Measurements of D^0-$\overline{D}^0$ mixing and searches for CP violation: HFAG combination of all data

A. J. Schwartz[1)]
(representing the HFAG charm group)
(Physics Department, University of Cincinnati, Cincinnati, Ohio 45221, USA)

Abstract We present world average values for D^0-$\overline{D}^0$ mixing parameters x and y, CP violation parameters $|q/p|$ and $\mathrm{Arg}(q/p)$, and strong phase differences δ and $\delta_{K\pi\pi}$. These values are calculated by the Heavy Flavor Averaging Group (HFAG) by performing a global fit to relevant experimental measurements. The results for x and y differ significantly from zero and are inconsistent with no mixing at the level of 6.7σ. The results for $|q/p|$ and $\mathrm{Arg}(q/p)$ are consistent with no CP violation. The strong phase difference δ is less than 45° at 95% C.L.

Key words mixing, CP violation

PACS 12.15.Ff, 11.30.Er, 13.25.Ft

1 Introduction

Mixing in the D^0-$\overline{D}^0$ system has been searched for for more than two decades without success—until last year. Three experiments −Belle,[1] Babar,[2] and CDF[3] − have now observed evidence for this phenomenon. These measurements can be combined with others to yield World Average (WA) values for the mixing parameters $x \equiv (m_1 - m_2)/\Gamma$ and $y \equiv (\Gamma_1 - \Gamma_2)/(2\Gamma)$, where m_1, m_2 and Γ_1, Γ_2 are the masses and decay widths for the mass eigenstates $D_1 \equiv p|D^0\rangle - q|\overline{D}^0\rangle$ and $D_2 \equiv p|D^0\rangle + q|\overline{D}^0\rangle$, and $\Gamma = (\Gamma_1 + \Gamma_2)/2$. Here we use the phase convention $CP|D^0\rangle = -|\overline{D}^0\rangle$ and $CP|\overline{D}^0\rangle = -|D^0\rangle$. In the absence of CP violation (CPV), $p = q = 1/\sqrt{2}$ and D_1 is CP-even, D_2 is CP-odd.

Such WA values have been calculated by the Heavy Flavor Averaging Group (HFAG)[4] in two ways: (a) adding together three-dimensional log-likelihood functions obtained from various measurements for parameters x, y, and δ, where δ is the strong phase difference between amplitudes $\mathcal{A}(\bar{D}^0 \to K^+\pi^-)$ and $\mathcal{A}(D^0 \to K^+\pi^-)$; and (b) doing a global fit to measured observables for x, y, δ, an additional strong phase $\delta_{K\pi\pi}$, and $R_D \equiv |\mathcal{A}(D^0 \to K^+\pi^-)/\mathcal{A}(D^0 \to K^-\pi^+)|^2$. For this fit, correlations among observables are accounted for by using covariance matrices provided by the experimental collaborations. The first method has the advantage that non-Gaussian errors are accounted for, whereas the second method has the advantage that it is easily expanded to allow for CPV. In this case three additional parameters are included in the fit: $|q/p|$, $\phi \equiv \mathrm{Arg}(q/p)$, and $A_D \equiv (R_D^+ - R_D^-)/(R_D^+ + R_D^-)$, where the $+\,(-)$ superscript corresponds to $D^0\,(\overline{D}^0)$ decays. When both methods are applied to the same set of observables, almost identical results are obtained. The observables used are from measurements of $D^0 \to K^+\ell^-\nu$, $D^0 \to K^+K^-/\pi^+\pi^-$, $D^0 \to K^+\pi^-$, $D^0 \to K^+\pi^-\pi^0$, $D^0 \to K^+\pi^-\pi^+\pi^-$, and $D^0 \to K_S^0\pi^+\pi^-$ decays, and from double-tagged branching fractions measured at the $\psi(3770)$ resonance.

Mixing in heavy flavor systems such as those of B^0 and B_s^0 is governed by the short-distance box diagram. In the D^0 system, however, this diagram is doubly-Cabibbo-suppressed relative to amplitudes dominating the decay width, and it is also GIM-suppressed. Thus the short-distance mixing rate is tiny, and D^0-$\overline{D}^0$ mixing is expected to be dominated by long-distance processes. These are difficult to calculate reliably, and theoretical estimates for x and y range over two-three orders of magnitude[5, 6].

With the exception of $\psi(3770) \to DD$ measurements, all methods identify the flavor of the D^0

Received 25 February 2008
1) E-mail: schwartz@physics.uc.edu

or $\overline{\mathrm{D}}^0$ when produced by reconstructing the decay $\mathrm{D}^{*+} \to \mathrm{D}^0\pi^+$ or $\mathrm{D}^{*-} \to \overline{\mathrm{D}}^0\pi^-$; the charge of the accompanying pion identifies the D flavor. For signal decays, $M_{D^*} - M_{\mathrm{D}^0} - M_{\pi^+} \equiv Q \approx 6$ MeV, which is relatively close to the threshold. Thus analyses typically require that the reconstructed Q be small to suppress backgrounds. For time-dependent measurements, the D^0 decay time is calculated via $(d/p)\times M_{\mathrm{D}^0}$, where d is the distance between the D^* and D^0 decay vertices and p is the D^0 momentum. The D^* vertex position is taken to be at the primary vertex[3] ($\bar{\mathrm{p}}\mathrm{p}$) or is calculated from the intersection of the D^0 momentum vector with the beamspot profile ($\mathrm{e}^+\mathrm{e}^-$).

2 Input observables

The global fit determines central values and errors for eight underlying parameters using a χ^2 statistic constructed from 26 observables. The underlying parameters are $x, y, \delta, R_{\mathrm{D}}, A_{\mathrm{D}}, |q/p|, \phi$, and $\delta_{\mathrm{K}\pi\pi}$. The parameters x and y govern mixing, and the parameters A_{D}, $|q/p|$, and ϕ govern CPV. The parameter $\delta_{\mathrm{K}\pi\pi}$ is the strong phase difference between the amplitude $\mathcal{A}(\mathrm{D}^0 \to \mathrm{K}^+\pi^-\pi^0)$ evaluated at $M_{\mathrm{K}^+\pi^-} = M_{\mathrm{K}^*(890)}$, and the amplitude $\mathcal{A}(\mathrm{D}^0 \to \mathrm{K}^-\pi^+\pi^0)$ evaluated at $M_{\mathrm{K}^-\pi^+} = M_{\mathrm{K}^*(890)}$.

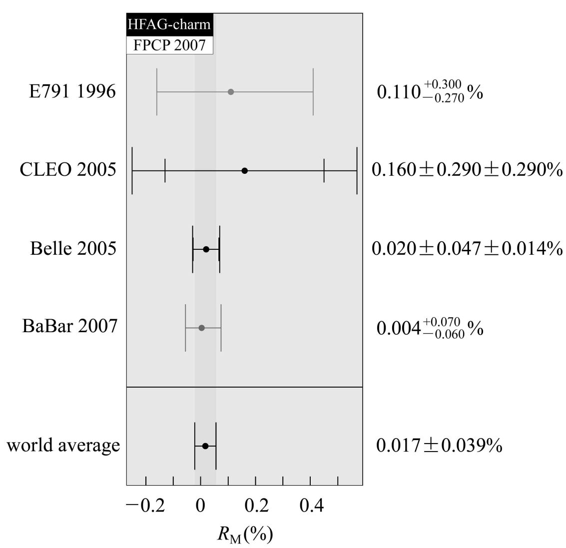

Fig. 1. WA value of R_{M} from Ref. [4], as calculated from $\mathrm{D}^0 \to \mathrm{K}^+\ell^-\nu$ measurements[7].

All input values are listed in Table 1. The observable $R_{\mathrm{M}} = (x^2+y^2)/2$ measured in $\mathrm{D}^0 \to \mathrm{K}^+\ell^-\nu$ decays[7] is taken to be the WA value[4] calculated by HFAG (see Fig. 1). The observables y_{CP} and A_{Γ} measured in $\mathrm{D}^0 \to \mathrm{K}^+\mathrm{K}^-/\pi^+\pi^-$ decays[1, 8] are also taken to be their WA values[4] (see Fig. 2). The observables from $\mathrm{D}^0 \to \mathrm{K}^0_{\mathrm{S}}\pi^+\pi^-$ decays[9] for no-CPV are HFAG WA values[4], but for the CPV-allowed case only Belle values are available. The $\mathrm{D}^0 \to \mathrm{K}^+\pi^-$ observables used are from Belle[10] and Babar[2], as these measurements have much greater precision than previously published $\mathrm{D}^0 \to \mathrm{K}^+\pi^-$ results. The $\mathrm{D}^0 \to \mathrm{K}^+\pi^-\pi^0$ and $\mathrm{D}^0 \to \mathrm{K}^+\pi^-\pi^+\pi^-$ results are from Babar[11], and the $\psi(3770) \to \mathrm{D}\mathrm{D}$ results are from CLEOc[12].

The relationships between the observables and the fitted parameters are listed in Table 2. For each set of correlated observables, we construct the difference vector $\boldsymbol{V}$, e.g., for $\mathrm{D}^0 \to \mathrm{K}^0_{\mathrm{S}}\pi^+\pi^-$ decays $\boldsymbol{V} = (\Delta x, \Delta y, \Delta|q/p|, \Delta\phi)$, where Δ represents the difference between the measured value and the fitted parameter value. The contribution of a set of measured observables to the χ^2 is calculated as $\boldsymbol{V}\cdot(M^{-1})\cdot\boldsymbol{V}^{\mathrm{T}}$, where M^{-1} is the inverse of the covariance matrix for the measurement. All covariance matrices used are listed in Table 1.

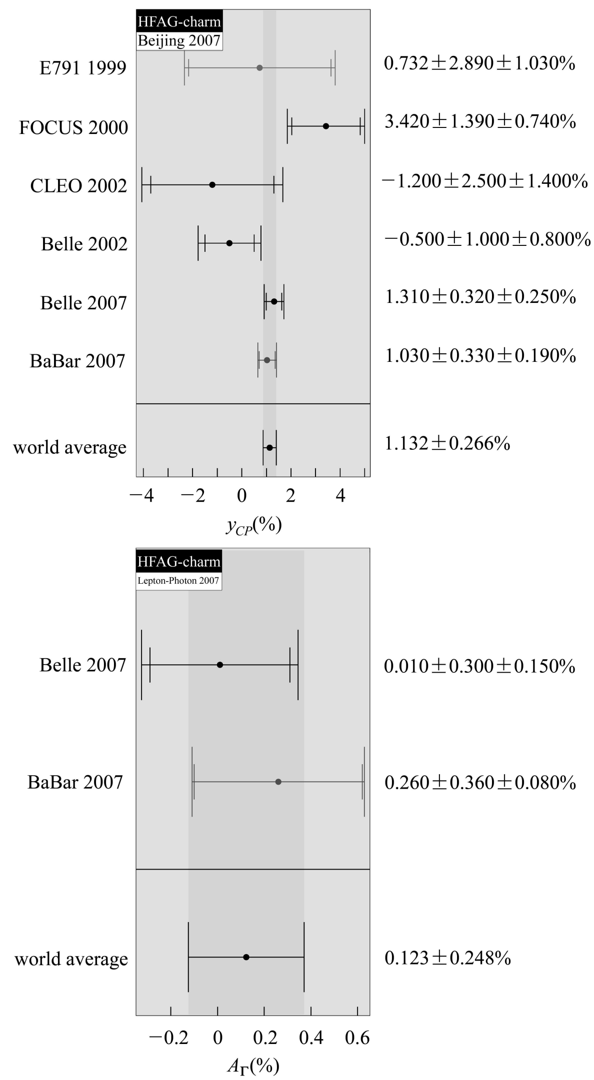

Fig. 2. WA values of y_{CP} (top) and A_{Γ} (bottom) from Ref. [4], as calculated from $\mathrm{D}^0 \to \mathrm{K}^+\mathrm{K}^-/\pi^+\pi^-$ measurements[1, 8].

Table 1. Observables used for the global fit, from Refs. [1, 2, 7—12].

observable	value	comment
y_{CP} A_Γ	$(1.132\pm0.266)\%$ $(0.123\pm0.248)\%$	WA $D^0\to K^+K^-/\pi^+\pi^-$ results[4]
x (no CPV) y (no CPV) $\lvert q/p\rvert$ (no direct CPV) ϕ (no direct CPV)	$(0.811\pm0.334)\%$ $(0.309\pm0.281)\%$ $0.95\pm0.22^{+0.10}_{-0.09}$ $(-0.035\pm0.19\pm0.09)$ rad	No CPV: WA $D^0\to K^0_S\pi^+\pi^-$ results[4]
x y $\lvert q/p\rvert$ ϕ	$(0.81\pm0.30^{+0.13}_{-0.17})\%$ $(0.37\pm0.25^{+0.10}_{-0.15})\%$ $0.86\pm0.30^{+0.10}_{-0.09}$ $(-0.244\pm0.31\pm0.09)$ rad	CPV-allowed: Belle $D^0\to K^0_S\pi^+\pi^-$ results. Correlation coefficients: $\begin{Bmatrix} 1 & -0.007 & -0.255\alpha & 0.216 \\ -0.007 & 1 & -0.019\alpha & -0.280 \\ -0.255\alpha & -0.019\alpha & 1 & -0.128\alpha \\ 0.216 & -0.280 & -0.128\alpha & 1 \end{Bmatrix}$ Note: $\alpha=(\lvert q/p\rvert+1)^2/2$ is a variable transformation factor
R_M	$(0.0173\pm0.0387)\%$	WA $D^0\to K^+\ell^-\nu$ results[4]
x'' y''	$(2.39\pm0.61\pm0.32)\%$ $(-0.14\pm0.60\pm0.40)\%$	Babar $D^0\to K^+\pi^-\pi^0$ result. Correlation coefficient $=-0.34$. Note: $x''\equiv x\cos\delta_{K\pi\pi}+y\sin\delta_{K\pi\pi}$, $y''\equiv y\cos\delta_{K\pi\pi}-x\sin\delta_{K\pi\pi}$.
R_M	$(0.019\pm0.0161)\%$	Babar $D^0\to K^+\pi^-\pi^+\pi^-$ result.
R_M y R_D $\sqrt{R_D}\cos\delta$	$(0.199\pm0.173\pm0.0)\%$ $(-5.207\pm5.571\pm2.737)\%$ $(-2.395\pm1.739\pm0.938)\%$ $(8.878\pm3.369\pm1.579)\%$	CLEOc results from "double-tagged" branching fractions measured in $\psi(3770)\to DD$ decays. Correlation coefficients: $\begin{Bmatrix} 1 & -0.0644 & 0.0072 & 0.0607 \\ -0.0644 & 1 & -0.3172 & -0.8331 \\ 0.0072 & -0.3172 & 1 & 0.3893 \\ 0.0607 & -0.8331 & 0.3893 & 1 \end{Bmatrix}$ Note: the only external input to these fit results are branching fractions.
R_D x'^{2+} y'^{+}	$(0.303\pm0.0189)\%$ $(-0.024\pm0.052)\%$ $(0.98\pm0.78)\%$	Babar $D^0\to K^+\pi^-$ results. Correlation coefficients: $\begin{Bmatrix} 1 & 0.77 & -0.87 \\ 0.77 & 1 & -0.94 \\ -0.87 & -0.94 & 1 \end{Bmatrix}$
A_D x'^{2-} y'^{-}	$(-2.1\pm5.4)\%$ $(-0.020\pm0.050)\%$ $(0.96\pm0.75)\%$	Babar $D^0\to K^+\pi^-$ results. Correlation coefficients same as above.
R_D x'^{2+} y'^{+}	$(0.364\pm0.018)\%$ $(0.032\pm0.037)\%$ $(-0.12\pm0.58)\%$	Belle $D^0\to K^+\pi^-$ results. Correlation coefficients: $\begin{Bmatrix} 1 & 0.655 & -0.834 \\ 0.655 & 1 & -0.909 \\ -0.834 & -0.909 & 1 \end{Bmatrix}$
A_D x'^{2-} y'^{-}	$(2.3\pm4.7)\%$ $(0.006\pm0.034)\%$ $(0.20\pm0.54)\%$	Belle $D^0\to K^+\pi^-$ results. Correlation coefficients same as above.

3 Fit results

The global fit uses MINUIT with the MIGRAD minimizer, and all errors are obtained from MINOS. Three separate fits are performed: (a) assuming CP conservation (A_D and ϕ are fixed to zero, $|q/p|$ is fixed to one); (b) assuming no direct CPV (A_D is fixed to zero); and (c) allowing full CPV (all parameters floated). The results are listed in Table 3. For the CPV-allowed fit, individual contributions to the χ^2 are listed in Table 4. The total χ^2 is 23.5 for $26-8=18$ degrees of freedom; this corresponds to a confidence level of 0.17.

Confidence contours in the two dimensions (x,y) or in $(|q/p|,\phi)$ are obtained by letting, for any point in the two-dimensional plane, all other fitted parameters take their preferred values. The resulting 1σ—5σ contours are shown in Fig. 3 for the CP-conserving case, and in Fig. 4 for the CPV-allowed case. The contours are determined from the increase of the χ^2 above the minimum value. One observes that the (x,y) contours for no-CPV and for CPV-allowed are almost identical. In both cases the χ^2 at the no-mixing point $(x,y)=(0,0)$ is 49 units above the minimum value; this has a confidence level corresponding to 6.7σ.

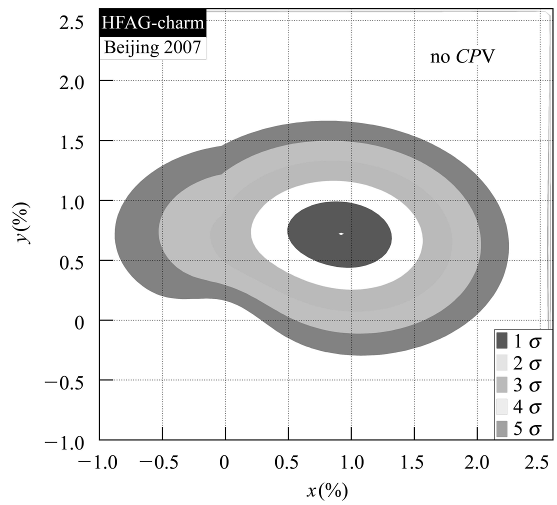

Fig. 3. Two-dimensional contours for mixing parameters (x,y), for no CPV.

Table 2. Left: decay modes used to determine fitted parameters x, y, δ, $\delta_{K\pi\pi}$, R_D, A_D, $|q/p|$, and ϕ. Middle: the observables measured for each decay mode. Right: the relationships between the observables measured and the fitted parameters.

decay mode	observables	relationship
$D^0\to K^+K^-/\pi^+\pi^-$	y_{CP} A_Γ	$2y_{CP}=(\lvert q/p\rvert+\lvert p/q\rvert)\,y\cos\phi\ -\ (\lvert q/p\rvert-\lvert p/q\rvert)\,x\sin\phi$ $2A_\Gamma=(\lvert q/p\rvert-\lvert p/q\rvert)\,y\cos\phi\ -\ (\lvert q/p\rvert+\lvert p/q\rvert)\,x\sin\phi$
$D^0\to K^0_S\pi^+\pi^-$	x y $\lvert q/p\rvert$ ϕ	
$D^0\to K^+\ell^-\nu$	R_M	$R_M=(x^2+y^2)/2$
$D^0\to K^+\pi^-\pi^0$ (dalitz plot analysis)	x'' y''	$x''=x\cos\delta_{K\pi\pi}+y\sin\delta_{K\pi\pi}$ $y''=y\cos\delta_{K\pi\pi}-x\sin\delta_{K\pi\pi}$
$D^0\to K^+\pi^-\pi^+\pi^-$	R_M	$R_M=(x^2+y^2)/2$
"double-tagged" branching fractions measured in $\psi(3770)\to DD$ decays	R_M y R_D $\sqrt{R_D}\cos\delta$	$R_M=(x^2+y^2)/2$
$D^0\to K^+\pi^-$	$R_D^+,\ R_D^-$ $x'^{2+},\ x'^{2-}$ $y'^+,\ y'^-$	$R_D=(R_D^++R_D^-)/2$ $A_D=(R_D^+-R_D^-)/(R_D^++R_D^-)$ $x'=x\cos\delta+y\sin\delta$ $y'=y\cos\delta-x\sin\delta$ $A_M\equiv(\lvert q/p\rvert^4-1)/(\lvert q/p\rvert^4+1)$ $x'^{\pm}=[(1\pm A_M)/(1\mp A_M)]^{1/4}(x'\cos\phi\pm y'\sin\phi)$ $y'^{\pm}=[(1\pm A_M)/(1\mp A_M)]^{1/4}(y'\cos\phi\mp x'\sin\phi)$

Table 3. Results of the global fit for different assumptions concerning CPV.

parameter	no CPV	no direct CPV	CPV-allowed	CPV-allowed 95% C.L.
$x(\%)$	$0.98^{+0.26}_{-0.27}$	$0.97^{+0.27}_{-0.29}$	$0.97^{+0.27}_{-0.29}$	[0.39, 1.48]
$y(\%)$	0.75 ± 0.18	$0.78^{+0.18}_{-0.19}$	$0.78^{+0.18}_{-0.19}$	[0.41, 1.13]
$\delta/(^\circ)$	$21.6^{+11.6}_{-12.6}$	$23.4^{+11.6}_{-12.5}$	$21.9^{+11.5}_{-12.5}$	[−6.3, 44.6]
$R_D(\%)$	0.335 ± 0.009	0.334 ± 0.009	0.335 ± 0.009	[0.316, 0.353]
$A_D(\%)$	–	–	-2.2 ± 2.5	[−7.10, 2.67]
$\lvert q/p\rvert$	–	$0.95^{+0.15}_{-0.14}$	$0.86^{+0.18}_{-0.15}$	[0.59, 1.23]
$\phi/(^\circ)$	–	$-2.7^{+5.4}_{-5.8}$	$-9.6^{+8.3}_{-9.5}$	[−30.3, 6.5]
$\delta_{K\pi\pi}/(^\circ)$	$30.8^{+25.0}_{-25.8}$	$32.5^{+25.0}_{-25.7}$	$32.4^{+25.1}_{-25.8}$	[−20.3, 82.7]

Table 4. Individual contributions to the χ^2 for the CPV-allowed fit.

observable	χ^2	$\sum\chi^2$
y_{CP}	2.06	2.06
A_Γ	0.10	2.16
$x_{K^0\pi^+\pi^-}$	0.20	2.36
$y_{K^0\pi^+\pi^-}$	1.94	4.30
$\lvert q/p\rvert_{K^0\pi^+\pi^-}$	0.00	4.30
$\phi_{K^0\pi^+\pi^-}$	0.46	4.76
$R_M(K^+\ell^-\nu)$	0.06	4.83
$x_{K^+\pi^-\pi^0}$	1.24	6.06
$y_{K^+\pi^-\pi^0}$	1.62	7.69
$R_M/y/R_D/\sqrt{R_D}\cos\delta$ (CLEOc)	5.59	13.28
$R_D^+/x'^{2+}/y'^+$ (Babar)	2.54	15.82
$R_D^-/x'^{2-}/y'^-$ (Babar)	1.75	17.57
$R_D^+/x'^{2+}/y'^+$ (Belle)	3.96	21.53
$R_D^-/x'^{2-}/y'^-$ (Belle)	1.43	22.95
$R_M(K^+\pi^-\pi^+\pi^-)$	0.49	23.45

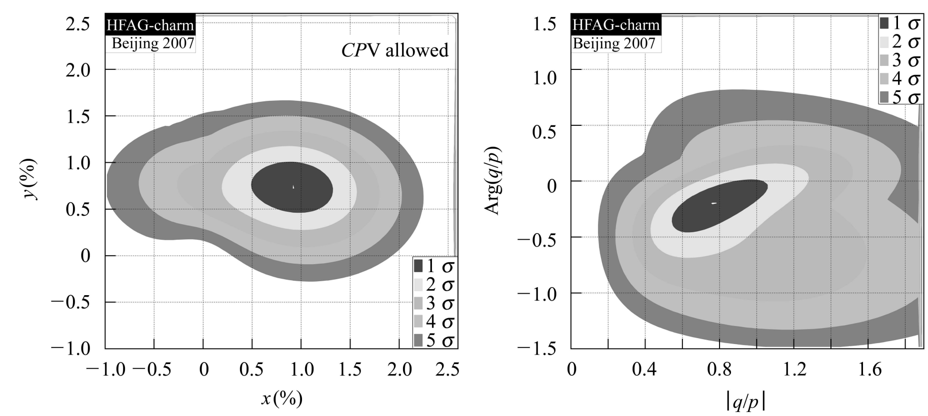

Fig. 4. Two-dimensional contours for parameters (x,y) (left) and $(|q/p|,\phi)$ (right), allowing for CPV.

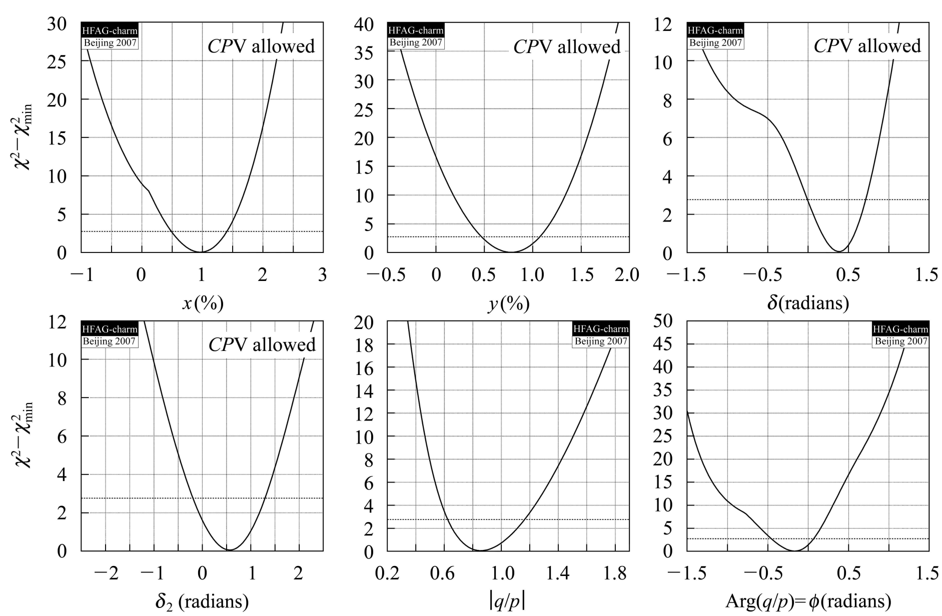

Fig. 5. The function $\Delta\chi^2=\chi^2-\chi^2_{\rm min}$ for fitted parameters x, y, δ, $\delta_{K\pi\pi}$, $|q/p|$, and ϕ. The points where $\Delta\chi^2=2.70$ (denoted by the dashed horizontal line) determine a 90% C.L. interval.

Thus, no mixing is excluded at this high level. In the $(|q/p|,\phi)$ plot, the point (1,0) is on the boundary of the 1σ contour; thus the data is consistent with CP conservation.

One-dimensional confidence curves for individual parameters are obtained by letting, for any value of the parameter, all other fitted parameters take their preferred values. The resulting functions $\Delta\chi^2 = \chi^2 - \chi^2_{\rm min}$ (where $\chi^2_{\rm min}$ is the minimum value) are shown in Fig. 5. The points where $\Delta\chi^2 = 2.70$ determine 90% C.L. intervals for the parameters as shown in the figure. The points where $\Delta\chi^2 = 3.84$ determine 95% C.L. intervals; these are listed in Table 3.

4 Conclusions

From the global fit results listed in Table 3 and shown in Figs. 4 and 5, we conclude the following:

1) the experimental data consistently indicate that D^0 mesons undergo mixing. The no-mixing point $x = y = 0$ is excluded at 6.7σ. The parameter x differs from zero by 3.0σ; the parameter y differs from zero by 4.1σ. The effect is presumably dominated by long-distance processes, which are difficult to calculate. Thus unless $|x| \gg |y|$ (see Ref. [5]), it may be difficult to identify new physics from mixing alone.

2) Since y_{CP} is positive, the CP-even state is shorter-lived, as in the K^0-$\overline{K}^0$ system. However, since x also appears to be positive, the CP-even state is heavier, unlike in the K^0-$\overline{K}^0$ system.

3) It appears difficult to accomodate a strong phase difference δ larger than 45°.

4) There is no evidence yet for CPV in the D^0-$\overline{D}^0$ system. Observing CPV at the level of sensitivity of the current experiments would indicate new physics.

References

1 Staric M et al. (Belle). Phys. Rev. Letts., 2007, **98**: 211803
2 Aubert B et al. (Babar). Phys. Rev. Letts., 2007, **98**: 211802
3 Aaltonen T et al. (CDF). arXiv: 0712.1567
4 http://www.slac.stanford.edu/xorg/hfag/charm/index.html
5 Bigi I, Uraltsev N. Nucl. Phys. B, 2001, **592**: 92
6 Petrov A A. Charm Physics: Theoretical Review. eConf, 2003, **C030603**. arXiv: hep-ph/0311371; Nucl. Phys. Proc. Suppl., 2005, **142**: 333
7 Aitala E M et al. (E791). Phys. Rev. Letts., 1996, **77**: 2384; Cawlfield C et al. (CLEO). Phys. Rev. D, 2005, **71**: 071101; Bitenc U et al. (Belle). Phys. Rev. D, 2005, **72**: 071101; Aubert B et al. (Babar). Phys. Rev. D, 2007, **76**: 014018
8 Aitala E M et al. (E791). Phys. Rev. Letts., 1999, **83**: 32; Link J M et al. (FOCUS). Phys. Letts. B, 2000, **485**: 62; Csorna S E et al. (CLEO). Phys. Rev. D, 2002, **65**: 092001; Abe K et al. (Belle). Phys. Rev. Letts., 2002, **88**: 162001; Aubert B et al. (Babar). arXiv: 0712.2249
9 Asner D M et al. (CLEO). Phys. Rev. D, 2005, **72**: 012001; arXiv: hep-ex / 0503045 (revised April 2007); ZHANG L M et al. (Belle). Phys. Rev. Letts., 2007, **99**: 131803
10 ZHANG L M et al. (Belle). Phys. Rev. Letts., 2006, **96**: 151801
11 Aubert B et al. (Babar). arXiv: hep-ex / 0607090; Lockman W (Babar). presented at LP'07, Daegu, S. Korea (13 August 2007); see also: TIAN X C et al. (Belle). Phys. Rev. Letts., 2005, **95**: 231801
12 Asner D M et al. (CLEOc). arXiv: 0802.2268