Lattice results on nucleon/roper properties*

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Abstract In this proceeding, I review the attempts to calculate the Nucleon resonance (including Roper as first radially excited state of nucleon and other excited states) using lattice quantum chromodynamics (QCD). The latest preliminary results from Hadron Spectrum Collaboration (HSC) with $m_\pi \approx 380$ MeV are reported. The Sachs electric form factor of the proton and neutron and their transition with the Roper at large Q^2 are also updated in this work.

Key words nucleon excited spectroscopy, electric and magnetic form factor, transposition form factor

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1 Introduction

Quantum chromodynamics (QCD) has been successful in describing many properties of the strong interaction. In the weak-coupling regime, we can rely on perturbation theory to work out the path integral which describes physical observables of interest. However, for long distances perturbative QCD no longer converges. Instead, we use a discretization of space and time in a finite volume to calculate these quantities from first principles numerically; such research forms the regime of lattice QCD.

To keep the systematic error due to discretization under control, one follows Symanzik improvement order by order in terms of the ultraviolet cutoff (a) for both the action and operators. However, the breaking of continuous (Euclidean) $SO(4)$ symmetry allows many new degrees of freedom, leading to various lattice actions that return to the same continuum action once the symmetry is restored. Thus, there exist many gauge and fermion actions for us to choose from. Today, most gauge actions used are $O(a^2)$-improved and leave small discretization effects ($O(a^3\Lambda^3_{\rm QCD})$) due to gauge choices. On the other hand, most fermion actions are only $O(a)$-improved and have systematic errors of $O(a^2\Lambda^2_{\rm QCD})$ that become dominant. For this reason, lattice calculations are generally distinguished according to the fermion action used. Differences among the actions are benign once all systematics are included, and the choice of fermion action is constrained by limits of computational and human power and by the main physics focus. The commonly used actions are: domain-wall fermions (DWF), overlap fermions, Wilson/clover fermions, twisted-Wilson fermions and staggered fermions.

Since the real world is effectively continuous and infinitely large, we will have to take limits of $a\rightarrow 0$ and $V\rightarrow\infty$ to eliminate the artifacts introduced in a discretized finite box. With the most state-of-the-art supercomputer, we are close but yet to simulate at the physical pion mass. Using calculations at multiple heavier pion masses, which are affordable for available computational resources, we can apply chiral perturbation theory to extrapolate quantities of interest to the physical limit. A recent work by the BMW collaboration[1] calculating multiple lattice spacings, volumes and pion masses as light as 180 MeV provided an excellent demonstration of how ground-state hadron masses with fully understood and controlled systematics are consistent with experiment. Such calculations with multiple pion masses also help to determine the low-energy constants of chiral effective theory.

A typical nucleon interpolating field used in lattice calculations is $\chi_{\rm N}=\sum_{\vec{x},a,b,c}{\rm e}^{{\rm i}\vec{p}\cdot\vec{x}}\epsilon^{abc}\left[u_a^T C\gamma_5 d_b\right]u_c$, and the nucleon two- and three-point Green functions are obtained from

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$$\Gamma^{(2)}(t_{\rm src},t)=\langle\chi_{\rm N}(t)\chi^{\dagger}_{\rm N}(t_{\rm src})\rangle$$
$$\Gamma^{(3)}(t_{\rm src},t,t_{\rm snk})=\langle\chi_{\rm N}(t_{\rm snk},\vec{p}_{\rm snk})\,\mathcal{O}(t,\vec{q})\,\chi^{\dagger}_{\rm N}(t_{\rm src},\vec{p}_{\rm src})\rangle,$$

where $\mathcal{O}$ is the operator of interest. For the vector (axial) current, the operator is $\mathcal{O}=\overline{\psi}\gamma_{\mu}(\gamma_5)\psi$. We calculate only the "connected" diagrams, which means the inserted quark current is contracted with the valence quarks in the baryon interpolating fields, as in the majority of lattice three-point calculations. For electromagnetic form factors, recent lattice studies[2] have found the disconnected diagrams contributes only a small amount, consistent with zero within the statistical error; therefore, we will defer the study of such diagrams to the future. For more details on lattice nucleon form factor calculations, please refer to a selection recent review articles[3—5] and references within.

2 Lattice roper-resonance calculation

Both in meson and baryon spectroscopy there are many experimentally observed excited states whose physical properties are poorly understood and could use theoretical input from LQCD to solidify their identification. Aside from masses, other excited-state quantities that could be computed on the lattice, such as form factors and coupling constants, would be useful to groups such as the Excited Baryon Analysis Center (EBAC) at Jefferson Lab, where dynamical reaction models have been developed to interpret experimentally observed properties of excited nucleons in terms of QCD[6, 7]. In certain cases, input from the lattice may be helpful in determining the composition of controversial states, which may be interpreted as ordinary hadrons, tetra- or pentaquarks, hadronic molecules or unbound resonances.

Among the excited nucleon states, the nature of the Roper resonance, N(1440) P_{11}, has been the subject of interest since its discovery in the 1960s. It is quite surprising that the rest energy of the first excited state of the nucleon is less than the ground-state energy of nucleon's negative-parity partner, the N(1535) S_{11}[8], a phenomenon never observed in meson systems. There are several interpretations of the Roper state, for example, as the hybrid state that couples predominantly to QCD currents with some gluonic contribution[9] or as a five-quark (meson-baryon) state[10].

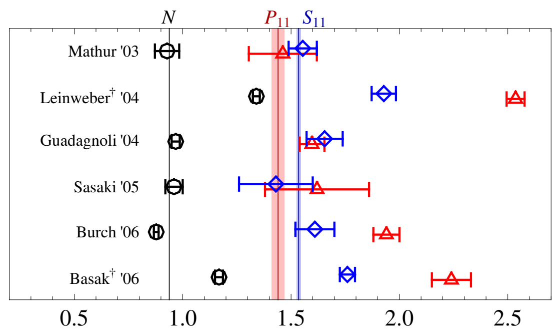

Fig. 1. Summary of previous lattice calculations with extrapolation to the physical pion mass point or the lowest simulated pion point (labeled as "†").

Table 1. Summary of existing published S_{11} and P_{11} calculations. Due to space limitations, we adopt these abbreviations for fermion actions: Domain-Wall Fermions[11—14] (DWF), Chirally Improved Dirac Operator[15, 16] (CIDO), Fat-Link Irrelevant Clover[17] (FLIC); and for the analysis methods: Variational Method[18, 19] (VM), Constrained Curve Fitting[20] (CCF), Maximum Entropy Method[21, 22] (MEM), Black Box Method[23—25] (BBM). For those works which do not perform extrapolation, we use the lightest pion mass to represent their results.

group	$N_{\rm f}$	$S_{\rm f}$	a_t^{-1}/GeV	M_π/GeV	L/fm	method	extrapolation
Basak et al.[26]	0	Wilson	6.05	0.49	2.35	VM	N/A
Burch et al.[27]	0	CIDO	1.68,1.35	0.35–1.1	2.4	VM	$a+bm_\pi^2$
Sasaki et al.[28]	0	Wilson	2.1	0.61–1.22	1.5,3.0	MEM	$\sqrt{a+bm_\pi^2}$
Guadagnoli et al.[29]	0	Clover[30]	2.55	0.51–1.08	1.85	BBM	$a+bm_\pi^2+cm_\pi^4$
Leinweber et al.[31]	0	FLIC	1.6	0.50–0.91	2.0	VM	N/A
Mathur et al.[32]	0	Overlap[33]	1.0	0.18–0.87	2.4,3.2	CCF	$a+bm_\pi+cm_\pi^2$
Sasaki et al.[34]	0	DWF	2.1	0.56–1.43	1.5	VM	$a+bm_\pi^2$

Early LQCD calculations using the quenched approximation[27—29, 31, 32, 34, 35], found the computed spectrum inverted relative to experiment, with P_{11} heavier than the S_{11}. Fig. 1 and Table 1 show a summary of parameters used in these works and the extrapolated masses at the physical pion mass. Only Ref. [32], using the lightest pion mass, seems to observe a potential mass reversal in their central values. There is agreement with experimental values within errors, since the masses of the P_{11} and S_{11}

are overlapping within their statistical errorbar. Furthermore, Ref. [36] summarizes most of the published even-parity LQCD results ($N_{\rm f}=0$) as a function of pion mass and found big discrepancies in the calculated nucleon first-excited mass, which created an even more chaotic atmosphere. We re-address the same issue on left-hand-side of Fig. 2 but sort the results by the lattice size; we find that the Roper masses are roughly inversely proportional to lattice size. If finite-volume effects dominate in these calculations, we could modified the axes in terms of the dimensionless quantity ML, as shown on the right-hand side of Fig. 2. Now we see a better agreement (or universality) among the LQCD Roper-mass calculations; the Roper masses agrees within 2 standard deviations of the numbers in Ref. [32]. This suggests finite-volume effects can be more severe for excited states than the ground states and that careful examination of such systematic errors is crucial. The examples given here are calculated in a vacuum with gluonic degrees of freedom only; that is, no sea fermion loops contribute to the ensembles. Future calculations should remove this approximation, and more results with $N_{\rm f}=2+1$ (degenerate up and down plus strange in the sea) should be available within the next few years.

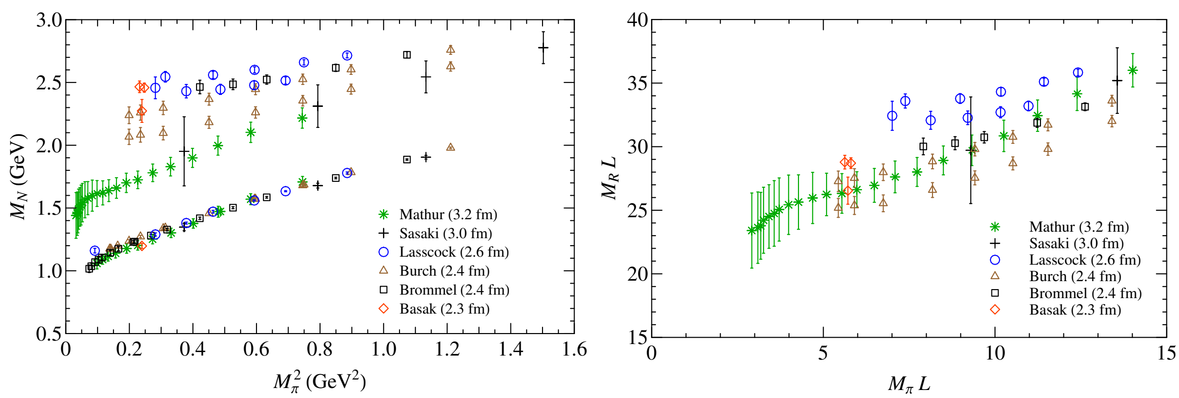

Fig. 2. Summary of published $N_{\rm f}=0$ LQCD calculations of the nucleon and Roper masses in GeV (left) and in terms of the dimensionless product of the Roper mass and lattice size L (right).

3 Excited nucleon states

In the previous section, we found how difficult it is to extract the Roper resonance. What if we want to get to even higher excited states with LQCD? Unfortunately, in Euclidean space, excited-state contributions to correlation functions decay faster than the ground state. Therefore at large times, the signals for excited states are swamped by the signals for lower-energy states. One way resolve this issue is to improve resolution in the temporal direction. An anisotropic lattice where the temporal lattice spacing is finer than spatial spacings can provide better resolution while avoiding the increase in computational cost associated with a similar reduction of all spacings.

Another method is to use the variational method[18, 19] on a matrix of source and sink operators to project more exactly onto the eigenstates of the Hamiltonian. To do this, we need a large number of operators that overlap well with excited states with desired quantum numbers.

Hadron Spectrum Collaboration (HSC) has been investigating interpolating operators projected into irreducible representations (irreps) of the cubic group[37, 38] in order to better calculate two-point correlators for nucleon spectroscopy. In the cubic group, for baryons, there are four two-dimensional irreps $G_{\rm 1g}, G_{\rm 1u}, G_{\rm 2g}$, $G_{\rm 2u}$ and two four-dimensional representations $H_{\rm g}$ and $H_{\rm u}$. (The subscripts "g" and "u" indicate positive and negative parity, respectively.) Each lattice irrep contains parts of many continuum states. The G_1 irrep contains $J=\frac{1}{2},\frac{7}{2},\frac{9}{2},\frac{11}{2},\cdots$ states, the H irrep contains the $J=\frac{3}{2},\frac{5}{2},\frac{7}{2},\frac{9}{2},\cdots$ states, and the G_2 irrep contains the $J=\frac{5}{2},\frac{7}{2},\frac{11}{2},\cdots$ states. The continuum-limit spins J of these states must be deduced by examining degeneracy patterns among the different O_h irreps.

Using these operators, we construct an $r\times r$ correlator matrix, $C_{ij}(t)$, where each element of the matrix is a two-point correlator composed from different operators O_i and O_j. Then we consider the generalized eigenvalue problem

$$C(t)\psi=\lambda(t,t_0)C(t_0)\psi, \tag{1}$$

where the selection of t_0 depends on the range of validity of our approximation of the correlators by the lowest r eigenstates. If t_0 is too large, the highest-lying states will have exponentially decreased too far to have good signal-to-noise ratio; if t_0 is too small, many states above the r we can determine will contaminate our extraction. Over some intermediate range in t_0, we should find consistent results.

If the eigenvector for this system is $|\alpha\rangle$, and α goes from 1 to r. Thus the correlation matrix can be approximated as

$$C_{ij}=\sum_{n=1}^{r}v_i^{k*}v_j^n \mathrm{e}^{-tE_n}, \tag{2}$$

with eigenvalues

$$\lambda_n(t,t_0)=\mathrm{e}^{-(t-t_0)E_n}, \tag{3}$$

by solving

$$C(t_0)^{-1/2}C(t)C(t_0)^{-1/2}\psi=\lambda(t,t_0)\psi\ . \tag{4}$$

The resulting eigenvalues $\lambda_n(t,t_0)$, called the principal correlators, are then further analyzed to extract the energy levels, E_n. Since they have been projected onto pure eigenstates of the Hamiltonian, each principal correlator should be fit well by a single exponential. The leading contamination due to higher-lying states is another exponential having higher energy; a two-state fit may help to remove this contamination.

Demonstrations of how these operators work using purely gluonic vacuum with light valence quarks for nucleon and delta spectroscopy are reported in Refs. [26, 39, 40]. Further calculations of isospin-$\frac{1}{2}$ excited nucleons in two-flavor QCD, using u and d quarks that have the same mass, are reported in Ref. [41] with 2 pion masses: 416(36) and 578(29) MeV. In this proceeding, we report a further step toward the goal of determining the spectrum of nucleon excited states on $2+1$-flavor lattices[42, 43]. Fig. 3 shows a preliminary results on nucleon spectroscopy with pion mass around 380 MeV. The calculation here is done with a new technique, called "distillation"[44], which has better signal-to-noise ratios than the conventional approach. We observe a similar distribution of states as the previous study. Further study on the larger volume and investigation of decay thresholds and potential two-particle states are underway.

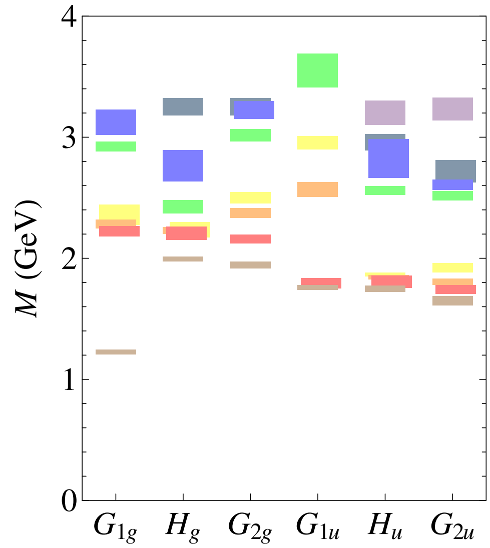

Fig. 3. Nucleon excited spectrum according to cubic-group irrep.

4 Form Factors

Studying the momentum-transfer (Q^2) dependence of the elastic electromagnetic form factors is important in understanding the structure of hadrons at different scales. There have been many experimental studies of these form factors on the nucleon. A recent such experiment, the Jefferson Lab double-polarization experiment (with both a polarized target and longitudinally polarized beam) revealed a nontrivial momentum dependence for the ratio G_E^p/G_M^p. This contradicts results from the Rosenbluth separation method, which suggested $\mu_p G_E^p/G_M^p\approx 1$. The contradiction has been attributed to systematic errors due to two-photon exchange that contaminate the Rosenbluth separation method more than the double-polarization. (For details and further references, see the recent review articles: Refs. [45—47]).

Lattice calculations can make valuable contributions to the study of nucleon form factors, since they allow access to both the pion-mass and momentum dependence of such form factors. Recently, the limitations of the largest-available Q^2 (in terms of the quality of the signal-to-noise ratios) has been overcome[48, 49]. An exploratory study using clover fermions extends the range of momentum transfer to 6 GeV^2, as shown in Fig. 4. The range of Q^2 lattice data available for the neutron has now exceeded that of experiment. Such calculations will provide interesting comparisons for data collected after the future 12 GeV upgrade at Jefferson Lab.

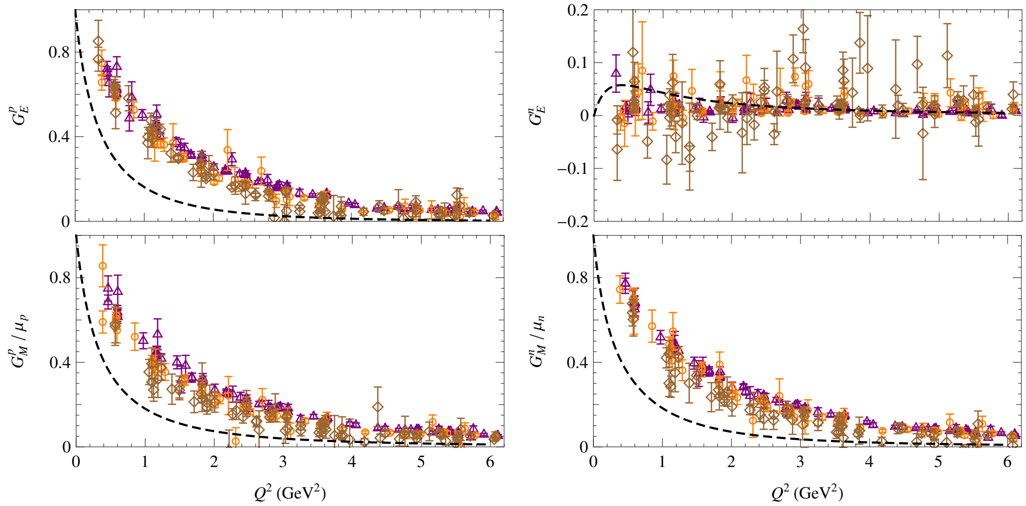

Fig. 4. Nucleon form factors with pion masses of 480, 720 and 1080 MeV. The dashed lines are plotted using experimental form-factor fit parameters[45, 50].

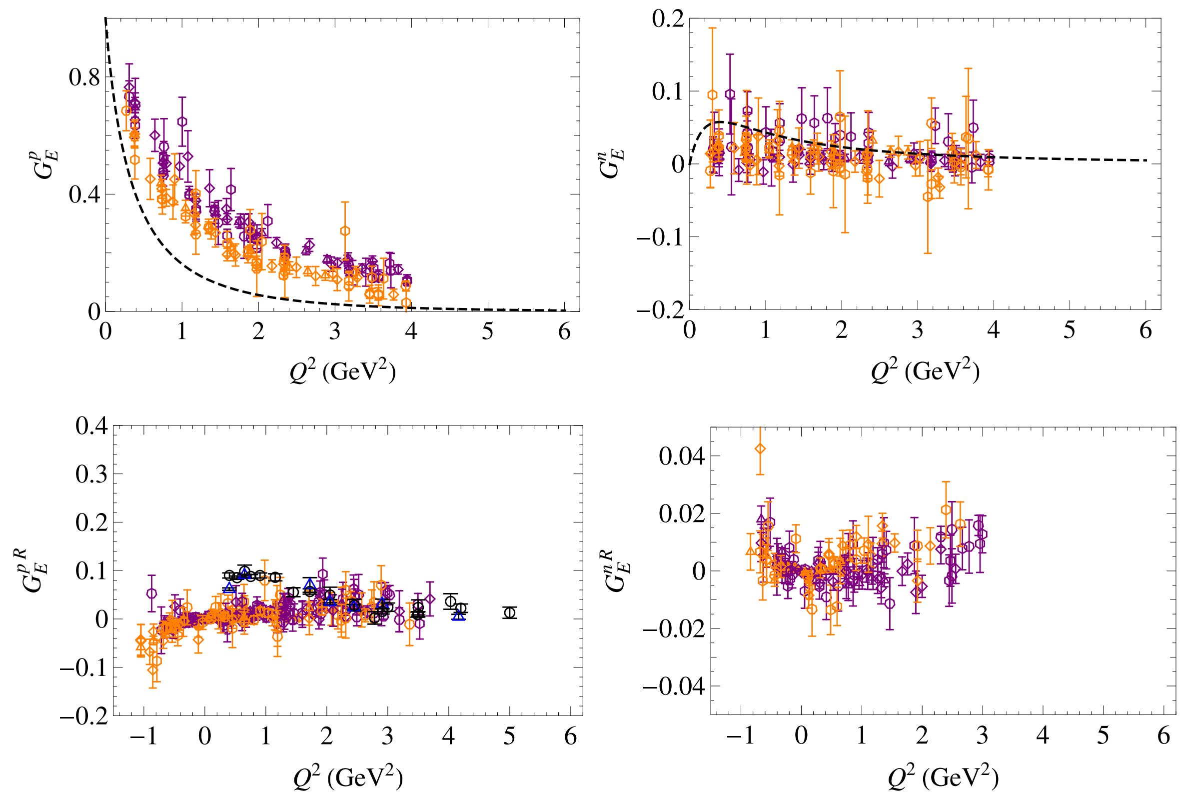

Fig. 5. Preliminary results on nucleon (top panels) and Roper-nucleon transition (bottom panels) G_{E} form factors with pion masses of 580 and 780 MeV on dynamical lattices. The dashed lines for the nucleon form factors are the same as Fig. 4, and the Roper-nucleon black circles and blue triangles are experimental form factors from CLAS and MAID.

We are also exploring the nucleon and Roper-nucleon form factors on $2+1$-flavor anisotropic lattices (Fig. 5)[42, 43]. The methodology is similar to what is described in Refs. [48, 49]. In this study, the range of Q^2 becomes smaller (as compared with the cases of Fig. 4) due to the change of the lattice spacing. The proton G_{E} moves closer to the experimental parametrization lines as the pion mass decreases. The Roper-nucleon transitions are too noisy to distinguished according to the pion-mass contribution.

Overall, we see a urgent need to improve signal-to-noise ratios to allow comparison with experimental data. The new technique of distillation[44] has shown great potential for getting better signal-to-noise ratios than the conventional approach. We are in the precess of extending the method to work on form-factor calculations. We hope soon to report better lattice calculations and extended our transition form factors to other excited nucleon states.

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