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Isospin effect on spontaneous fission half-lives of even-even nuclei *

CHEN Yu-Qing(陈玉庆)[1;1)] REN Zhong-Zhou(任中洲)[1,2]
XU Chang(许昌)[1] GUO Yan-Qing(郭艳青)[1]

1 (Department of Physics, Nanjing University, Nanjing 210008, China)
2 (Center of Theoretical Nuclear Physics National Laboratory of Heavy-Ion Accelerator of Lanzhou, Lanzhou 730000, China)

Abstract By using a new five-parameter formula derived from the WKB approximation, we systematically calculate the spontaneous fission half-lives of even-even nuclei with Z=90—108. The isospin effect is taken into account in the new formula. The calculated half-lives agree well with the experimental data. In addition, we predict the spontaneous fission half-lives of superheavy nuclei with Z=108—114. Our predictions may provide references for future experiments.

Key words isospin effect, WKB approximation, spontaneous fission half-lives

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1 Introduction

The spontaneous fission is an important decay mode for elements heavier than thorium[1]. There are various theoretical approaches for calculating the half-lives of spontaneous fission[2—5]. Due to the complexity of the fission process and the uncertainties of the height and the shape of the fission barrier, it is difficult to describe the spontaneous fission half-lives in the microscopical model[5]. In 1955, Swiatecki and his coworkers proposed a semi-empirical formula for spontaneous fission half-lives[3]. Recently, Xu et al proposed a formula of spontaneous fission half-lives based on the Viola-Seaborg formula [6—8]. In both of these formulae, the isospin effect was not included. However, many recent studies show that there are strong isospin effects in the nuclear fission [9—13]. So it is interesting to investigate the isospin effect on the spontaneous fission half-lives.

In this paper, we investigate the isospin effect on the fission barriers. In the framework of WKB approximation, a new formula for spontaneous fission half-lives of even-even nuclei is proposed. An isospin-dependent term is included in this new formula. It is valuable to see whether the isospin effect is important for nuclei far from the long-lived line $N=Z+52$.

2 Theoretical framework

It is well known that the spontaneous fission is a problem of multi-dimensional barrier penetration. It is difficult to solve this complex problem microscopically. Within the framework of Wentzel-Kramers-Brillouin (WKB) approximation, the penetration coefficient P can be written as[2,14—16]

$$P \approx \exp\left(-\frac{2}{\hbar}\int_{r_{\rm b}}^{r_{\rm a}}\sqrt{2\mu(V-E)}{\rm d}r\right) \simeq \exp(-2\kappa\sqrt{2\mu E_{\rm f}}/\hbar), \qquad (1)$$

where $\mu \approx M/2$ is the reduced mass, E denotes the decay energy; V denotes the interaction potential; $E_{\rm f}$ is the height of the fission barrier; $r_{\rm a}$ and $r_{\rm b}$ are two

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1) E-mail: chenyq127@yahoo.com.cn

classical turning points. κ denotes the fission width. E_{f} can be expressed as $E_{\mathrm{f}} = F(\chi)S_{\mathrm{sur}}$ in the liquid-drop model. $F(\chi)$ is a function of the fission parameter $\chi.S_{\mathrm{sur}}$ represents the surface potential. It has been proved experimentally that the height of the fission barrier doesn't vary with the fission parameter χ in the actinide elements. So the function $F(\chi)$ can be considered as a constant for simplicity. In order to investigate the isospin effect on the height of the fission barrier, we adopt the following form for the surface energy[17],

$$S_{\mathrm{sur}} = a_{\mathrm{s}}(1 - k_{\mathrm{s}}(\frac{N-Z}{A})^2)A^{2/3}. \quad (2)$$

Z, N and A are the neutron, charge and mass numbers of the parent nuclei, respectively. a_{s} and k_{s} are constant parameters. Substituting Eq. (1) and Eq. (2) into the following formula

$$T_{sf} = \frac{ln2}{nP}. \quad (3)$$

The formula for the spontaneous fission half-lives in the first order of approximation can be expressed as

$$\log_{10}(T_{\mathrm{sf}}) \simeq c_0 + c_1\frac{(N-Z)}{A}. \quad (4)$$

c_0 and c_1 are constants. In order to obtain more accurate results, the term $(Z-98)(Z-104)(A-232)^2$ is included in Eq. (4) for the following reasons: (1) there are two or more fission modes for nuclei with $Z > 98$; (2) the height of the fission barrier decreases to less than 1 MeV for $Z > 104$[17]; (3) considering the dependence of spontaneous fission half-lives on mass, the correction $(A-232)^2$ is introduced where 232 is the mass of the longest fission half-life nucleus ^{232}Th. The terms $(N-Z)^2/A$ and $(N-Z)^3/A$ as the higher order corrections are also introduced. So the new formula can be written as

$$\log_{10}(T_{\mathrm{sf}}) = c_0 + c_1\frac{(N-Z)}{A} + c_2\frac{(N-Z)^2}{A} + c_3\frac{(N-Z)^3}{A} + c_4\frac{(Z-98)(Z-104)(A-232)^2}{A}. \quad (5)$$

By fitting the experimental data of even-even nuclei listed in Table 1[18—21], the optimal values of the parameters are $c_0 = -230.21$, $c_1 = 1116.10$, $c_2 = 17.19$, $c_3 = -0.33$ and $c_4 = 0.07$.

Table 1. Logarithm of spontaneous fission half-lives (in years) of even-even nuclei.

nucleus	$\log_{10}T_{\mathrm{exp}}$	$\log_{10}T_{\mathrm{Swia}}$	$\log_{10}T_{\mathrm{Xu}}$	$\log_{10}T_{\mathrm{Eq(5)}}$	nucleus	$\log_{10}T_{\mathrm{exp}}$	$\log_{10}T_{\mathrm{Swia}}$	$\log_{10}T_{\mathrm{Xu}}$	$\log_{10}T_{\mathrm{Eq(5)}}$
^{232}Th	21.08	21.08	20.41	20.30	^{246}Fm	−6.60	−6.56	−6.39	−4.67
^{232}U	15.41	15.69	14.45	14.11	^{248}Fm	−2.94	−2.88	−2.65	−2.23
^{234}U	16.18	16.18	15.79	15.73	^{250}Fm	−0.10	−0.63	−0.55	−0.82
^{236}U	16.40	16.36	16.30	16.40	^{252}Fm	2.10	0.22	0.08	−0.47
^{238}U	15.91	16.26	16.11	16.07	^{254}Fm	−0.20	−0.29	−0.59	−1.23
^{236}Pu	9.18	10.46	9.94	10.16	^{256}Fm	−3.48	−2.13	−2.43	−3.13
^{238}Pu	10.68	11.41	11.47	11.94	^{258}Fm	−9.93	−5.27	−5.27	−6.20
^{240}Pu	11.06	11.78	12.01	12.70	^{260}Fm	−8.90	−9.68	−9.00	−10.48
^{242}Pu	10.83	11.57	11.71	12.40	^{250}Fm	−10.10	−10.45	−10.05	−8.56
^{244}Pu	10.82	10.81	10.71	11.02	^{252}No	−6.54	−6.12	−5.93	−6.17
^{240}Cm	6.28	5.53	5.47	6.27	^{254}No	−3.04	−3.48	−3.63	−4.78
^{242}Cm	6.85	6.94	7.18	7.98	^{256}No	−4.77	−2.48	−2.96	−4.45
^{244}Cm	7.12	7.47	7.76	8.64	^{258}No	−9.42	−3.09	−3.76	−5.20
^{246}Cm	7.26	7.17	7.34	8.21	^{260}No	−7.47	−5.28	−5.89	−7.07
^{248}Cm	6.62	6.03	6.06	6.66	^{262}No	−8.80	−9.00	−9.18	−10.10
^{250}Cm	4.05	4.10	4.07	3.94	^{254}Rf	−11.14	−13.51	−12.98	−11.34
^{238}Cf	−8.18	−11.55	−13.99	−8.57	^{256}Rf	−9.71	−8.56	−8.47	−8.79
^{240}Cf	−4.00	−6.14	−7.17	−4.05	^{258}Rf	−8.35	−5.54	−5.94	−7.23
^{242}Cf	−1.33	−1.95	−2.16	−0.48	^{260}Rf	−8.20	−4.41	−5.23	−6.68
^{246}Cf	3.26	2.88	3.11	3.65	^{262}Rf	−7.18	−5.13	−6.16	−7.19
^{248}Cf	4.51	3.58	3.72	4.14	^{258}Sg	−9.04	−15.65	−15.01	−11.80
^{250}Cf	4.23	3.17	3.17	3.52	^{260}Sg	−9.65	−10.10	−10.08	−8.82
^{252}Cf	1.93	1.68	1.62	1.76	^{262}Sg	−9.32	−6.72	−7.32	−6.78
^{254}Cf	−0.78	−0.86	−0.79	−1.18	^{264}Sg	−8.93	−5.47	−6.55	−5.72
^{256}Cf	−4.64	−4.44	−3.93	−5.33	^{266}Sg	−6.00	−6.30	−7.59	−5.67
^{242}Fm	−9.60	−18.34	−19.48	−12.46	^{264}Hs	−9.20	−10.64	−10.59	−4.64
^{244}Fm	−8.98	−11.70	−11.93	−8.09					

3 Numerical results and discussions

By using the new formula expressed as Eq. (5), we systematically calculate the spontaneous fission half-lives of even-even nuclei with Z=90—108 listed in Table 1. $T_{\rm exp}$ denotes the experimental spontaneous fission half-life. The numerical results $T_{\rm Swia}$ are calculated from the formula proposed by Swiatecki et al. $T_{\rm Xu}$ represents the numerical result from the formula proposed by Xu et al. $T_{\rm Eq(5)}$ denotes the numerical result obtained from the new formula of Eq. (5).

On the whole, our results listed in Table 1 agree with the experimental data. In the following, we pay our attention to the deviations between theoretical results and the experimental data. The average deviations are denoted by the values $S = \sum_n |\log_{10}(T_{\rm exp}) - \log_{10}T_{\rm the}|/n$. Here, $\log_{10}T_{\rm the}$ denote $\log_{10}T_{\rm Swia}, \log_{10}T_{\rm Xu}$ and $\log_{10}T_{\rm Eq(5)}$; n is the number of the nuclei. Because the fission mechanism alters around $Z = 100$ where the shell effect manifests itself. It is valuable to investigate the systematic behavior of the deviations around $Z = 100$. So we choose the nuclei with $Z = 96 - 102$ as our research objects. For Cm isotopes, the maximum deviation is 1.5 and the minimum deviation is 0.01 and the average deviation is only 0.63. For Cf isotopes, the maximum, the minimum, the average deviations are 0.85, 0.06 and 0.45, respectively. For Fm isotopes, the maximum deviation is 3.73 for ^{258}Fm and the minimum deviation is 0.35 for ^{256}Fm. As to No isotopes, the largest deviation is 4.22 for ^{258}No and the smallest deviation is 0.32 for^{256}No. It is obvious that the deviations become large for Fm and No isotopes. The main reason is that the dominant decay mode is no longer asymmetric but symmetric in the fission process for Fm and No isotopes[22]. For Cm, Cf, Fm and No, the average deviation is 0.75. This means that the calculations by Eq. (5) agree with the experimental data within a factor of $10^{0.75}$.

In order to see the deviations more clearly, we plot the variation of the deviations as a function of the neutron number N for nuclei with $Z = 100 - 106$ in Fig. 1. It is shown that deviations are close to zero between the experimental values and the calculated ones from Eq. (5). A larger deviation emerges for ^{252}Fm due to the effect of the sub-shell at $N = 152$. The largest deviation is 4.22 for ^{258}No. The reason may be that the main fission mode altered from asymmetry to symmetry. Due to many uncertainties in the fission process, Möller et al[5] consider that such a large deviation is acceptable. The deviations vary unsmoothly with the number of the neutrons. However, the systematic behavior of the deviations may be helpful for obtaining more reliable prediction for nuclei far from long-lived line $N = Z + 52$. So we try to investigate the systematic behavior that may be found in the further research.

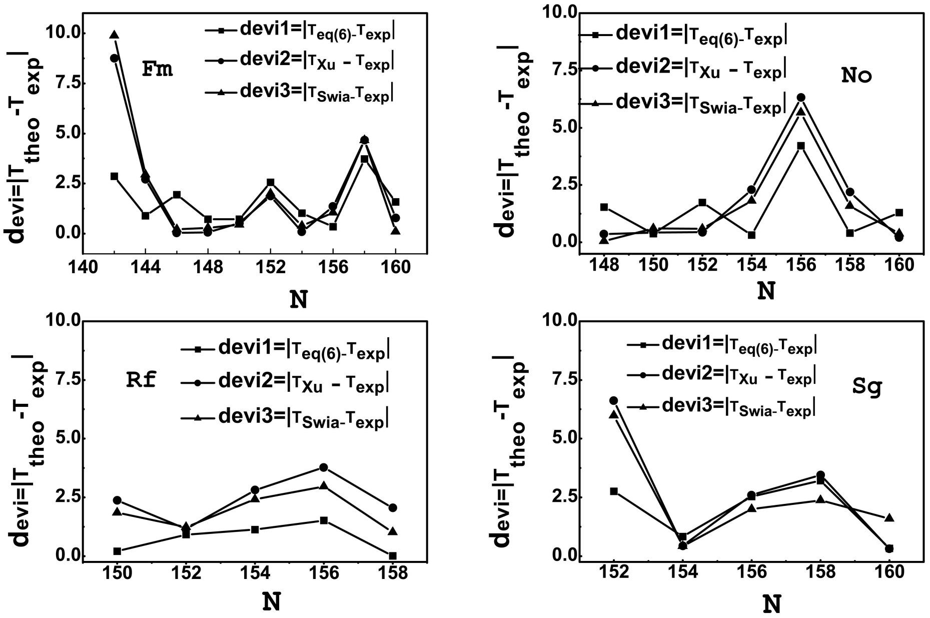

Fig. 1. The variations of the deviations between experimental values and calculated ones with neutron N for Fm, No, Rf and Sg isotopes.

Table 2. Logarithm of spontaneous fission half-lives (in years) of the superheavy nuclei.

nucleus	$\log_{10}T_{\rm Swia}$	$\log_{10}T_{\rm Xu}$	$\log_{10}T_{\rm Eq(5)}$	nucleus	$\log_{10}T_{\rm Swia}$	$\log_{10}T_{\rm Xu}$	$\log_{10}T_{\rm Eq(5)}$
$^{254}108$	−66.51	−65.20	−31.46	$^{264}110$	−26.13	−24.30	−5.06
$^{256}108$	−50.17	−48.30	−24.58	$^{258}112$	−122.41	−122.25	−35.84
$^{258}108$	−36.47	−34.61	−18.41	$^{260}112$	−96.67	−94.65	−25.56
$^{260}108$	−25.35	−23.91	−13.01	$^{262}112$	−74.19	−71.22	−15.77
$^{262}108$	−16.76	−15.97	−8.40	$^{264}112$	−54.92	−51.69	−6.51
$^{258}110$	−71.04	−68.83	−26.59	$^{258}114$	−193.62	−199.94	−45.65
$^{260}110$	−53.20	−50.64	−18.75	$^{260}114$	−158.76	−160.83	−32.96
$^{262}110$	−38.25	−35.87	−11.56	$^{262}114$	−127.58	−126.71	−20.56

In addition, we present the predictions of the spontaneous fission half-lives for some superheavy nuclei which are unavailable experimentally at present. The theoretical half-lives of spontaneous fission for nuclei with Z=108—114 are listed in Table 2. It is obvious that our results $\log_{T_{\rm Eq(5)}}$ are smaller than $\log_{10}T_{\rm Swia}$ and $\log_{10}T_{\rm Xu}$. However, our results are in agreement with the values calculated from the dynamical model[23, 24]. Further experiments are needed for testing these theoretical calculations.

4 Summary

In the framework of WKB approximation, a new formula with the isospin effect included is proposed. By using this formula, we systematically investigate the spontaneous fission half-lives of heavy and superheavy nuclei for Z=90—114. Available experimental spontaneous fission half-lives are well reproduced by the new formula. Compared with the theoretical results derived from the formulae proposed by Swiatecki et al and by Xu et al, our results are closer to the experimental data for the nuclei far from the long half-lives line $N=Z+52$. It implies that the isospin effect is important for nuclei far from the long-lived line. We also predict the spontaneous fission half-lives of even-even superheavy nuclei for Z=108—114. These predictions may provide references for future experiments.

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