# Bounds on the magnetic moment of the τ-neutrino via the process $e^+e^- \to \nu\bar{\nu}\gamma$

C. Aydın[1) ] M. Bayar N. Kılıç

(Karadeniz Technical University Department of Physics, 61080 Trabzon, Turkey)

**Abstract** Using Breit-Wigner resonance relation, bounds on the magnetic moment of the tau-neutrino are calculated through the reaction $e^+e^- \to \nu\bar{\nu}\gamma$ at the neutral boson pole in the framework of a superstring-inspired $E_6$ model which has one extra low-energy neutral gauge boson and a LRSM.

**Key words** neutrino, magnetic moment, Z-boson, SM, LRSM, superstring-inspired $E_6$

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## 1 Introduction

The question of whether the neutrinos are Dirac or Majorana particles is one of the most important issues in particle physics, astrophysics and cosmology. The properties of neutrinos have become the subject of an increasing research effort over the last years. The search for the neutrino mass, magnetic moment, dipole moment and anapole moment is of great significance for the choice of theory of elementary particles and for understanding the phenomena such as supernova dynamics, stellar evolution and the production of neutrino by the sun[1].

The Standard Model (SM) describes many phenomena up to the energies that can be reached today. Of all the particles of the SM, neutrinos are the least known. It is known that the neutrinos are massless in the SM. Neutrinos seem to be likely candidates for carrying features of physics beyond the SM. The purpose of the extended theories is to explain some fundamental aspects, for example, the neutrino mass, the neutrino oscillations, the neutrino magnetic moment, etc. which are not clarified in the frame of the SM. In many extensions of the Standard Model a neutrino acquires a nonzero mass. Massive neutrinos are Dirac or Majorana neutrinos. These neutrinos have different electromagnetic properties. Dirac neutrino has three form factors which are charge, magnetic moment and anapole moment since the electric dipole moment is zero in a $CP$ conserving theory[2]. If there is no neutrino mixing, Majorana neutrino has only one form factor which is the anapole moment[3]. Once neutrino mixing is taken into account, then there are magnetic and electric transition moments as well. In this manner the neutrinos seem to be likely candidates for carrying features of physics beyond the Standard Model. Apart from masses and mixings, magnetic moments, electric dipole moments and anapole moments are also signs of new physics. These were calculated by many authors in many different models[4].

As is known, there are a number of possible physical processes involving a neutrino with a magnetic moment. Among these are the ν-e scattering, the spin-flavor precession in an external magnetic field, the plasmon decay, and the neutrino decay.

In 1994, Gould and Rothstein[5] reported a bound on the tau neutrino magnetic moment which they obtained through the analysis of the process $e^+e^- \to \nu\bar{\nu}\gamma$ , near the $Z_0$-resonance by considering a massive tau- neutrino and using the Standard Model $Ze^+e^-$ and $Z\nu\bar{\nu}$ couplings. At low center of mass energy $s \ll M_{Z0}^2$, the dominant contribution of that process involves the exchange of virtual photon[6]. The dependence on the magnetic moment comes to a direct coupling to the virtual photon and then the observed photon is a result of the initial state bremsstrahlung.

In 2001, Aydemir and Sever[7] calculated the same process in the framework of a class of $E_6$ inspired models with a light additional neutral vector boson. Detailed discussion on $E_6$ inspired model can be found in Ref. [8].

In 2004, a bound on the tau- neutrino magnetic

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1) E-mail: coskun@ktu.edu.tr

moment (and the tau- neutrino dipole moment) has been reported by Gutierrez-Rodriguez, Hermander-Ruiz and Del Rio-De Santiago[9] through the analysis of the process $e^+e^- \to \nu\bar{\nu}\gamma$ in the framework of the left-right symmetric model, based on the $SU(2)_R \times SU(2)_L \times U(1)$ gauge group. Detailed discussion on LRSM can be found in Ref. [10]. They did their analysis near the resonance of the $Z_1 (s = M^2_{Z_1})$. Thus their results are independent of the mass of the additional heavy $Z_2$ gauge boson which appears in these kinds of models. Therefore, they have the mixing angle $\phi$ between the left-right neutral bosons as the only additional parameter besides the SM parameters.

Our aim in this study is to analyse the reaction $e^+e^- \to \nu\bar{\nu}\gamma$ in different models (beyond SM) using Breit-Wigner resonance form for the resonance contribution. The Breit-Wigner formula is good representation of data when an isolated state dominates the cross-section, but in many cases the non-resonant scattering also occurs and other terms have to be added to the amplitude in order to extract reliable resonance parameters from experimental data. We had made this work for the first time originally in $E_6$ using Breit-Wigner resonance relation. After our work[11], Rodriguez et al. calculated the tau- neutrino magnetic moment and dipole moment in $E_6$[12].

At higher $s$, near the Z $(Z_0, Z_1, Z_\theta)$ pole-where Z is $Z_0$ for SM, $Z_1$ for LRSM and $Z_\theta$ for $E_6 - s = M_Z^2$, the dominant contribution for $E_\gamma > 10$ GeV involves the exchange of a Z boson. The dependence on the magnetic moment now comes from the radiation of the photon observed by the neutrino and antineutrino in the final state. The Feynman diagrams which give the most important contribution to the cross section are shown in Fig. 1.

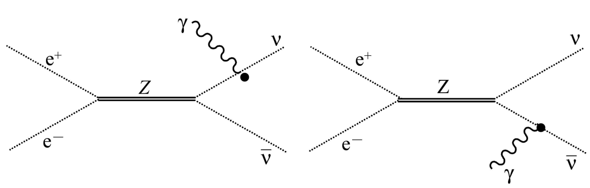


Fig. 1. The Feynman diagrams contributing to the process $e^+e^- \to \nu\bar{\nu}\gamma$.

We calculate the total cross section of the process $e^+e^- \to \nu\bar{\nu}\gamma$ using the Breit-Wigner resonance form for the resonance contribution[13]

$$\sigma = \frac{4\pi\,(2J+1)\,\Gamma_{Z\to e^+e^-}\Gamma_{z\to\nu\bar{\nu}\gamma}}{(s-M_Z)^2 + M_Z^2\Gamma_Z^2} \tag{1}$$

where $\Gamma_{Z\to e^+e^-}$ in the decay rate of Z to the channel $Z \to e^+e^-$, $\Gamma_{Z\to\nu\bar{\nu}\gamma}$ in the decay rate Z to the channel $Z_{\to}\nu\bar{\nu}\gamma$ and $\Gamma_Z$ is total the width of Z.

This paper is organized as follows: In Sect. 2, we present the calculations of the process $e^+e^- \to \nu\bar{\nu}\gamma$ in the framework of a superstring-inspired $E_6$ model and LRSM. Finally, we present our results and discussions in Sect. 3.

## 2 The total cross section

1) Width of section $Z_\theta \to e^+e^-$

In this section, we calculate the total width of the reaction $Z_\theta \to e^+e^-$ in the context of a superstring-inspired $E_6$ model. The expression for the Feynman amplitude $M$ of the process $Z_\theta \to e^+e^-$ is given by

$$M = -\frac{\mathrm{i}g}{\cos\theta_W}\bar{u}\gamma^\mu\,(C'_V - C'_A\gamma_5)\,\nu\,\varepsilon^\lambda_\mu \tag{2}$$

where

$$C'_V = X^{1/2}\left(\frac{1}{\sqrt{6}}\cos\varphi + \frac{1}{\sqrt{10}}\sin\varphi\right),$$

$$C'_A = 2X^{1/2}\frac{\sin\varphi}{\sqrt{10}}, \quad X = \frac{g_\theta^2}{g^2+g'^2}\left(\frac{M_{Z_0}}{M_{Z_\theta}}\right)^2,$$

$x_W = \sin^2\theta_W$, $\theta_W$ is the electroweak mixing angle, $\varphi$ is the mixing angle between $Z_\psi$ and $Z_\chi$[8], and $\varepsilon^\lambda_\mu$ is the polarization vector of the boson $Z_\theta$.

The expression for the total width of the process $Z_\theta \to e^+e^-$, due only to the $Z_\theta$ boson exchange, according to the diagrams depicted in Fig. 1, and using the expression for the amplitude given in Eq. (2) is

$$\Gamma_{Z_\theta\to e^+e^-} = \frac{\alpha\,M_{Z_\theta}}{12\,x_W\,(1-x_W)}\,(C'^2_V + C'^2_A)\ . \tag{3}$$

2) Width of $Z_\theta \to \nu\bar{\nu}\gamma$

The expression for the Feynman amplitude of the process $Z_\theta \to \nu\bar{\nu}\gamma$ is due only to the $Z_\theta$ boson exchange, as shown in the diagrams in Fig. 1.

The expression for the Feynman amplitude $M$ of the process $Z_\theta \to \nu\bar{\nu}\gamma$ is given by

$$\begin{aligned} M \ = \ & \frac{\mathrm{i}g}{4\cos\theta_W}\kappa k^\nu \varepsilon^\mu_\gamma \bar{u}(q_1)\Bigg[\frac{1}{(k+q_1)^2 - m_\nu^2}\sigma_{\mu\nu}\times \\ & (\not{k} + \not{q}_1 + m_\nu)\not{\varepsilon}(a' - b'\gamma_5) + \\ & \frac{1}{(k+q_2)^2 - m_\nu^2}\not{\varepsilon}(a' - b'\gamma_5)\times \\ & (\not{k} + \not{q}_2 + m_\nu)\sigma_{\mu\nu}\Bigg]\nu(q_2) \end{aligned} \tag{4}$$

where $k$ is the photon momentum,

$$a' = b' = X^{1/2}\left(-\frac{1}{\sqrt{6}}\cos\varphi + \frac{3}{\sqrt{10}}\sin\varphi\right),$$

$q_1$ is the neutrino momentum, $q_2$ is the antineutrino momentum, $\varepsilon^\lambda_\gamma$ and $\varepsilon^\lambda_Z$ are the polarization vectors of photon and of the boson Z, respectively.

After a long and straightforward calculation, we obtain for the total width of $Z_\theta \to \nu\bar{\nu}\gamma$

$$\Gamma_{Z_\theta\to\nu\bar{\nu}\gamma} = \int\limits_{E_\gamma}\int\limits_{\theta}\frac{\alpha\kappa^2}{96\pi^2 x_W(1-x_W)E_Z}\times \Bigg[(a'^2+b'^2)\times \left(s-2\left(E_Z-\sqrt{E_Z^2-M_{Z_\theta}^2}\cos\theta\right)E_\gamma\right)+ \frac{2a'^2}{3}E_\gamma^2\left(E_Z-\sqrt{E_Z^2-M_{Z_\theta}^2}\cos\theta\right)^2\Bigg]\times E_\gamma \mathrm{d}E_\gamma\sin\theta\,\mathrm{d}\theta \quad (5)$$

In the Z boson rest frame, we have

$$\Gamma_{Z_\theta\to\nu\bar{\nu}\gamma} = \int\limits_{E_\gamma}\int\limits_{\theta}\frac{\alpha\kappa^2}{96\pi^2 x_W(1-x_W)M_{Z_\theta}}\times \left[(a'^2+b'^2)(s-2\sqrt{s}E_\gamma)+\frac{2}{3}a'^2E_\gamma^2\right]\times E_\gamma\mathrm{d}E_\gamma\sin\theta\,\mathrm{d}\theta \quad (6)$$

where $E_\gamma$ and $\theta$ are the energy and scattering angle of the photon, $M_{Z_\theta}$ is the neutral $Z_\theta$ boson mass.

The substitution of Eqs. (3) and (6) in Eq. (1) gives

$$\sigma = \frac{\alpha^2\kappa^2}{96\pi^2x_W^2(1-x_W)^2M_{Z_\theta}^2\Gamma_{Z_\theta}^2}(C_V^2+C_A^2)\times \int\limits_{E_\gamma}\int\limits_{\theta}\left[2\left(s-2\sqrt{s}E_\gamma\right)+\frac{2}{3}E_\gamma^2\right]E_\gamma\mathrm{d}E_\gamma\sin\theta\,\mathrm{d}\theta \quad (7)$$

where

$$C_V = X\left(\frac{1}{\sqrt{6}}\cos\varphi+\frac{1}{\sqrt{10}}\sin\varphi\right)\times \left(-\frac{1}{\sqrt{6}}\cos\varphi+\frac{3}{\sqrt{10}}\sin\varphi\right)$$

$$C_A = 2X\frac{\sin\varphi}{\sqrt{10}}\left(\frac{-\cos\varphi}{\sqrt{6}}+\frac{3}{\sqrt{10}}\sin\varphi\right).$$

Similarly, in LRSM, we obtain the following relations

$$\sigma = \frac{\alpha^2\kappa^2}{192\pi^2x_W^2(1-x_W)^2M_{Z_1}^2\Gamma_{M_Z}^2}\times \left[\frac{1}{2}(a^2+b^2)-4a^2x_W+8a^2x_W^2\right]\times \int\limits_{E_\gamma}\int\limits_{\theta}\left[(a^2+b^2)(s-2\sqrt{s}E_\gamma)+\frac{2a^2}{3}E_\gamma^2\right]\times E_\gamma\mathrm{d}E_\gamma\sin\theta\mathrm{d}\theta \quad (8)$$

where $s_\varphi=\sin\phi$, $c_\phi=\cos\phi$, $\phi$ is the mixing angle between $Z_1$ and $Z_2$ in the LRSM, $r_W=\sqrt{\cos2\theta_W}$, $a=c_\phi-\frac{s_\phi}{r_w}$, $b=c_\phi+r_w s_\phi$.

## 3 Numerical calculations and discussion

The L3 Collaboration evaluated the selection efficiency using detector-simulated $e^+e^-\to\nu\bar{\nu}\gamma$ events. A total of 14 events was found by the selection. As was discussed in Ref. [5] $N=\sigma\cdot L$ is less than 14, with $L$=137 $\mathrm{pb}^{-1}$, according to the data reported by the L3 Collaboration Ref. [14]. In our calculations we used the following numerical values $x_W=\sin^2\theta_W=0.2314$, $\Gamma_Z$=2.49 GeV, and assumed that all $\Gamma_Z(Z_0,Z_1,Z_\theta)$ is to be the same (Variations of the $\Gamma_{Z_\theta}$ are taken in the range from 0.15 to 2.0 times $\Gamma_Z$ in the results of the CDF Collaboration[15]. So we take $\Gamma_{Z_\theta}=\Gamma_Z$ as a special case of this variation). The $\theta$ and $E_\gamma$ here change from 44.5° to 135.5° and from 15 GeV to $\sqrt{s}/2$ respectively. Using the above numerical values we have obtained the results in Table 1 for the LRSM and in Table 2 for the superstring-inspired $E_6$ model in which assumed all $U(1)$ couplings are to be the same, i.e., $\frac{g_\theta^2}{g^2+g'^2}=\frac{5}{3}\sin^2\theta_W$. We see that the contribution of the mixing angle to the magnetic moment is very small in LRSM .

Table 1. Bounds on the tau-neutrino magnetic moment for the different $Z_1$ resonance, i.e., $s=(M_{Z_1}^2)$ for $N$=14 and $L$=137 $\mathrm{pb}^{-1}$.

| $\phi$ | $\kappa/(10^{-6}\mu_B)$ | | |
|---|---|---|---|
| | 91.18 GeV | 161 GeV | 183 GeV |
| −0.009 | 6.83 | 3.55 | 1.51 |
| −0.005 | 6.84 | 3.55 | 1.51 |
| −0.004 | 6.84 | 3.55 | 1.51 |
| 0.000 | 6.85 | 3.56 | 1.52 |
| 0.004 | 6.86 | 3.56 | 1.52 |

Upper limits on the tau neutrino magnetic moment reported in the literature are given $\mu_{\nu_\tau}\leqslant 3.3\times10^{-6}\mu_B$ (90% C.L.) from a sample of $e^+e^-$ annihilation events collected with the L3 detector[13] at the $Z_1$ resonance corresponding to an integrated luminosity of 137 $\mathrm{pb}^{-1}$; $\mu_{\nu_\tau}\leqslant 2.7\times10^{-6}\mu_B$ (95% C.L.) at $q^2=M_{Z_1}^2$ from the measurements of the $Z_1$ invisible width at LEP[14]; $\mu_{\nu_\tau}\leqslant 1.83\times10^{-6}\mu_B$ (90% C.L.) from the analysis of $e^+e^-\to\nu\bar{\nu}\gamma$ in a class of $E_6$ inspired model[7]. A.M.Cooper et al. obtained from the BEBC beam dump experiment which was the data limit the possible magnetic moment of tau neutrino to be blow $5.4\times10^{-7}\mu_B$[16]; from the order of $\mu_{\nu_\tau}<O(1.1\times10^{-6}\mu_B)$ Akama et al.[17] derived and applied model independent bounds on the anomalous magnetic moment; the DONUT Collaboration reported the tau-neutrino magnetic moment as $\mu_{\nu_\tau}<3.9\times10^{-7}\mu_B$[18, 19].

We have seen that $\kappa$ values given in Table 1 are in good agreement, but the results of Table 2 are bigger than $10^{-6}\mu_B$.

Table 2. Bounds on the tau-neutrino magnetic moment for different mixing angle $\varphi$ and different $M_{Z_\theta}$ values for $N$=14 and $L$=137 $pb^{-1}$.

| $\varphi$ | $\kappa/(10^{-6}\mu_B)$ | | |
|---|---|---|---|
| | 91.18 GeV | 161 GeV | 183 GeV |
| 0° | 27.1 | 48.9 | 58 |
| 37.8° | 39.3 | 70 | 82 |
| 90° | 6.8 | 12 | 14.4 |
| 127.8° | 9.2 | 16.4 | 19.4 |

Table 3. Bounds on the magnetic moment of the tau-neutrino for different mixing angle $\varphi$ and different $M_{Z_\theta}$ values for $N$=3 and $L$=29.4 $fb^{-1}$.

| $\varphi$ | $\kappa/(10^{-6}\mu_B)$ | | |
|---|---|---|---|
| | 400 GeV | 500 GeV | 638.26 GeV |
| 0° | 6.9 | 8.6 | 10.9 |
| 37.8° | 6.3 | 7.7 | 6.3 |
| 90° | 1.6 | 2 | 2.5 |
| 127.8° | 2.2 | 2.7 | 3.4 |

$Z_0$ and $Z_\theta$ contributions in Ref. [7] and $Z_1$ and $Z_\theta$ contributions in Ref. [12] are taken together. The contribution of $Z_0$ or $Z_1$ is $\dfrac{\sigma}{\kappa^2} \sim 10^{-26}$ cm$^2$ and the contribution $Z_\theta$ is $\dfrac{\sigma}{\kappa^2} \sim 10^{-36}$cm$^2$ in the total cross section[7, 12]. From this point of view, we think that the contribution of $Z_\theta$ is not a dominant factor in both calculations[7, 12] for the total cross-section. Using the above $N$ and $L$ values in Refs. [7, 12], we have obtained $\mu \sim 10^{-6}\mu_B$ for $Z_0$ or $Z_1$, $\mu \sim 10^{-4}\mu_B$ for $Z_\theta$.

The tau-neutrino magnetic moment are determined about $10^{-7}\mu_B$ or $10^{-6}\mu_B$ in several work done before. It is seen that the values given in Table 2 are about $10^{-5}\mu_B$. We have not gained the results of Refs. [7, 12], especially for $\varphi$ = 37.8° and $M_{Z_\theta} = 7M_{Z_0}$.

For that reason, we conclude that it is not suitable to use the values $N$=14 and $L$=137 $pb^{-1}$ in which Refs. [7, 12] are used for $Z_\theta$ . If one uses $L$=29.4 $fb^{-1}$[20] instead of $L$=137 $pb^{-1}$ and $N$=3, we obtain the values shown in Table 3. These values are in agreement with the literature values.

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