Bipartite entanglement in a two-qubit Heisenberg XXZ chain under an inhomogeneous magnetic field

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Abstract This paper investigates the bipartite entanglement of a two-qubit Heisenberg XXZ chain under an inhomogeneous magnetic field. By the concept of negativity, we find that the inhomogeneity of the magnetic field may induce entanglement and the critical magnetic field is independent of J_z. We also find that the entanglement is symmetric with respect to a zero magnetic field. The anisotropy parameter J_z may enhance the entanglement.

Key words bipartite entanglement, negativity, Heisenberg XXZ chain

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1 Introduction

Quantum entanglement plays a central role in quantum information processing (QIP)[1—5] and condensed matter physics. The main motivation behind such interest is twofold: On the one hand, entanglement is a unique quantum mechanical resource that plays a key role in many of the most interesting applications of quantum computation and quantum information. On the other hand, entanglement is a unique measure of the quantum correlation of a pure state in condensed matter physics and may bring new insights.

The quantum entanglement in solid systems such as spin chains[6—22] is an important emerging field. Spin chains are natural candidates for the realization of entanglement compared with other physics systems. The Heisenberg spin system, which may be a suitable candidate to simulate the relation between qubits in a quantum computer, is an extensively studied solid-state system which is simple but realistic. The computational results of the preceding work show that the amount of pairwise entanglement between two spins can be modified by varying the strength of the temperature or the external magnetic fields.

In this paper, we study the bipartite entanglement in a two-qubit Heisenberg XXZ chain under an inhomogeneous magnetic field by applying the concept of negativity. We will concentrate on the dependence of entanglement on various parameters such as the external magnetic field, the anisotropy parameter, as well as the temperature.

2 General formalism

The Hamiltonian of the N-qubit anisotropic Heisenberg XXZ model in an inhomogeneous magnetic field is[11]

$$H = \frac{1}{2}\sum_{i=1}^{N}\big[J\sigma_i^x\sigma_{i+1}^x + J\sigma_i^y\sigma_{i+1}^y + J_z\sigma_i^z\sigma_{i+1}^z + (B+b)\sigma_i^z + (B-b)\sigma_{i+1}^z\big], \tag{1}$$

where J and J_z are the real coupling coefficients. The coupling constants $J>0$ and $J_z>0$ correspond to the anti-ferromagnetic case, and $J<0$ and $J_z<0$ to the ferromagnetic case. $B\geqslant 0$ is restricted, and the magnetic fields acting on the two spins have been parameterized in such a way that b controls the degree of inhomogeneity. We consider here the Hamiltonian for the N=2 case.

Now, we briefly introduce the definition of the

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negativity[23] for a state ρ. A non-entangled state has necessarily a positive partial transpose (PPT)[24] according to the Peres-Horodecki criterion. In the case of two spins of one half (1/2,1/2), and the case of (1/2, 1) mixed spins, a PPT is sufficient. Negativity was firstly introduced by Vidal and Werner and is defined as

$$N(\rho)=\frac{||\rho^{T_2}||_1-1}{2}\ , \tag{2}$$

where the norm of the trace ρ^{T_2} is equal to the sum of the absolute values of the eigenvalues of ρ^{T_2}, and T_2 denotes the partial transpose of ρ with respect to the second subsystem. The state at thermal equilibrium[25] is represented by Gibb's density operator $\rho(T)=Z^{-1}\exp(-H/k_{\rm B}T)$, where $Z=\mathrm{tr}[\exp(-H/k_{\rm B}T)]$ is the partition function, $k_{\rm B}$ is the Boltzmann's constant and is set to be 1 hereafter.

From the fact that the partial transpose does not change the trace of a state and $\mathrm{tr}(\rho)$=1, it is straightforward to check that the negativity is equivalent to the absolute value of the sum of the negative eigenvalues of ρ^{T_2}. That is, the bipartite entanglement between sites 1 and the others can be measured by means of the negativity N_{1-n}[23]

$$N_{1-n}=\sum\nolimits_i|(\mu_{1-n})_i|. \tag{3}$$

3 Results and discussion

In the standard basis $\{|11\rangle, |10\rangle, |01\rangle, |00\rangle\}$, we can get the eigenvalues and eigenstates of this system. This is in accordance with Ref. [11].

$$E_1=(J_z/2-B),\quad E_2=(J_z/2+B),$$

$$E_3=-Jz/2-\nu,\quad E_4=-Jz/2+\nu,$$

$$|\psi_1\rangle=|00\rangle,\ |\psi_2\rangle=|11\rangle,$$

$$|\psi_3\rangle=\frac{1}{\sqrt{1+\chi^2/J^2}}\left(\frac{\chi}{J}|10\rangle+|01\rangle\right), \tag{4}$$

$$|\psi_4\rangle=\frac{1}{\sqrt{1+\mu^2/J^2}}\left(\frac{\mu}{J}|10\rangle+|01\rangle\right),$$

where $\nu=\sqrt{b^2+J^2}$, $\chi=b-\nu$, $\mu=b+\nu$. And the partial transposed density matrix of this system can be written as

$$\rho_{12}^{T_1}=\frac{1}{Z}\times$$

$$\begin{pmatrix} \mathrm{e}^{-E_2/kT} & 0 & 0 & -\omega \\ 0 & \mathrm{e}^{J_z/2kT}(\alpha-\beta) & 0 & 0 \\ 0 & 0 & \mathrm{e}^{J_z/2kT}(\alpha+\beta) & 0 \\ -\omega & 0 & 0 & \mathrm{e}^{-E_1/kT} \end{pmatrix}, \tag{5}$$

where $Z=\mathrm{e}^{-E_2/kT}(1+\mathrm{e}^{2B/kT})+2\mathrm{e}^{(J_z+B)/kT}\cosh(\nu/kT)$, $\alpha=\cosh(\nu/kT)$, $\beta=b\sinh(\nu/kT)$ and $\omega=\mathrm{e}^{J_z/2kT}J\sinh(\nu/kT)$. It is very tedious to write the eigenvalues of $\rho_{12}^{T_1}$ and here we give some numerical results and discuss them in detail.

Case 1: J_z=0. This model becomes a XX spin chain. In Fig. 1; we give the results at different temperatures for the nonuniform magnetic field (Fig. 1(a), B=0) and the uniform magnetic field (Fig. 1(b), b=0). It can be seen that with increasing temperature the negativity strongly decreases due to the mixing of the maximally entangled state with other states. The maximum entanglement occurs of course at T=0 (ground-state entanglement) with a value of 0.5 in this case. The negativity always decreases with an increase of B and b. However, the curve line will emerge at a stable value with the

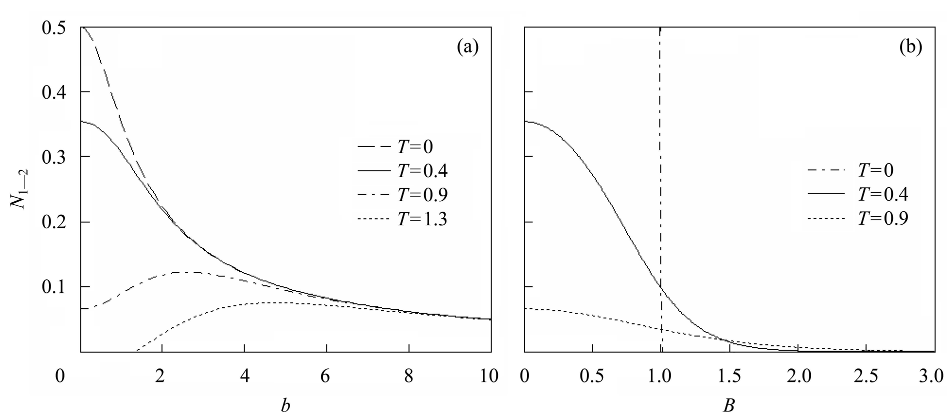

Fig. 1. The negativity N_{1-2} versus b (left) and B (right) for different temperature (J=1).

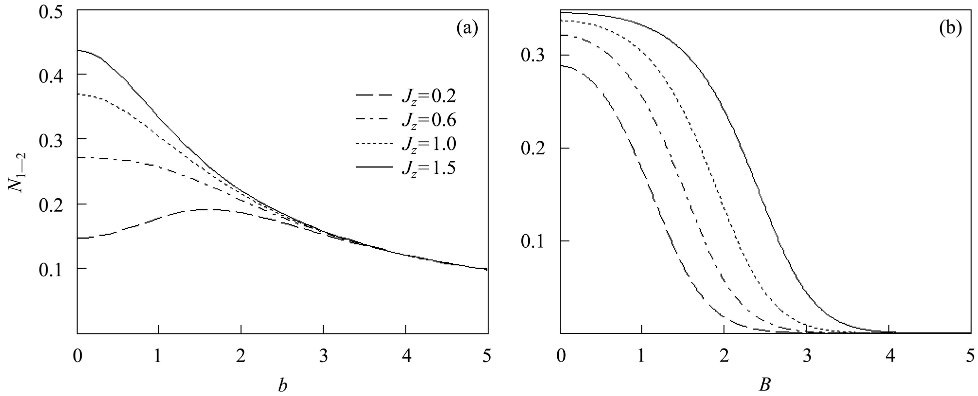

Fig. 2. The negativity N_{1-2} versus b (left) and B (right) for different J_z.

increase of the magnetic field. When T=1.3, the inhomogeneity of the magnetic field will induce entanglement and the value will increase with the increase of b. Obviously entanglement is reduced by higher temperatures, however the inhomogeneity can induce it. At the same time, we also note that the entanglement is symmetric with respect to a zero magnetic field.

Case 2: $J_z \neq 0$. We give here negativities N_{1-2} calculated for different anisotropy parameters J_z versus the inhomogeneous (J=1, T=0.5, B=1) and versus the uniform magnetic field (J=1, T=0.5, b=1). N_{1-2} falls off gradually with the increasing value of B and b. One observes that for higher values of J_z, the entanglement of this system is stronger than in case 1. It can also be seen that the bipartite entanglement decreases with the increasing value of B and approaches zero at some B value (Fig. 2(b)). In Fig. 2 also the combination occurs when $J_z = 1$. The model then corresponds to a XXX spin chain. From the two figures we notice that different apects of the inhomogeneity of the magnetic field seem to play a role.

4 Conclusions

In this paper, we have studied the properties of the bipartite entanglement in the two-qubit Heisenberg XXZ chain under the influence of an inhomogeneous magnetic field. We obtained some numerical results of this model by investigating the negativity. Our results show that the negativity exists for both the anti-ferromagnetic and the ferromagnetic case. We found that the inhomogeneity of the magnetic field will induce entanglement and the entanglement is enhanced by increasing the anisotropy parameter J_z.

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