B_s\to\gamma\gamma$, one can work in the weak binding approximation and assume that both the b and the s quarks are at rest in the B_s meson, and that the b quark carries most of the meson energy. Its four velocity can then be treated as equal to that of B_s. Hence one may write the b quark momentum as $p_b=m_b v$, where v is the common four velocity of b and B_s. We have

$$\begin{aligned} p_b\cdot k_1 &= m_b v\cdot k_1=\frac{1}{2}m_b m_{B_s}=p_b\cdot k_2,\\ p_s\cdot k_1 &= (p-k_1-k_2)\cdot k_1= -\frac{1}{2}m_{B_s}(m_{B_s}-m_b)=p_s\cdot k_2\ . \end{aligned} \tag{21}$$

The amplitude of $B_s\to\gamma\gamma$ is calculated by means of the following relations

$$\begin{aligned} \langle 0|\bar{s}\gamma_\mu\gamma_5 b|B_s(P)\rangle &= -\mathrm{i}f_{B_s}P_\mu\ ,\\ \langle 0|\bar{s}\gamma_5 b|B_s(P)\rangle &= \mathrm{i}f_{B_s}M_B\ , \end{aligned} \tag{22}$$

where f_{B_s} is the B_s meson decay constant which is about 200 MeV.

The total amplitude[6] is now separated into a CP-even and a CP-odd part

$$T(B_s\to\gamma\gamma)=M^+F_{\mu\nu}F^{\mu\nu}+\mathrm{i}M^-F_{\mu\nu}\tilde{F}^{\mu\nu}\ . \tag{23}$$

We find that

$$M^+ = \frac{-4\sqrt{2}\alpha G_F}{9\pi}f_{B_s}m_{b_s}V_{ts}^*V_{tb}\times\left(\frac{m_b}{m_{B_s}}BK(m_b^2)+\frac{3C_7}{8\bar{\Lambda}}\right), \tag{24}$$

with $B=-(3C_6+C_5)/4$, $\bar{\Lambda}=m_{B_s}-m_b$, and

$$M^- = \frac{4\sqrt{2}\alpha G_F}{9\pi}f_{B_s}m_{b_s}V_{ts}^*V_{tb}\times\left(\sum_q A_q J(m_q^2)+\frac{m_b}{m_{B_s}}BL(m_b^2)+\frac{3C_7}{8\bar{\Lambda}}\right), \tag{25}$$

where

$$\begin{aligned} A_u &= (C_3-C_5)N_c+(C_4-C_6),\\ A_d &= \frac{1}{4}\left[(C_3-C_5)N_c+(C_4-C_6)\right],\\ A_c &= (C_1+C_3-C_5)N_c+(C_2+C_4-C_6),\\ A_s &= \frac{1}{4}\left[(C_3+C_4-C_5)N_c+(C_3+C_4-C_6)\right], \end{aligned} \tag{26}$$

$$A_s = \frac{1}{4}\left[(C_3+C_4-C_5)N_c+(C_3+C_4-C_6)\right]. \tag{27}$$

The functions $J(m^2)$, $K(m^2)$ and $L(m^2)$ are defined by

$$\begin{aligned} J(m^2) &= I_{11}(m^2),\\ K(m^2) &= 4(I_{11}(m^2)-I_{00}(m^2)),\\ L(m^2) &= I_{00}(m^2), \end{aligned} \tag{28}$$

with

$$I_{pq}(m^2)=\int_0^1\mathrm{d}x\int_0^{1-x}\mathrm{d}y\frac{x^p y^q}{m^2-2xyk_1\cdot k_2-\mathrm{i}\varepsilon}\ . \tag{29}$$

The decay width for $B_s\to\gamma\gamma$ is simply

$$\Gamma(B_s\to\gamma\gamma)=\frac{m_{B_s}^3}{16\pi}\left(|M^+|^2+|M^-|^2\right). \tag{30}$$

In the SM $C_2 = C_2(M_W) = 1$ (the other Wilson coefficients are zero), we find $\Gamma(B_s \to \gamma\gamma) = 1.28 \times 10^{-10}$ eV which amounts to a branching ratio of $Br(B_s \to \gamma\gamma) = 2.8 \times 10^{-7}$ for the given $\Gamma_{B_s}^{\rm total} = 4.48 \times 10^{-4}$ eV. In the numerical calculations we use as input parameters $M_W = 80.22$ GeV, $\alpha_s(m_Z) = 0.117$, $m_c = 1.5$ GeV, $m_b = 4.8$ GeV and $|V_{tb}V_{ts}^*|^2/|V_{cb}|^2 = 0.95$, respectively. The present experimental limit[22] on the decay $B_s \to \gamma\gamma$ is

$$Br(B_s \to \gamma\gamma) \leqslant 8.6 \times 10^{-6} , \qquad (31)$$

which is far from the theoretical results. So, we can not put any constraint on the masses of PGBs. The constraints on the masses of $P^{\pm}$ and $P_8^{\pm}$ can be obtained from the decay[21] $B \to s\gamma$: $m_{P_8^{\pm}} > 400$ GeV.

3 Summary

Figures 3 and 4 show the mass dependence of $Br(B_s \to \gamma\gamma)$ on $P_8^{\pm}$ ($P^{\pm}$) for different values of $m_{P^{\pm}}$ ($m_{P_8^{\pm}}$). From Figs. 3 and 4, one can see that the new physics contributions can lead to appreciable changes of the SM predictions which may be enhanced by about one to two orders of magnitude in a reasonable mass range for the PGBs. This provides a strong new physical signal from the technicolor model. The new physics effects will become smaller with increasing mass of the $P_8^{\pm}$ and $P^{\pm}$, which is consistent with the so-called decoupling theorem for heavy enough non-standard bosons. From Eqs. (16), (17) and (18) one can see that the functions B, D and E go to zero as x, $y \to \infty$. The branching ratios as shown in Fig. 3 change much faster than those in Fig. 4, which reveals that the color octet $P_8^{\pm}$ provides the dominant new physics contribution.

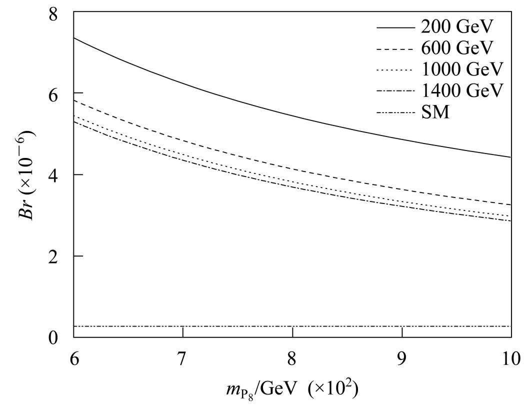

Fig. 3. $Br(B_s \to \gamma\gamma)$ versus the mass of $P_8^{\pm}$ for different values of $m_{P^{\pm}}$.

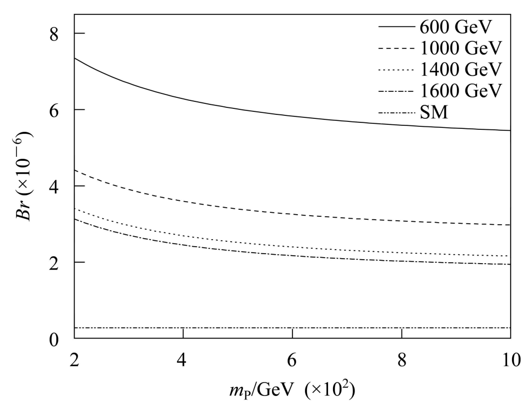

Fig. 4. $Br(B_s \to \gamma\gamma)$ versus the mass of $P^{\pm}$ for different values of $P_8^{\pm}$.

As a conclusion, the new physics contribution to the rare decay of $B_s \to \gamma\gamma$ from the PGBs can be rather large in magnitude, and may be detected in future precision experiments.

The author Qin Xiu-Mei thanks Dr. Huo-WJ and Dr. Zhou-HQ for their help and the instruction.

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