Design of magnet and control of the beam emittance for Penning H^- ion source

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Abstract The design requirement and principle of the deflection magnet for Magnetron and Penning H^- ion source are discussed. It is proved that there exists a maximum emittance for the beam that may be transformed by the magnet into a state with equal Twiss parameters of $\alpha_r=\alpha_y$ and $\beta_r=\beta_y$, which is the requisite condition to get a minimum emittance at the entrance of RFQ after transporting by a LEBT with solenoids. For this maximum emittance, the corresponding magnetic field gradient index is 1.

Key words H^- ion source, Magnetron ion source, Penning ion source, magnetic field gradient index

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1 Introduction

The Magnetron and Penning H^- ion sources[1, 2] are widely used in accelerators. Usually the ions are extracted from a long slit and the ion source is disposed in a 90° deflection magnet with a special magnetic field gradient index $n=-\dfrac{r}{B}\dfrac{\partial B}{\partial y}$. In practice, the magnetic field gradient index n is generally designed to 1[1, 2]. Following the ion source two or three solenoids are often used in a low energy beam transport line (LEBT) and then the beam is injected into a Radio-Frequency Quadrupole accelerator (RFQ), which is generally designed to accept an axial symmetric input beam with the same values of α and β as well as the same beam emittances in the two transverse phase planes. However, the beam extracted from the above kinds of ion sources is generally of the different values of α and β as well as the different emittances in the two transverse phase planes. The previous research results[3, 4] show that even when the initial transverse beam has different emittances in the two transverse phase planes, but if the initial transverse beam has the same values of α and β in the two phase planes, a beam with minimum and equal emittances at the entrance of RFQ can be still got through controlling the coupling and the focusing of the solenoids in LEBT. The deflection magnet and the extraction gap of ion source may be specially designed to accomplish this objective that transforms the extracted beam with unequal values of α and β into a beam with equal values of α and β in the two planes. In this paper, the requirements for the magnetic field gradient index n and the distance of extraction gap to get a beam with the same values of α and β in the two planes at the exit of the deflection magnet are obtained.

2 Singular particle model to approximately get the requirements for the magnetic field gradient index n and the extracted beam divergence

First, a singular particle model is used to analytically deduce the requisite magnetic field gradient index n and the extracted beam divergence to get an axial symmetric beam, i.e., the beam has the same values of α and β as well as the same beam emittances in the two transverse phase planes at the exit of the deflection magnet. In the model, assuming that: (1) the beam is extracted from a slit with the length of $2r_1=2a$ and the width of $2y_1=2b$, having a beam divergence of r_1' and y_1' in the two transverse planes, respectively. (2) The beam is a laminar flow, ie., the ion trajectories do not cross. (3) Hard-edge approximation is used for the magnetic field. The magnet deflection angle is θ and the bending radius

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is R. When a linear approximation is used, the beam envelope, (r_2, y_2), and the beam divergence, (r'_2, y'_2), which is also the amplitude and the slope of the most outer ion of the beam, are determined by the following equations at the exit of magnet[5]:

$$\begin{pmatrix} r_2 \\ r'_2 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} r_1 \\ r'_1 \end{pmatrix} = \begin{pmatrix} \cos(\sqrt{1-n}\theta) & \frac{R}{\sqrt{1-n}}\sin(\sqrt{1-n}\theta) \\ -\frac{\sqrt{1-n}}{R}\sin(\sqrt{1-n}\theta) & \cos(\sqrt{1-n}\theta) \end{pmatrix} \begin{pmatrix} r_1 \\ r'_1 \end{pmatrix}, \tag{1}$$

$$\begin{pmatrix} y_2 \\ y'_2 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} y_1 \\ y'_1 \end{pmatrix} = \begin{pmatrix} \cos(\sqrt{n}\theta) & \frac{R}{\sqrt{n}}\sin(\sqrt{n}\theta) \\ -\frac{\sqrt{n}}{R}\sin(\sqrt{n}\theta) & \cos(\sqrt{n}\theta) \end{pmatrix} \begin{pmatrix} y_1 \\ y'_1 \end{pmatrix}. \tag{2}$$

In getting the above equations, we have assumed that the magnetic field gradient n satisfies the condition $0 < n \leqslant 1$, i.e., the deflection magnet provides the ion beam with weak focus.

Assuming a parallel beam (beam waist or beam belly) with a circular cross section at the output is required, i.e., $r_2 = y_2$, $r'_2 = 0$, and $y'_2 = 0$. Then, from Eqs. (1) and (2), the following equations can be deduced:

$$\frac{\cos\left[n^{1/2}\theta\right]}{\cos\left[(1-n)^{1/2}\theta\right]} = \frac{y_1}{r_1} , \tag{3}$$

$$r_2 = \frac{r_1}{\cos\left[(1-n)^{1/2}\theta\right]} , \tag{4}$$

$$y_2 = \frac{y_1}{\cos\left[n^{1/2}\theta\right]} , \tag{5}$$

$$r'_1 = \frac{r_1}{R}\left[1-n\right]^{1/2}\tan\left[(1-n)^{1/2}\theta\right], \tag{6}$$

$$y'_1 = \frac{y_1}{R} n^{1/2}\tan\left[n^{1/2}\theta\right]. \tag{7}$$

From the above equations, one can conclude that: (1) In order to obtain a parallel beam (beam waist or beam belly) with a circular cross section at the output, the requisite magnetic field gradient index n is determined by the aspect ratio of slit. For an example, R=8 cm, $\theta = 90°$, $a = 5$ mm and $b = 0.3$ mm are used by RAL[2]. In this case, $n \approx 0.932$ should be used. (2) A beam with definite divergent angle in r and y direction should be extracted from the ion source. For the ion source used in RAL, from Eqs. (3), (6) and (7), the got beam divergence is $r'_1 \approx 0.4°$ and $y'_1 \approx 4.17°$. As is known, for a very long slit without the end aberration, the beam divergence r'_1 in the long direction of the slit, is determined by the following equation:

$$r'_1 \approx \sqrt{\frac{kT_i}{eV_a}} . \tag{8}$$

In which, T_i is the ion plasma temperature, k the Boltzmann constant and V_a the extraction voltage. For stable plasma, the ion energy is usually of the order $kT_i \approx 1$ eV. For the RAL ion source, V_a=17 keV, then the beam divergence in r direction is $r'_1 \approx 0.44°$. It approximates the requisite value. For plasma with a definite density and beam current, y'_1 is determined by the distance of the extraction gap for a certain extraction electrode. That means that, in order to get the needed beam, the distance of extraction gap should be adjusted to an optimum value.

3 Numerical solution for the magnetic field gradient index *n* and the extracted beam divergence

Singular particle model is very beneficial to analytically deduce the requisite magnetic field gradient index n and the extracted beam divergence. However, in reality, the beam emittance will never be zero and the beam is also not a laminar flow. Moreover, the beam extracted from the ion source generally has different emittances in the two transverse dimensions. For example, the measured ratio of beam emittance in the two transverse dimensions is $\varepsilon_r/\varepsilon_y \approx 1.07-1.39$ for the RAL ion source[6, 7]. In this case, as pointed in Refs. [3, 4], after transported by the magnet, a beam with the same values of α and β in the two phase planes at the exit of magnet is the optimum choice. That is to say, we require:

$$\alpha_{2r} = \alpha_{2y} , \tag{9}$$

$$\beta_{2r} = \beta_{2y} . \tag{10}$$

As is known, transformation of the Twiss parameters is determined by the following equation[8]:

$$\begin{pmatrix} \beta_2 \\ \alpha_2 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} M_{11}^2 & -2M_{11}M_{12} & M_{12}^2 \\ -M_{11}M_{21} & 1+2M_{12}M_{21} & -M_{22}M_{12} \\ M_{21}^2 & -2M_{22}M_{21} & M_{22}^2 \end{pmatrix} \times \begin{pmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{pmatrix}, \tag{11}$$

where $\beta = X^2/\varepsilon$, $\gamma = \phi^2/\varepsilon$, and $\alpha = -\sqrt{\phi^2 X^2 - \varepsilon^2}/\varepsilon$ (Assuming the beam is a divergent beam), here X and ϕ are the maximum dimension (envelope) and the maximum divergent angle of the beam, respectively, ε stands for the beam emittance in r or y direction.

From Eqs. (1), (2), and (9)—(11), we may obtain the following equations:

$$\cos^2(\sqrt{1-n}\theta)\beta_{1r}-\frac{2R}{\sqrt{1-n}}\sin(\sqrt{1-n}\theta)\cos(\sqrt{1-n}\theta)\alpha_{1r}+\frac{R^2}{1-n}\sin^2(\sqrt{1-n}\theta)\gamma_{1r}=\cos^2(\sqrt{n}\theta)\beta_{1y}-\frac{2R}{\sqrt{n}}\sin(\sqrt{n}\theta)\cos(\sqrt{n}\theta)\alpha_{1y}+\frac{R^2}{n}\sin^2(\sqrt{n}\theta)\gamma_{1y}\ , \tag{12}$$

$$\frac{\sqrt{1-n}}{R}\sin(\sqrt{1-n}\theta)\cos(\sqrt{1-n}\theta)\beta_{1r}+\left[1-2\sin^2(\sqrt{1-n}\theta)\right]\alpha_{1r}-\frac{R}{\sqrt{1-n}}\sin(\sqrt{1-n}\theta)\cos(\sqrt{1-n}\theta)\gamma_{1r}=\frac{\sqrt{n}}{R}\sin(\sqrt{n}\theta)\cos(\sqrt{n}\theta)\beta_{1y}+\left[1-2\sin^2(\sqrt{n}\theta)\right]\alpha_{1y}-\frac{R}{\sqrt{n}}\sin(\sqrt{n}\theta)\cos(\sqrt{n}\theta)\gamma_{1y}\ , \tag{13}$$

In the above Eq. (12) and Eq. (13), the known parameters are: the deflection angle $\theta=\pi/2$, the bending radius R, the slit length $2a$ and width $2b$. The beam emittance, ε_r and ε_y may be also known through measurement. There are still three unknown parameters: the magnetic field gradient index n, the maximum beam divergent angle from source in long slit edge direction ϕ_{1r} and in narrow slit edge direction ϕ_{1y}. As mentioned above, the divergent angle in long edge direction is determined through the ion temperature. So it is reasonable and without losing generality to assume that the beam phase ellipse in r direction is an upright ellipse. That means:

$$\alpha_{1r}=0\ , \tag{14}$$

$$\phi_{1r}=\frac{\varepsilon_r}{a}\ . \tag{15}$$

Based on Eqs. (14) and (15), Eqs. (12) and (13) may be rewritten as:

$$\sin(2\sqrt{1-n}\theta)\left[\frac{\sqrt{1-n}}{2R}\frac{a^2}{\varepsilon_r}-\frac{R}{2\sqrt{1-n}}\frac{\varepsilon_r}{a^2}\right]=\sin(2\sqrt{n}\theta)\left[\frac{\sqrt{n}}{2R}\frac{b^2}{\varepsilon_y}+\mathrm{ctg}(2\sqrt{n}\theta)\alpha_{1y}-\frac{R}{2\sqrt{n}}\frac{\varepsilon_y(1+\alpha_{1y}^2)}{b^2}\right], \tag{16}$$

$$\cos^2(\sqrt{1-n}\theta)\frac{a^2}{\varepsilon_r}+\frac{R^2}{1-n}\sin^2(\sqrt{1-n}\theta)\frac{\varepsilon_r}{a^2}=\cos^2(\sqrt{n}\theta)\frac{b^2}{\varepsilon_y}-\frac{R}{\sqrt{n}}\sin(2\sqrt{n}\theta)\alpha_{1y}+\frac{R^2}{n}\sin^2(\sqrt{n}\theta)\frac{\varepsilon_y(1+\alpha_{1y}^2)}{b^2}\ . \tag{17}$$

By numerical solution of the Eqs. (16) and (17), the requisite optimum n and ϕ_{1y}that result in $\alpha_{2r}=\alpha_{2y}$ and $\beta_{2r}=\beta_{2y}$ may be obtained.

However, it is discovered that there exists a maximum emittance within which Eq. (16) and Eq. (17) are solvable, as shown in Fig. 1 (for the case a = 5 mm, b = 0.3 mm, R = 8 cm and $\theta=\pi/2$). As shown in the figure, for the maximum emittance, the corresponding magnetic field index is 1. In Fig. 1, the relation between the required beam divergent angle ϕ_{1y} and the ratio of the initial emittance $\varepsilon_y/\varepsilon_r$ is also given.

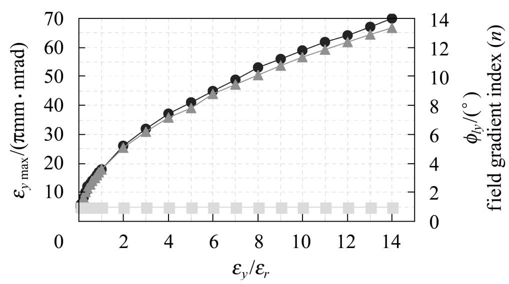

Fig. 1. The got maximum initial emittance ε_y (dot), beam divergence ϕ_{1y} (triangle) and field gradient index n (square) versus the ratio of the initial emittance $\varepsilon_y/\varepsilon_r$.

Physically, a prerequisite for a solvable Eq. (16) and Eq. (17) is that the field gradient index n must satisfy $n\leqslant 1$. For $n=1$, Eq. (16) and Eq. (17) may be rewritten as follows:

$$R\theta\frac{\varepsilon_r}{a^2}=\alpha_{1y}\ , \tag{18}$$

$$\frac{a^2}{\varepsilon_r}+R^2\theta^2\frac{\varepsilon_r}{a^2}=\frac{R^2\varepsilon_y(1+\alpha_{1y}^2)}{b^2}\ , \tag{19}$$

From Eq. (18), one can see that, α_{1y}, as well as ϕ_{1y}, are only determined by ε_r for a certain source. Substituting Eq. (18) into Eq. (19), we may get:

$$\varepsilon_r\varepsilon_y=\left(\frac{ab}{R}\right)^2\ . \tag{20}$$

Let $k=\varepsilon_y/\varepsilon_r$, the following relations may be got:

$$\varepsilon_r=\frac{ab}{R\sqrt{k}}\ , \tag{21}$$

$$\varepsilon_y=\frac{\sqrt{k}ab}{R}\ . \tag{22}$$

It is just the same result got by numerical calculation. To demonstrate this more clearly, in Fig. 2, we redraw Fig. 1 by changing the abscissa from $\varepsilon_y/\varepsilon_r$ to $\sqrt{\varepsilon_y/\varepsilon_r}$.

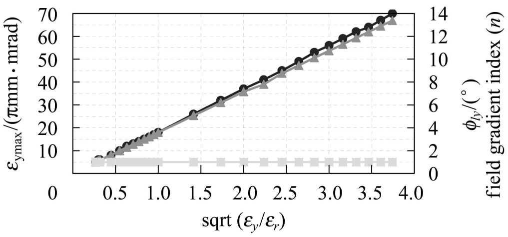

Fig. 2. The got maximum initial emittance ε_y (dot), beam divergence ϕ_{1y} (triangle) and field gradient index n (square) versus the root of the ratio of the initial emittance $\sqrt{\varepsilon_y/\varepsilon_r}$.

Figure 2 clearly shows that both the maximum beam emittance $\varepsilon_{y\max}$ and the beam divergence angle ϕ_{1y} in y direction are linearly proportional to $\sqrt{\varepsilon_y/\varepsilon_r}$. In addition, the physical meaning of Eq. (21) and Eq. (22) could be also well understood through the singular particle model. From Eq. (1) and Eq. (2), let $n=1$, then we have: $r_{2\max}=a$, $r'_{2\max}=r'_{1\max}$, $y_{2\max}=Ry'_{1\max}$, $y'_{2\max}=-b/R$. By using Eq. (15), one gets $r'_{1\max}=\varepsilon/a$. For a symmetric beam output, it is required that $r'_{2\max}=y'_{2\max}$, then we have: $\varepsilon=ab/R$. That means, for the symmetric output beam, the emittance is determined by ab/R.

For the smaller emittance (as an example, let $\varepsilon_r=5$ πmm·mrad), the requisite field gradient index n and divergent angle of extraction beam is shown in the following Fig. 3.

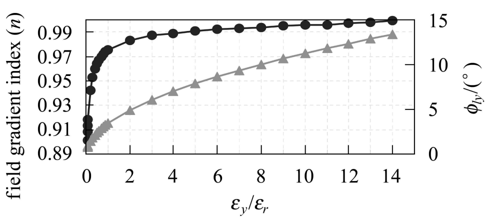

Fig. 3. In the case of ε_r=5 πmm·mrad, the got field gradient index n (dot) and the required divergent angle ϕ_{1y} (triangle) versus the ratio of the initial emittance $\varepsilon_y/\varepsilon_r$.

Usually the beam emittance in y direction is less than that in r direction ($\varepsilon_y/\varepsilon_r \leqslant 1$). So, from Fig. 1, one can see that the permissive maximum emittance $\varepsilon_{y\max}^{\text{perm}}$ in y direction to get $\alpha_{2r}=\alpha_{2y}$ and $\beta_{2r}=\beta_{2y}$ is less than 18.5 πmm·mrad. Unfortunately, the actual beam emittance is generally in the order of 166 πmm·mrad, much larger than $\varepsilon_{y\max}^{\text{perm}}$. That means it is impossible to get a complete requisite beam for the case of $0<n\leqslant 1$. Certainly, here the factor of emittance growth due to the space charge force in the beam transporting process is not taken into account. In practice, the deflection magnet with $n\leqslant 1$ is widely used in Magnetron[1] and Penning H$^-$ ion sources. However, from the above discussion, it could be deduced that in order to get a complete requisite beam with larger emittance, the magnetic field gradient n should be larger than 1.

4 Other factors related with the design of magnet

4.1 Beam centering

To the first order approximation, the deflection magnet can be treated as a magnet with an effective hard edge. In order to center the beam at the designed axis, a certain beam injection site must be matched by a definite magnetic edge. That means that, for a certain injection site, as well as the extractor site, if the magnetic edge is wrong, assuming an error of Δx for example, then the output beam must have a declination angle δ with respect to the axis when the beam is output at the axis; or the output beam must have a displacement, Δy, relative to the axis, when the beam is output parallel to the axis. For an estimation of the order, we have $\delta\approx\Delta x/R$ and $\Delta y\approx\Delta x$. Usually it is not easy to design and construct a magnet with the requisite magnetic edge, so a properly deposited thick iron plate[1] or some magnetic diaphragm[2] is used to cut the spreading magnetic field and form the requisite edge. In addition, the source place and the beam injection angle should be repeatedly and accurately chosen.

4.2 Extracting voltage stability or magnetic field stability

As is known, the variation of beam momentum, $\Delta p/p$, will cause the variation in beam position, Δr, and beam angle divergence, $\Delta r'$, at the exit of magnet. Linear approximation is used again, we have:

$$\begin{cases} \Delta r_2=\dfrac{\Delta p}{p}\dfrac{R}{1-n}\left\{1-\cos\left[(1-n)^{1/2}\theta\right]\right\}, \\ \Delta r'_2=\dfrac{\Delta p}{p}\dfrac{\sin\left[(1-n)^{1/2}\theta\right]}{(1-n)^{1/2}}, \end{cases} \tag{23}$$

In order to estimate the order of this effect, we assume the output emittance diagram is an upright ellipse. The effect on the emittance diagram resulting from these variation is shown in the following Fig. 4.

The original beam emittance for a stable extraction voltage is: $\varepsilon=r_2r'_2$. The relative emittance variation caused by the momentum variation is determined by the following equation:

$$\frac{\Delta\varepsilon}{\varepsilon}\approx\frac{r_2\Delta r'_2+r'_2\Delta r_2}{r_2r'_2}\ . \tag{24}$$

According to the above design parameter, we assume:

n =0.94, $\varepsilon_n = 1$ πmm·mrad, $\varepsilon = 166$ πmm·mrad, r_2=5.45 mm, r'_2=30.5 mrad. For some characteristic momentum variation value, the got emittance variation value due to the momentum variation is shown in the following Table 1.

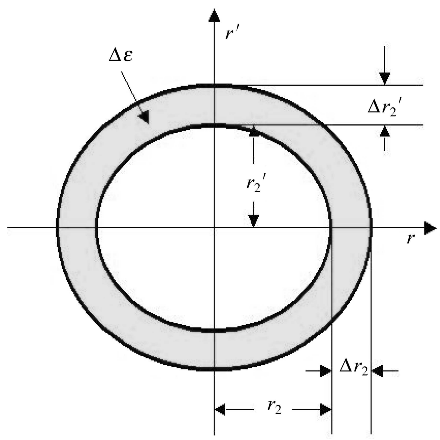

Fig. 4. The schematic diagram of beam emittance change.

Table 1. The emittance variation caused by the momentum change.

$\Delta p/p$	Δr_2/mm	$\Delta r'_2$/mrad	$\Delta\varepsilon/\varepsilon$
1×10^{-3}	0.098	1.53	0.068
2.5×10^{-3}	0.24	3.83	0.17
5×10^{-3}	0.49	7.66	0.34
7.5×10^{-3}	0.73	11.5	0.51
1×10^{-2}	0.98	15.3	0.68

If $\Delta\varepsilon/\varepsilon \leqslant 20\%$ is required, $\Delta p/p$ should be less than $\sim 3\times10^{-3}$. As is known, the extraction energy variation ΔE, the extraction voltage variation ΔV and the beam momentum variation $\Delta p/p$, satisfy the following relation $\Delta E/E \propto \Delta V/V \propto 2\Delta p/p$. Usually the extraction energy variation, as well as the extraction voltage variation is determined by the high voltage slewing on the output capacitor of the power supply within the pulse width for a pulsed extraction beam. Thus the capacitor, C, must satisfy the following expression:

$$C \geqslant \frac{I\tau}{2V}\frac{p}{\Delta p} \ . \qquad (25)$$

Where I is the total loading current of the power supply, τ is the beam pulse width. For a typical value, $I = 0.5$ A, τ=1.0 ms, V=17 kV, then $C \geqslant 4.9$ μF.

5 Summary and discussion

Based on the different beam emittance value, the magnetic field gradient index n and the extraction gap relating with the beam divergence can be specially designed to transform the extracted beam with unequal values of α and β into a beam with equal values of α and β in the two transverse planes. With such an input beam, a beam with minimum and equal emittances at the entrance of RFQ can be got through controlling the coupling and the focusing of the solenoids in LEBT. The numerical solution results show that, there exists a maximum emittance value for the beam extracted from the ion source when the field gradient index n is less than or equal to 1 as generally used for the Magnetron and Penning H^- ion sources. Unfortunately, this maximum emittance is greatly less than the measured beam emittance. In practice, the beam emittance is generally measured at a site located in LEBT downstream the exit of the magnet. This measured emittance is definitely larger than the beam emittance at the exit of ion source after the long distance transportation due to the space charge force. On the other hand, the beam with a large emittance can still be matched by increasing the magnetic field gradient index from the present value $n \leqslant 1$ to $n > 1$. In this case, the beam size in y direction will increase further due to magnet defocusing in y direction. So the magnet pole gap must be also designed to a comparatively larger value.

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