Quark model study of multiquark states*

PING Jia-Lun(平加伦)[1;1)] DENG Cheng-Rong(邓成荣)[2]
HUANG Hong-Xia(黄虹霞)[1] WANG Fan(王凡)[3]

1 (Physics Department, Nanjing Normal University, Nanjing 210097, China)
2 (School of Mathematics and Physics, Chongqing Jiaotong University, Chongqing 400074, China)
3 (Joint Center for Particle Nuclear Physics and Cosmology, Nanjing University and
Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210093, China)

Abstract The six- and four-quark systems are studied in the framework of constituent quark models. It is emphasized that the color confinement used in multiquark system should be different from the one used in two- or three-quark system. For six-quark system, we look for ΔΔ and NΔ dibaryon resonances by calculating NN scattering phase shifts with explicit coupling to these dibaryon channels in quark delocalization and color screening model. The model gives a good description of low-energy NN properties and predicts $IJ^P = 03^+$ and 01^+ ΔΔ resonances, which can be promising candidates for the isoscalar ABC structure reported by the CELSIUS-WASA Collaboration. For tetraquark system, a flux-tube quark model with multi-body confinement interaction is employed to study Y(2175) as a tetraquark state. The Y(2175) with diquark-antidiquark structure has energy 2174 MeV which is very consistent with experimental data. The calculation shows that multi-body confinement potential may play a vital role in the multiquark system.

Key words quark model, dibaryon, tetraquark, color confinement, multibody interaction

PACS 12.39.-x, 14.20.Pt, 13.75.Cs

1 Introduction

Since Jaffe predicted the H-particle in 1977[1], the interest in multiquark system has persisted. All the quark models, including lattice QCD calculations, predict that there should be multiquark systems $(q\bar{q})^2$, $q^4\bar{q}$, q^6, quark gluon hybrids $q\bar{q}g$, q^3g, and glueballs, in addition to the $q\bar{q}$ mesons and q^3 baryons. However, up to now there has been no well established experimental candidate of these multiquark states. Recently, the CELSIUS-WASA Collaboration has reported preliminary results on the ABC anomaly in the production cross section of the $pn\to d\pi^0\pi^0$ reaction that suggests the presence of an isoscalar $J^P = 1^+$ or 3^+ subthreshold ΔΔ resonance, with resonance mass estimated at $\sim$2.36 GeV and a width of $\sim$ 80 MeV[2]. The relatively large binding energy involved gives an object that is much closer to these interesting multiquark states than a loosely bound system like the deuteron. The Belle, BaBar and other experimental collaborations have also been reported a large number of candidates for open and hidden charm meson states, which can not be explained by $q\bar{q}$ meson and suggested tetraquark states[3].

Quantum chromodynamics (QCD) is widely accepted as the fundamental theory of the strong interaction. Direct applications of QCD using lattice methods have recently been made in the study of low-energy hadronic interactions, including the tetraquark system [4] and the nucleon-nucleon (NN) interaction[5]. However, QCD-inspired quark models are still the main tool for detailed studies of the hadron-hadron interaction and multiquark system. The commonly used quark model is the constituent quark model, where the complicated interactions between current quarks is approximately transformed into dynamic properties of quasiparticles (constituent quark) and the residual interactions between quasiparticles. The two-body color confinement has to be imposed by hand. The constituent quark model gives a good description of properties of hadrons: 2-

Received 7 August 2009
* Supported by National Natural Science Foundation of China (10775072) and Research Fund for the Doctoral Program of Higher Education of China (20070319007)
1) E-mail: jlping@njnu.edu.cn

quark meson and 3-quark baryon, because of their unique color structures. Applying to nucleon-nucleon scattering, a reasonable agreement with experimental data is still possible after including the σ-meson, although there is a controversy about its effect when taking it as ππ S-wave resonance[6]. The agreement is also due to the fact that the nucleon-nucleon scattering is not sensitive to the detail of short-range part of the nuclear force. From recent lattice QCD calculation, one find that the color dependent two body confinement interaction is consistent with the lattice QCD results only for two and three quark systems in color singlet states but inconsistent with the many body interaction obtained for multiquark systems[7]. For multiquark systems and color octet hadrons, quark pairs are not always in color antisymmetric state but also color symmetric ones. The color factor $\lambda_i\cdot\lambda_j$ will give rise to anti-confinement interaction for symmetric quark pairs[8]. There is no sound theoretical reason to extend the color dependent two body confinement interaction, with Casimir scaling $\lambda\cdot\lambda$, to multiquark system.

To study multiquark system, the color confinement should be modified or multi-body interaction should be used. In 1990s, a new quark model: quark delocalization and color screening model (QDCSM), is proposed, where the color confinement is modified when applying to quark-pair belonging to different baryons. The model has been successfully applied to describe nucleon-nucleon and hyperon-nucleon scattering[9]. The first part of the article is extend our past calculation of NN phase shifts to the resonance region near the ΔΔ and NΔ thresholds by including these dibaryon channels in coupled-channel calculations and try to understand the CELSIUS-WASA result on the ABC anomaly. The second part of the article is to study a tetraquark state by using string-like multi-body interaction. The structure of the article is as follows. The brief introduction to QDCSM and the calculation of NN scattering are given in Sec. II. Sec. III presents our flux-tube quark model and its application on tetraquark system. The summary is given in the last section.

2 QDCSM and NN scattering phase-shift calculation

The detail of QDCSM can be found in Ref. [9], here only the hamiltonian and the wavefunctions are given.

$$H = \sum_{i=1}^{6}\left(m_i+\frac{p_i^2}{2m_i}\right)-T_{\rm c}+\sum_{i<j}\left[V^{\rm OGE}(r_{ij})+V^{\pi}(r_{ij})+V^{\rm CON}(r_{ij})\right], \tag{1}$$

$$V^{\rm OGE}(r_{ij}) = \frac{1}{4}\alpha_s\lambda_i\cdot\lambda_j\left[\frac{1}{r_{ij}}-\frac{\pi}{m_{\rm q}^2}\left(1+\frac{2}{3}\sigma_i\cdot\sigma_j\right)\delta(r_{ij})-\frac{3}{4m_{\rm q}^2r_{ij}^3}S_{ij}\right],\quad S_{ij}=\frac{\vec{\sigma}_i\cdot\vec{r}_{ij}\vec{\sigma}_j\cdot\vec{r}_{ij}}{r_{ij}^2}-\frac{1}{3}\ \vec{\sigma}_i\cdot\vec{\sigma}_j,$$

$$V^{\pi}(r_{ij}) = \frac{1}{3}\alpha_{\rm ch}\frac{\Lambda^2}{\Lambda^2-m_\pi^2}m_\pi\left\{\left[Y(m_\pi r_{ij})-\frac{\Lambda^3}{m_\pi^3}Y(\Lambda r_{ij})\right]\sigma_i\cdot\sigma_j+\left[H(m_\pi r_{ij})-\frac{\Lambda^3}{m_\pi^3}H(\Lambda r_{ij})\right]S_{ij}\right\}\tau_i\cdot\tau_j,$$

$$V^{\rm CON}(r_{ij}) = -a_{\rm c}\vec{\lambda}_i\cdot\vec{\lambda}_j\begin{cases} r_{ij}^2 & \text{if } i,j \text{ occur in the same baryon orbit,}\\ \dfrac{1-{\rm e}^{-\mu r_{ij}^2}}{\mu} & \text{if } i,j \text{ occur in different baryon orbits,}\end{cases} \tag{2}$$

where all symbols have their usual meaning. Eq. (2) shows the modification of color confinement, the confinement is screened when the interacting quark-pair belongs to different baryons, whereas no screening for quark-pair in the same baryon.

Quark delocalization in QDCSM is realized by assuming the single particle orbital wavefunctions of QDCSM as a linear combination of left and right Gaussians, the single particle orbital wavefunctions of the ordinary quark cluster model,

$$\begin{aligned}
\psi_\alpha(\vec{S}_i,\epsilon) &= \left(\phi_\alpha(\vec{S}_i)+\epsilon\phi_\alpha(-\vec{S}_i)\right)/N(\epsilon),\\
\psi_\beta(-\vec{S}_i,\epsilon) &= \left(\phi_\beta(-\vec{S}_i)+\epsilon\phi_\beta(\vec{S}_i)\right)/N(\epsilon),\\
N(\epsilon) &= \sqrt{1+\epsilon^2+2\epsilon{\rm e}^{-S_i^2/4b^2}}.\\
\phi_\alpha(\vec{S}_i) &= (\pi b^2)^{-3/4}\,{\rm e}^{-\frac{1}{2b^2}(\vec{r}_\alpha-\vec{S}_i/2)^2}\\
\phi_\beta(-\vec{S}_i) &= (\pi b^2)^{-3/4}\,{\rm e}^{-\frac{1}{2b^2}(\vec{r}_\beta+\vec{S}_i/2)^2}.
\end{aligned} \tag{3}$$

To calculate the NN scattering phase-shifts, The resonanting-group method (RGM) is used. The detail of RGM can be found in Ref. [10]. Here only calculated results are shown. The model parameters used in the calculation is shown in Table 1. The phase shifts for NN $I=0,J=3$ and 1 are displayed in Figs. 1 and 2. The experimental data are taken from SP07[11]. In the low energy region ($E_{\rm cm}<400$ MeV), the quark models give good phase shifts, agrees with experimental data.

Table 1. Parameters of QDCSM.

$m_{\rm u,d}$/MeV	b/fm	$a_{\rm c}$/(MeV$\cdot$ fm^{-2})	μ/fm^{-2}
313	0.60	18.55	1.00
$\alpha_{\rm s}$	m_π/MeV	$\alpha_{\rm ch}$	Λ/fm^{-1}
0.996	138	0.027	4.2

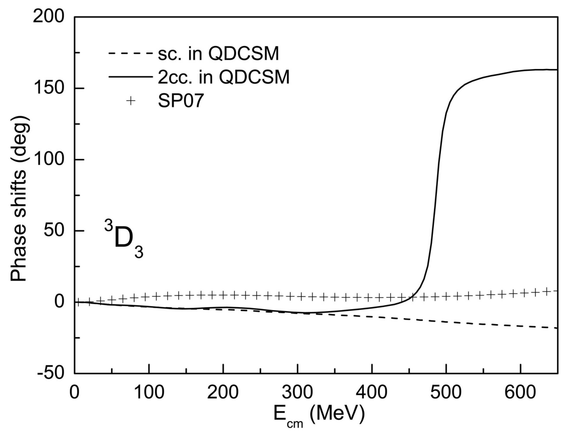

Fig. 1. $^3D_3^{\rm NN}$ phase shifts calculated for a single channel (sc) and two coupled channels (2cc): $^3D_3^{\rm NN}$, $^7S_3^{\Delta\Delta}$.

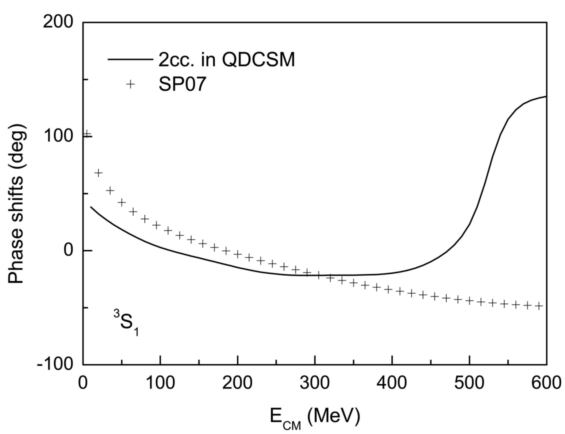

Fig. 2. $^3S_1^{\rm NN}$ phase shifts calculated with two coupled color-singlet channels: $^3S_1^{\rm NN}$, $^3S_1^{\Delta\Delta}$.

For $I=0,J=3$, The single channel calculation of $^7S_3^{\Delta\Delta}$ shows that the state is a bound-state with binding energy 188 MeV respect to two-Δ threshold. Coupling to the $^3D_3^{\rm NN}$ channel causes this bound state to change into an elastic resonance where the phase shift, shown in Fig. 1, rises through $\pi/2$ at a resonance mass that has been shifted up by 2 MeV. The result shows that the mass shift is dominated by the NN scattering states below the pure bound-state mass rather than those above it.

For $I=0,J=1$, the single channel calculation of $^3S_1^{\Delta\Delta}$ shows that the state is also a bound-state with binding energy 349 MeV respect to two-Δ threshold. The coupling to the $^3S_1^{\rm NN}$ channel now pushes up the bound $^3S_1^{\Delta\Delta}$ mass of the QDSCM by about 300 MeV, so that it becomes a resonance at 2408 MeV. This very large mass shift is caused by the presence of a lower-mass state, the deuteron, in the admixed $^3S_1^{\rm NN}$ channel.

The state $IJ^P=12^+$ is also studied in the model. The pure $^5S_2^{\rm N\Delta}$ state shows bound state behavior with the mass at 2167 MeV. Its mass is pushed up only a little: 2168 MeV by coupling to the $^1D_2^{\rm NN}$ continuum. The resulting NN phase shifts are shown in Fig. 3.

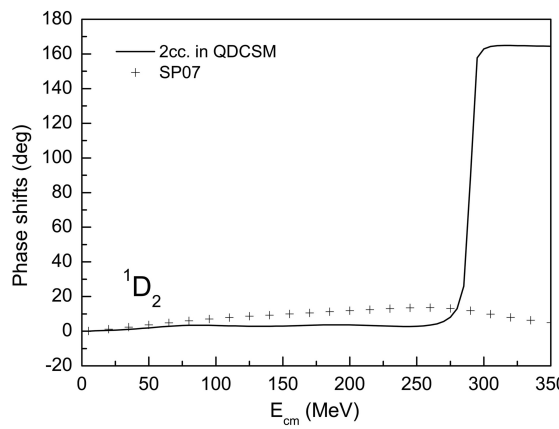

Fig. 3. NN $^1D_2$ phase shifts calculated with two coupled color-singlet channels: $^1D_2^{\rm NN}$, $^5S_2^{\rm N\Delta}$.

Table 2. Masses and widths (in MeV) of dibaryons.

	$IJ=03^+$		$IJ=01^+$		$IJ=12^+$	
	M	Γ	M	Γ	M	Γ
1c	2276	–	2115	–	2167	–
2cc	2278	17/33	2408	70/136	2168	4/117

Table 2 collects the masses and widths of the obtained dibaryons. The width after the slash is the total width, which is the sum of elastic width and the inelastic with caused by decaying Δs[12]. From the mass and width, it suggests that the ABC effect may originates from a dibaryon resonance in the $^3S_1^{\rm NN}$ channel. However, the difficult of the explanation comes from the fact that the NN scattering described by SP07 is highly elastic so that $\sigma_{{\rm d}\pi\pi}$ is far too small. The situation for the $^3D_3^{\rm NN}$ channel is more promising but the mass of the dibaryon is too small compared to the peak in the ABC effect. So additional experimental knowledge and theoretical studies of NN properties in the NN resonance region are needed.

3 Multi-body interaction and tetraquark state

3.1 The flux-tube quark model

Lattice QCD suggested that the color confinement in the multiquark system is a multi-body interaction[7]. So it is interesting to construct constituent quark model with the multi-body interaction to study multiquark system. In the following we construct a flux-tube quark model and apply it to tetraquark state. Fig. 4 shows a tetraquark state with two strange quarks (solid circles) and two anti-strange quarks (open circles). $\boldsymbol{r}_i$ is quark's position, $\boldsymbol{y}_i$ represents a junction where three flux tubes meet. A thin line connecting a quark (antiquark) and a junction represents a fundamental, i.e. color triplet, representation and a thick line connecting two junctions is for a color sextet or others representations, namely a compound string. The different types of string may have differing stiffness[13, 14]. Color coupling satisfying overall color singlet are $[[\mathrm{ss}]_{\bar{3}}[\overline{\mathrm{ss}}]_3]_1$ and $[[\mathrm{ss}]_6[\overline{\mathrm{ss}}]_{\bar{6}}]_1$, subscripts represent color dimensions.

In the flux-tube model with quadratic confinement (linear confinement is replaced by quadratic one for simplicity), the confinement potential of the tetraquark state has the following form,

$$V^{\mathrm{C}} = k\left[(\boldsymbol{r}_1-\boldsymbol{y}_1)^2+(\boldsymbol{r}_2-\boldsymbol{y}_1)^2+(\boldsymbol{r}_3-\boldsymbol{y}_2)^2+(\mathbf{r}_4-\boldsymbol{y}_2)^2+\kappa(\boldsymbol{y}_1-\boldsymbol{r}_2)^2\right], \qquad (4)$$

where k is the stiffness of 3-dimension string which is determined by meson spectrum, $k\kappa$ is the stiffness for the compound string. It can be determined by the the dimension of the color representation and it is set 1 for convenience in the present calculation.

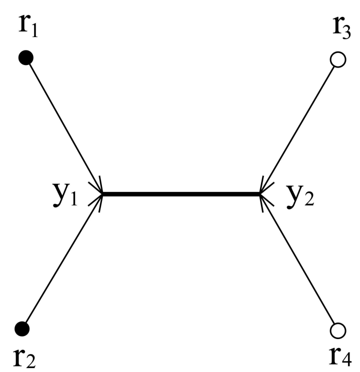

Fig. 4. Diquark-antidiquark state.

For the given quark positions $\boldsymbol{r}_i$, the coordinates of junctions $\boldsymbol{y}_i$ are fixed by minimizing the confinement potential Eq.(4) with respect to $\boldsymbol{y}_i$. Then by introducing canonical coordinates $\boldsymbol{Q}_i$,

$$\begin{aligned}
\boldsymbol{Q}_1 &= \sqrt{\frac{m}{2}}(\boldsymbol{r}_1-\boldsymbol{r}_2),\\
\boldsymbol{Q}_2 &= \sqrt{\frac{m}{2}}(\boldsymbol{r}_3-\boldsymbol{r}_4),\\
\boldsymbol{Q}_3 &= \sqrt{\frac{m}{4}}(\boldsymbol{r}_1+\boldsymbol{r}_2-\boldsymbol{r}_3-\boldsymbol{r}_4),\\
\boldsymbol{Q}_4 &= \sqrt{\frac{m}{4}}(\boldsymbol{r}_1+\boldsymbol{r}_2+\boldsymbol{r}_3+\boldsymbol{r}_4).
\end{aligned} \qquad (5)$$

the kinetic and potential energy of the system can be rewritten as

$$T = \frac{1}{2}\sum_{i=1}^{4}\dot{\boldsymbol{Q}}_i^2, \qquad (6)$$

$$V^{\mathrm{C}} = \frac{k}{m}\left(Q_1^2+Q_2^2+\frac{\kappa}{1+\kappa}\boldsymbol{Q}_3^2\right). \qquad (7)$$

Taking into account potential shift, the confinement potential V^{C} used in the calculation takes the following form

$$V^{\mathrm{C}} = k\left[\left(\frac{(\boldsymbol{r}_1-\boldsymbol{r}_2)^2}{2}-\Delta\right)+\left(\frac{(\boldsymbol{r}_3-\boldsymbol{r}_4)^2}{2}-\Delta\right)+\frac{\kappa}{1+\kappa}\left(\left(\frac{\boldsymbol{r}_1+\boldsymbol{r}_2}{2}-\frac{\boldsymbol{r}_3+\boldsymbol{r}_4}{2}\right)^2-\Delta\right)\right]. \qquad (8)$$

The other parts of the hamiltonian are the same as the chiral quark model[15] without σ-meson exchange and with color confinement replaced by the above equation.

3.2 Wavefunctions

Y(2175) is a resonance observed by Babar Collaboration in the process $\mathrm{e}^+\mathrm{e}^- \rightarrow \phi \mathrm{f}_0(980)$ via initial-state radiation[16]. The Breit-Wigner mass is $M = 2.175\pm0.010\pm0.015$ GeV, and width is narrow $\Gamma = 0.058\pm0.016\pm0.020$ GeV. It is claimed as an isospin singlet, and its spin-parity is determined to be $J^{PC} = 1^{--}$. It was also confirmed by BES collaboration in the process $\mathrm{J}/\psi \rightarrow \eta\phi \mathrm{f}_0(980)$[17]. One of the interpretation of this resonance is the tetraquark state $\mathrm{ss}\overline{\mathrm{ss}}$[18]. In the present work, the state Y(2175) is taken as a tetraquark state and as an example of applying the above flux-tube model. Because of its quantum numbers $J^{PC} = 1^{--}$, we assume that the state Y(2175) is the P-wave excitation state of a di-quark and an anti-diquark. The structure of the state is shown in Fig. 4. The Jacobi coordinates is defined as,

$$\boldsymbol{r} = \boldsymbol{r}_1-\boldsymbol{r}_2, \quad \boldsymbol{R} = \boldsymbol{r}_3-\boldsymbol{r}_4, \qquad (9)$$

$$\boldsymbol{X}=\frac{\boldsymbol{r}_1+\boldsymbol{r}_2}{2}-\frac{\boldsymbol{r}_3+\boldsymbol{r}_4}{2}. \tag{10}$$

The total wave function of the state can be written as follows,

$$\Phi_{IJ_\mathrm{T}M_\mathrm{T}}=\left[\left[\left[\phi^G_{l_1m_1}(\boldsymbol{r})\Psi_{s_1m_{s_1}}\right]_{J_1M_1}\left[\psi^G_{l_2m_2}(\boldsymbol{R})\Psi_{s_2m_{s_2}}\right]_{J_2M_2}\right]_{J_{12}M_{12}}\chi^\mathrm{G}_{LM}(\boldsymbol{X})\right]_{J_\mathrm{T}M_\mathrm{T}}\left[\Psi_{c_1}\Psi_{c_2}\right]_c\left[\Psi_{I_1}\Psi_{I_2}\right]_I, \tag{11}$$

where the bracket [ ] means $SU(2)$ or $SU(3)$ Clebsh-Gordan coefficients coupling, subscripts I, s and c represent isospin, spin and color indices respectively. The parity $P=(-1)^{l_1+l_2+L}=-1$ is obtained by setting $l_1=0$, $l_2=0$ and $L=1$. Color singlet limits color coupling to $(c_1,c_2)=(6,\bar{6})\rightarrow 1$ and $(c_1,c_2)=(\bar{3},3)\rightarrow 1$. For spin part, two combinations are used, $(s_1,s_2)=(0,0)\rightarrow 0$ and $(s_1,s_2)=(1,1)\rightarrow 0$.

The energy of Y(2175) is obtained by solving the four-body Schrödinger equation

$$(H-E)\Phi_{IJ_\mathrm{T}M_\mathrm{T}}=0 \tag{12}$$

and the equation is solved by employing Gaussian Expansion Method (GEM)[19], which has been proven to be a powerful method to solve few-body problem. In GEM, the spatial wave function is expanded by gaussians with different size,

$$\phi^G_{lm}(\boldsymbol{r})=\sum_{n=1}^{n_\mathrm{max}}c_nN_{nl}r^l\mathrm{e}^{-\nu_nr^2}Y_{lm}(\hat{\boldsymbol{r}}), \tag{13}$$

$$\psi^G_{LM}(\boldsymbol{R})=\sum_{N=1}^{N_\mathrm{max}}c_NN_{NL}R^L\mathrm{e}^{-\zeta_NR^2}Y_{LM}(\hat{\boldsymbol{R}}), \tag{14}$$

$$\chi^\mathrm{G}_{\beta\gamma}(\boldsymbol{X})=\sum_{\alpha=1}^{\alpha_\mathrm{max}}c_\alpha N_{\alpha\beta}X^\beta\mathrm{e}^{-\omega_\alpha X^2}Y_{\beta\gamma}(\hat{\boldsymbol{X}}). \tag{15}$$

The Gaussian size parameters are taken as geometric progression

$$\nu_n=\frac{1}{r_n},\ r_n=r_1a^{n-1},\ a=\left(\frac{r_{n_\mathrm{max}}}{r_1}\right)^{\frac{1}{n_\mathrm{max}-1}}. \tag{16}$$

3.3 Numerical results and discussion

The model parameters (see Table 3) are determined by fitting meson spectra. The mass of meson is also obtained by solving the Schrödinger equation with GEM. The results are shown in Table 4. Clearly a good description of meson spectra is obtained.

With fixed model parameters, the energy of tetraquark state is calculated. The calculated results are converged with n_max=7, $N_\mathrm{max}=7$ and $\alpha_\mathrm{max}=7$. Minimum and maximum ranges of the bases are, respectively, 0.1 fm and 2.0 fm for coordinates r and R, and 0.1 fm and 2.0 fm for coordinates X. Eventually, the model space is constructed by about 700 basis functions.

Table 3. Model parameters. The meson masses take the experimental values.

quark masses	$m_\mathrm{u,d}$/MeV	313
	m_s/MeV	520
Goldstone bosons	Λ_π/fm^{-1}	4.2
	$\Lambda_\mathrm{K,eta}$/fm^{-1}	5.2
	$g^2_\mathrm{ch}/(4\pi)$	0.54
	$\theta_\mathrm{P}/(^\circ)$	-15
confinement	k/(MeV·fm^{-2})	213.3
	Δ/fm^2	0.50
OGE	α_0	4.25
	$\hat{r}_0$/(MeV·fm)	30.85
	μ_0/MeV	36.98
	Λ_0/fm	0.113

Table 4. Numerical results for meson spectrum.

meson	π	K	ρ	K*	ω	ϕ
mass/MeV	139	502	761	897	735	1023
$\sqrt{\langle r^2\rangle}$/fm	0.57	0.60	1.05	0.96	1.02	0.85

The calculation shows that the channel with the lowest energy is $(c_1,c_2)(s_1,s_2)=(6,\bar{6})(1,1)$, the energy is 2188 MeV, which is very close to experimental value. The channel coupling calculation reduces the energy of the state to 2174 MeV, which is consistent with experimental value. For comparison, the chiral quark model is used and the energy is 2387 MeV, much larger than experimental value. Furthermore, by calculating the distance between any two quarks, the spatial structure of Y(2175) can be obtained. The distances between any two quarks are shown in the following,

$$\langle\boldsymbol{r}^2_{12}\rangle=\langle\boldsymbol{r}^2_{34}\rangle=1.0\ \mathrm{fm}, \tag{17}$$

$$\langle\boldsymbol{r}^2_{13}\rangle=\langle\boldsymbol{r}^2_{14}\rangle=\langle\boldsymbol{r}^2_{23}\rangle=\langle\boldsymbol{r}^2_{24}\rangle=1.6\ \mathrm{fm}. \tag{18}$$

The spatial structure of the Y(2175) is shown in the Fig. 5. The decay of the Y(2175) into color singlet hadrons requires the breakup of the nonplanar flux-tube structure into conventional color singlet hadrons, which involves flux-tube structure rearrangement which is similar to the structure transformation in isomer. This might be the reason of the narrow width of the Y(2175), quantitative calculation is needed.

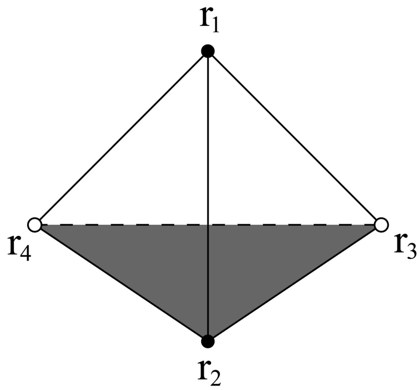

Fig. 5. Spacial structure of the Y(2175).

The above calculation is also applied to other light tetraquark system. For states $qq\bar{q}\bar{q}$ with quantum numbers $I^GJ^{PC} = 0^+0^{++}$, where q represents u or d quark, the lowest mass is 596 MeV, which can be identified as $f_0(600)$. The energy of the first radial excited state is 1036 MeV, which is close to the mass of state $f_0(980)$. For state $qs\bar{q}\bar{s}$ with quantum numbers $J^{PC} = 1^{--}$, the energy is 1715 MeV, which is consistent with the state X(1576) with experiment value $1576^{+49}_{-55}(\text{stat})^{+98}_{-91}(\text{syst})$[20], so X(1576) should be a tetraquark state in our model which is consistent with Karliner and Lipkin's work[21].

4 Summary

In the framework of the quark delocalization, color screening model, the NN scattering phase shifts up to ΔΔ or NΔ threshold are calculated by incorporating ΔΔ or NΔ states. For $IJ^P = 01^+$, 03^+ and 12^+ channels, resonance structures appeared. The predicted ΔΔ $IJ^P = 01^+$ resonance at 2390 MeV that is very close to the ABC peak seen at 2410 MeV for the reaction $pd \rightarrow {}^3He\pi^0\pi^0$[2]. In the flux-tube quark model, where the multi-body interaction is used, the tetraquark state is investigated. The calculation shows that states $f_0(600)$, $f_0(980)$, X(1576), Y(2175) could be tetraquark states and the masses can be reproduced in the flux-tube model. Generally in the conventional quark model, these states have higher masses. Although lattice QCD cannot give definite answer to the existence of multiquark state up to now, it really gives us some indications on the interaction among quarks. For multiquark system, it suggests the existence of multi-body confinement. In this sense, the multi-body interaction is indispensable for the multiquark system. How to model the multi-body confinement in the quark model is an interesting problem. Further study is needed.

References

1 Jaffe R L. Phys. Rev. Lett., 1977, **38**: 195—198
2 Bashkanov M, Bargholtz C, Berlowski M et al. Phys. Rev. Lett., 2009, **102**: 052301: 1—4
3 Aubert B et al (BABAR collaboration). Phys. Rev. Lett., 2003, **90**: 242001: 1—7; Choi S K et al.(Belle collaboration). Phys. Rev. Lett., 2003, **91**: 262001: 1—6; Evdokimov A V et al (SELEX collaboration). Phys. Rev. Lett., 2004, **93**: 242001: 1—5; Aubert B et al (BABAR collaboration). Phys. Rev. Lett., 2005, **95**: 142001: 1—5; Jaffe R L, Phys. Rep., 2005, **409**: 1—45
4 Suganuma H, Tsumura K, Ishii N, Okiharu F. Prog. Theor. Phys., 2007, **168**(Suppl): 168—172
5 Savage M J. arXiv: nucl-th/0612063v2
6 Kaiser N, Grestendorfer S, Weise W. Nucl. Phys. A, 1998, **637**: 395—420; Oset E, Toki H, Mizobe M, Takahashi T. Prog. Theor. Phys., 2000, **103**: 351—365; Kaskulov M M, Clement H, Phys. Rev. C, 2004, **70**: 014002: 1—12
7 Alexandrou C, Forcrand P, Tsapalis A. Phys. Rev. D, 2002, **65**: 054503: 1—7; Takahashi T T, Suganuma H, Nemoto Y, Matsufuru H. Phys. Rev. D, 2002, **65**: 114509: 1—19; Okiharu F, Suganuma H, Takahashi T T. Phys. Rev. Lett., 2005, **94**: 192001: 1—4; Alexandrou C, Koutsou G. Phys. Rev. D, 2005, **71**: 014504: 1—12; Okiharu F, Suganuma H, Takahashi T T. Phys. Rev. Lett., 2005, **94**: 192001: 1—4; Ishii N, Aoki S, Hatsuda T. Phys. Rev. Lett., 2007, **99**: 022001: 1—4
8 Dmitrasinovic V. Phys. Rev. D, 2003, **67**: 114007: 1—12.
9 WANG F, WU G H, TENG L J, Goldman T. Phys. Rev. Lett., 1992, **69**: 2901—2904; WU G H, PING J L, TENG L J, WANG F, Goldman T. Nucl Phys. A, 2000, **673**: 279—297; CHEN L Z, PANG H R, HUANG H X et al. Phys. Rev. C, 2007, **76**: 014001: 1—4
10 Wheeler J A. Phys. Rev., 1937, **52**: 1107—1122; TANG Y C, LeMere M, Thompsom D R. Phys. Rep., 1978, **47**: 167—223; Kamimura M. Prog. Theor. Phys. Suppl., 1977, **62**: 236—294
11 Arndt R A, Briscoe W J, Strakovsky I I, Workman R L. Phys. Rev. C, **76**, 025209: 1—10
12 PING J L, HUANG H X, PANG H R, WANG F, WONG C W. Phys. Rev. C, 2009, **79**: 024001: 1—12
13 Johnson K, Thorn C B. Phys. Rev. D, 1976, **13**: 1934—1939; Semay C. Eur. Phys. J. A, 2004, **22**: 353—354
14 Bali G S. Phys. Rev. D, 2000, **62**: 114503: 1—11
15 Entem D R, Fernández F, Valcarce A. Phys. Rev. C, 2000, **62**: 034002: 1—18
16 Aubert B et al (BABAR collaboration). Phys. Rev. D, 2006, **74**: 091103: 1—9; Phys. Rev. D, 2007, **76**: 012008: 1—33
17 Ablikim et al (BES collaboration). Phys. Rev. Lett., 2008, **100**: 102003: 1—5
18 ZHU S L. Int. J. Mod. Phys. E, 2008, **17**: 283—322
19 Hiyama E, Kino Y, Kamimura M. Prog. Part. Nucl. Phys., 2003, **51**: 223—307
20 Ablikim et al (BES collaboration). Phys. Rev. Lett., 2006, **97**: 142002: 1—5
21 Lipkin H, Karliner M. arXiv: hep-ph/0607093