Collectivity under random two-body interactions*

ZHAO Yu-Min(赵玉民)[1,2,3;1)]

1 (Department of Physics, Shanghai Jiaotong University, Shanghai 200240, China)
2 (Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000, China)
3 (Nishina Center, RIKEN, Hirosawa 2-1, Wako-shi, Saitama 351-0198, Japan)

Abstract In this paper we study collective motion under random two-body interactions in the fermion dynamical symmetry Model (FDSM). It is found that a Hamiltonian with the $SO(8)$ symmetry of the FDSM does not give generic vibration and rotation under random interactions while that with the $SP(6)$ symmetry does.

Key words random interactions, regular structure, collective motion, symmetry

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Many features of low-lying states of atomic nuclei are frequently observed in other systems such as metallic clusters. It is interesting to investigate whether or not these features are robust when interactions become more and more arbitrary.

In 1998 the spin zero ground state dominance under random interactions was pointed out by Johnson, Bertsch and Dean in Ref. [1]. This discovery has attracted much attention among nuclear physicists since then. It also stimulated many studies of other regular patterns such as odd-even staggering of binding energies, energy centroids with various quantum numbers, collectivity, etc., in the presence of random interactions. One refers Refs. [2—4] for recent reviews.

Collective behavior of low-lying excited states is one of attractive topics among many works of regularities under the TBRE. For details, we refer to Refs. [1, 5—9]. In Ref. [6], it was shown that collective motions including vibration and rotation arise dominantly for sd boson systems in the presence of a TBRE Hamiltonian. On the other hand, as noticed in many papers, neither vibrational motion nor rotational motion arises dominantly in fermion systems under the TBRE. In particular, rotational spectra has very few cases among 1000 runs of the TBRE Hamiltonian. It was then conjectured that certain constraints on model space and/or Hamiltonian might be necessary to obtain a generic rotational behavior in fermion systems[7]. In Ref. [10], it was shown that rotational motion does not arise from the SD-pair truncated shell model space with the TBRE but from a hamiltonian with quadrupole correlation properly enhanced. Velázquez and Zuker suggested in Ref. [11] displaced TBRE or randomizing the single-particle energies with the two-body matrix elements fixed, to obtain rotational structure for fermions.

The purpose of this talk is to report our recent work[12] of collective behavior of fermion systems in the presence of random interactions. Because both vibrational motion and rotational motion are generic in sd-boson systems, do these motions appear dominantly also within the framework of the Fermion dynamical symmetry model (FDSM)[13, 14], where one has a similar Hamiltonian and space as the IBM?

In this paper we concentrate on rotational spec-

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1) E-mail: ymzhao@sjtu.edu.cn

tra in FDSM systems under random interactions. Because there are two types of symmetries in the FDSM, the $SP(6)$ and the $SO(8)$, which contains different subgroups—the $SP(6)$ has the $SU(3)$ subgroup but the $SO(8)$ has not, it is interesting to investigate whether or not there exist any essential differences in obtaining collective motion between these two symmetric cases.

The FDSM originated from a toy model suggested by Ginocchio[13] and further developed by Wu and collaborators[14]. The nucleon SD pairs follow either the $SP(6)$ or the $SO(8)$ symmetry (depending on the shell), with either k active or i active in the so-called k-i basis. The FDSM Hamiltonian is as follows (See Eqs. (3.27) and (4.1c) of Ref. [14]).

$$H = G_0 S^\dagger S + G_2 D^\dagger \cdot D + \sum_r B_r P^r \cdot P^r \ , \qquad (1)$$

$$r = \begin{cases} 0,1,2 & \text{for } k \text{ active,} \\ 0,1,2,3 & \text{for } i \text{ active .} \end{cases}$$

Details of the FDSM can be found in Refs. [13, 14].

We calculate low-lying states for systems which respect the FDSM symmetries by using random interactions. We use Gaussian type random values for two-body interaction parameters G_0, G_2 and B_r. Widths for particle-particle interaction strengths G_0 and G_2, and those for particle-hole type interaction strengths B_r are taken to be equal in this paper. Similar to Ref. [6], we do not make difference between neutrons and protons. All results are based on 1000 runs of the TBRE. We investigate the distribution of R, defined by $E_{4_1^+}/E_{2_1^+}$, an indicator of collectivity for systems–R is 2 for a vibrator and 10/3 for a rotor. We concentrate on only cases with spin zero ground states which are obtained by using random interactions when we calculate R values.

In Fig. 1, we present our calculated results of R values for same systems with spin zero ground states among the random ensemble. One sees that $SO(8)$ systems do not give dominantly vibrational and rotational spectra, because R values do not exhibit any sharp peaks at R=2 and R=10/3. In particular, statistics of R larger than 3 is very small. This situation is different from that of sd bosons studied in Ref. [6]. In Fig. 1, one sees that most R values locate between 1.6 and 3.1, similar to those of fermion systems obtained by shell model calculations by using a two-body random ensemble[10], where R has a very broad distribution.

Figure 2 presents the distribution of R for $SP(6)$ systems. We see that results of R distribution for systems with the $SP(6)$ symmetry are very similar to those of sd-boson systems. There are two peaks around $R = 2$ (although a bit scattered) and 10/3. We calculate ratio $R' = 6_1^+ \to 2_1^+$ for $N = 5,6$ and 7, as shown in the right column of Fig. 5. We see R' has a sharp peak at $R' = 7$, which indicates again the rotational spectra.

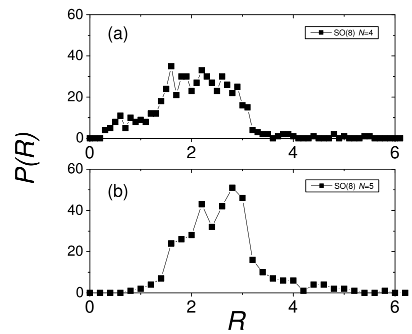

Fig. 1. Distribution of $R = E_{4_1^+}/E_{2_1^+}$ for systems with the $SO(8)$ symmetry and spin zero ground states under random interactions. (a) N=4; (b) N=5. One sees that R distributes in a broad range instead of concentrating on specific values.

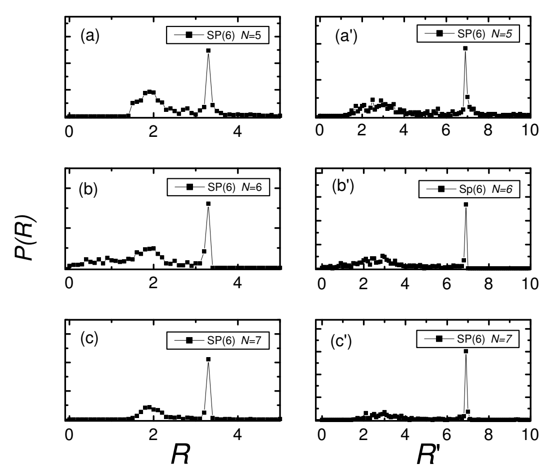

Fig. 2. Distribution of $R = E_{4_1^+}/E_{2_1^+}$ and $R' = E_{6_1^+}/E_{2_1^+}$ of systems with the $SP(6)$ symmetry. We include only cases in which ground states have zero spin under random interactions. (a) and (a′) N=5; (b) and (b′) N=6; (c) and (c′) N=7. There are many cases with $R \sim 3.3$ and $R' \sim 7$ (sharp peaks in the figures), indicating that rotational spectra are dominantly favored in low-lying states of such systems under random interactions.

In this paper we have studied collective motion in low-lying states under random interactions within the framework of the Fermion Dynamical Symmetry Model (FDSM). Although the FDSM Hamiltonian is very similar to that of the sd interacting boson model (IBM), with monopole and quadrupole correlation properly emphasized, we have shown in this paper that collective motion for these two models is different under random interactions. The FDSM has two symmetry groups: the $SP(6)$ and the $SO(8)$. The $SP(6)$ case presents dominantly collective motions including vibration and rotation as the IBM, while the $SO(8)$ case is very different.

There are no dominant vibrational or rotational spectra peaks for systems with the $SO(8)$ symmetry, for which most low-lying states obtained by random interactions lie between vibrational and rotational spectra, with the ratio $R = E_{4_1^+}/E_{2_1^+}$ locating 1.6 to 3.1 in most cases. For the $SP(6)$ symmetry case, one sees two clear peaks of R: one around 2 corresponding to vibration and the other sharper one around 10/3 corresponding to rotation. Corresponding ratio $R' = E_{6_1^+}/E_{2_1^+}$ also shows one sharp peak at 7 corresponding to that of rotational spectra and one scattered peak around 3 (indication of vibration). Calculated results of $B(\mathrm{E2})$ values are also consistent with the above suggestion. Without details we note that overlaps between wavefunctions of states (with spin zero ground states) of systems with the $SP(6)$ symmetry by using random interactions and corresponding the $SU(3)$ wavefunctions are very close to 1 in most cases when $R(4/2) \approx 10/3$, which demonstrates explicitly the rotational motion of systems with the $SP(6)$ symmetry under random interactions.

It is interesting to point out the implications of main results in this paper, i.e., we obtained different collective behaviors in low-lying states for the $SO(8)$ case and the $SP(6)$ case of the FDSM by using random interactions. Because the $SO(8)$ symmetry of the FDSM has no rotational limit (i.e., the $SU(3)$ subgroup) while the $SP(6)$ symmetry does, our calculations demonstrate that the existence of the $SU(3)$ subgroup chain in the Hamiltonian is essential in obtaining rotational motion by random interactions. These results are robust with respect to different widths for particle-hole and particle-particle interactions G_J and B_r. Thus our results suggest that the dynamical symmetries in the Hamiltonian are relevant in obtaining collective motion of low-lying states within the Fermion Dynamical Symmetry Model in the presence of random interactions.

Finally, we note that we recently discovered a strong linear correlation between sorted eigenvalues and diagonal matrix elements of given random matrices[15]. We also made progress in evaluating the lowest eigenvalues of two-body random ensemble[16]. These by-products are important for future applications.

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