Two-pion interferometry for a partially coherent evolution source*

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Abstract We give the formulas of two-pion Hanbury-Brown-Twiss (HBT) correlation function for a partially coherent evolution pion-emitting source, using quantum probability amplitudes in a path-integral formalism. The multiple scattering of the particles in the source is taken into consideration based on Glauber scattering theory. Two-pion interferometry with effects of the multiple scattering and source collective expansion is examined for a partially coherent source of hadronic gas with a finite baryon density and evolving hydrodynamically. We do not find observable effect of either the multiple scattering or the source collective expansion on HBT chaotic parameter.

Key words HBT correlation function, partially coherent evolution source, multiple scattering, source collective expansion

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1 Introduction

The goal of the study of high-energy heavy-ion collisions is to obtain the information of the particle-emitting sources produced in the collisions. Pion interferometry is an important tool for studying the space-time structure and the coherence of the emitting source[1—3]. As a produced pion propagates in the source, it is subject to the collective motion of the source, which can be described by a long-ranged mean-field interaction, and the multiple scattering with the particles in the source, which can be described by short-ranged interactions. Recently, Wong put forward formulas of two-pion interferometry for a chaotic evolution particle-emitting source, using quantum probability amplitudes in a path-integral formalism[4—6]. Based on these formulas, Zhang et al. calculated the two-pion correlation function for a chaotic expanding source of hadronic gas with a finite baryon density and investigated the effects of multiple scattering and source collective expansion on HBT radius[7]. They found that the effects of multiple scattering and source collective expansion lead to HBT radius between the radii extracted from the initial source configuration and the source freeze-out configuration[7]. In this article, we shall follow the work mentioned above to derive the two-pion HBT correlation function for a partially coherent evolution source and investigate the effects of multiple scattering and source collective expansion on the HBT interferometry results for the source.

2 Theory

The two-pion HBT correlation function is defined as

$$C(\boldsymbol{k}_1,\boldsymbol{k}_2)=\frac{P(\boldsymbol{k}_1,\boldsymbol{k}_2)}{P(\boldsymbol{k}_1)P(\boldsymbol{k}_2)}, \tag{1}$$

where $P(\boldsymbol{k})$ is the probability for observing a pion with momentum $\boldsymbol{k}$ (single-particle momentum distribution), and $P(\boldsymbol{k}_1,\boldsymbol{k}_2)$ is the probability for observing two pions with momentum $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ (two-particle momentum distribution). For a partially coherent source, the 4-dimension source density can be

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expressed as[1]

$$\rho(x) = \rho_\chi(x) + \rho_c(x)\,, \tag{2}$$

where $\rho_\chi(x)$ and $\rho_c(x)$ are the densities of the chaotic and coherent components of the source. We assume that the density of the chaotic component is proportional to the total density of the partially coherent source[1]

$$\rho_\chi(x) = \chi\,\rho(x)\,, \tag{3}$$

where χ is called the chaotic factor, and the density of the coherent component is then

$$\rho_c(x) = (1-\chi)\,\rho(x)\,. \tag{4}$$

2.1 Single-particle momentum distribution

In order to obtain the single-pion momentum distribution $P(\boldsymbol{k})$, we need to know the probability amplitude $\Psi(\boldsymbol{k},x_d)$, for a pion to be produced from the source with momentum $\boldsymbol{k}$, and to arrive at the detection point x_d. The probability amplitude of a pion produced at the source point x with momentum $\boldsymbol{\kappa}$ and detected at x_d with momentum $\boldsymbol{k}$ can be expressed as

$$\psi(\boldsymbol{\kappa}x \to \boldsymbol{k}x_d) = A(\boldsymbol{\kappa}(x),x)\mathrm{e}^{\mathrm{i}\phi_0(x)}K(\boldsymbol{\kappa}x \to \boldsymbol{k}x_d)\,, \tag{5}$$

where $A(\boldsymbol{\kappa}(x),x)$ and $\phi_0(x)$ are the production amplitude and phase, and $K(\boldsymbol{\kappa}x \to \boldsymbol{k}x_d)$ is the propagation probability amplitude which can be expressed as[4—6]

$$K(\boldsymbol{\kappa}x \to \boldsymbol{k}x_d) = \int \mathrm{d}\{x'\}\mathrm{e}^{\mathrm{i}S(\boldsymbol{\kappa}x\to\boldsymbol{k}x_d;x')}\,, \tag{6}$$

where $S(\boldsymbol{\kappa}x \to \boldsymbol{k}x_d;x')$ is the propagation action along a possible path $\{x'\}$ from x to x_d, and $\int \mathrm{d}\{x'\}\cdots$ denotes the sum over all the possible paths. In Eq. (6), the dominant contribution in the sum is from the trajectory along the classical path $\{x'_c\}$, for the other trajectory contributions may cancel out each other in a great degree[1, 4—6]. Based on the Glauber scattering theory[8], each collision of the pion with the medium particles in the source will add a phase factor to the propagation probability amplitude[4—6]. While the source collective expansion will change the pion momentum from $\boldsymbol{\kappa}$ to $\boldsymbol{k}$ when it propagates in the source. Therefore, under the approximation of classical path one has[4—6]

$$K(\boldsymbol{\kappa}x \to \boldsymbol{k}x_d) = \mathrm{e}^{\mathrm{i}S(\boldsymbol{\kappa}x\to\boldsymbol{k}x_d;x'_c)} = \mathrm{e}^{\mathrm{i}\overline{\phi}_s(x\to x_f;x'_c)} \times \exp\left\{-\mathrm{i}\int_x^{x_f}\boldsymbol{\kappa}(x'_c)\cdot\mathrm{d}x'_c - \mathrm{i}\boldsymbol{k}\cdot(x_d-x_f)\right\}, \tag{7}$$

where $\overline{\phi}_s(x \to x_f;x'_c)$ is the phase shift associated with the multiple scattering, which is the sum over all phase shifts of individual collisions of the pion with the medium particles along the classical path $\{x'_c\}$ in the source, x_f is the freeze-out point on the integral path, and $\boldsymbol{\kappa}(x')$ is the momentum of the pion in the source, $\boldsymbol{\kappa}(x'_f) = \boldsymbol{k}$.

First we consider the extended source as a discrete source. The probability amplitude of a pion with momentum $\boldsymbol{k}$ produced from the source and detected at x_d, $\Psi(\boldsymbol{k},x_d)$, can be obtained by summing $\psi(\boldsymbol{\kappa}x \to \boldsymbol{k}x_d)$ over all x source points,

$$\Psi(\boldsymbol{k},x_d) = \sum_x A(\boldsymbol{\kappa}(x),x)\mathrm{e}^{\mathrm{i}\phi_0(x)}\mathrm{e}^{\mathrm{i}\overline{\phi}_s(x\to x_f;x'_c)} \times \exp\left\{-\mathrm{i}\int_x^{x_f}\boldsymbol{\kappa}(x'_c)\cdot\mathrm{d}x'_c - \mathrm{i}\boldsymbol{k}\cdot(x_d-x_f)\right\}. \tag{8}$$

For a continuous extended source, the summation should be transcribed as an integral over x, $\sum_x \to \int \mathrm{d}^4x\,\rho(x)$. We divid the summation in Eq. (8) into the coherent part $\sum^c$ and the chaotic part $\sum^\chi$. For simplicity we let the production phases associated with the coherent source be zero[1] in the following derivation, and denote $\overline{\phi}_s(x \to x_f;x'_c)$ as $\overline{\phi}_s(x)$ and x'_c as x'. Eq. (8) becomes

$$\begin{aligned}
\Psi(\boldsymbol{k},x_d) = &\\
&\left\{\left[\sum_x{}^{c} A(\boldsymbol{\kappa}(x),x)\,\mathrm{e}^{\mathrm{i}\phi_0^c(x)} + \sum_x{}^{\chi} A(\boldsymbol{\kappa}(x),x)\mathrm{e}^{\mathrm{i}\phi_0^\chi(x)}\right] \times\right.\\
&\left.\mathrm{e}^{\mathrm{i}\overline{\phi}_s(x)}\exp\left\{-\mathrm{i}\int_x^{x_f}\boldsymbol{\kappa}(x')\cdot\mathrm{d}x' - \mathrm{i}\boldsymbol{k}\cdot(x_d-x_f)\right\}\right\} =\\
&\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot x_d}\left\{\left[\int\mathrm{d}^4x\,\rho_c(x)A(\boldsymbol{\kappa}(x),x) + \sum_x{}^{\chi} A(\boldsymbol{\kappa}(x),x)\times\right.\right.\\
&\left.\left.\mathrm{e}^{\mathrm{i}\phi_0^\chi(x)}\right]\mathrm{e}^{\mathrm{i}\overline{\phi}_s(x)}\exp\left\{-\mathrm{i}\int_x^{x_f}\boldsymbol{\kappa}(x')\cdot\mathrm{d}x' + \mathrm{i}\boldsymbol{k}\cdot x_f\right\}\right\} =\\
&\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot x_d}\left\{(1-\chi)\widetilde{\rho}(\boldsymbol{k}) + \sum_x{}^{\chi} A(\boldsymbol{\kappa}(x),x)\mathrm{e}^{\mathrm{i}\phi_0^\chi(x)}\times\right.\\
&\left.\mathrm{e}^{\mathrm{i}\overline{\phi}_s(x)}\exp\left\{-\mathrm{i}\int_x^{x_f}\boldsymbol{\kappa}(x')\cdot\mathrm{d}x' + \mathrm{i}\boldsymbol{k}\cdot x_f\right\}\right\},
\end{aligned} \tag{9}$$

where

$$\widetilde{\rho}(\boldsymbol{k}) = \int\mathrm{d}^4x\rho(x)A(\boldsymbol{\kappa}(x),x)\mathrm{e}^{\mathrm{i}\overline{\phi}_s(x)} \times \exp\left\{-\mathrm{i}\int_x^{x_f}\boldsymbol{\kappa}(x')\cdot\mathrm{d}x' + \mathrm{i}\boldsymbol{k}\cdot x_f\right\}. \tag{10}$$

The single-pion momentum distribution $P(\boldsymbol{k})$ is the absolute square of the amplitude $\Psi(\boldsymbol{k},x_d)$. From Eq. (9), we expand the absolute square of the probability amplitude into terms which are independent of ϕ_0^χ and terms which contain ϕ_0^χ. Terms which depend on the production phase ϕ_0^χ give zero contribution because of the randomness of the production phases associated with the chaotic source. Therefore, we have

$$P(\boldsymbol{k}) = \left|\Psi(\boldsymbol{k},x_d)\right|^2 = (1-\chi)^2\left|\widetilde{\rho}(\boldsymbol{k})\right|^2 + \chi\int \mathrm{d}^4x\, \mathrm{e}^{-2\mathrm{Im}\overline{\phi}_s(x)}\rho(x)A^2(\boldsymbol{\kappa}(x),x)\,. \quad (11)$$

It can be seen that the effect of multiple scattering behaves as an absorption factor, $\mathrm{e}^{-2\mathrm{Im}\overline{\phi}_s(x)}$, to the source density $\rho(x)$[4—7].

2.2 Two-particle momentum distribution

We now derive the probability amplitude, $\Psi(\boldsymbol{k}_1,x_{d1};\boldsymbol{k}_2,x_{d2})$, of two pions with momentum $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ produced from the source and detected at x_{d1} and x_{d2}. Because of the Bose-Einstein statistics of identical bosons, the wave function of two-pions must be symmetrical with respect to the interchange of the labels of source points x_1 and x_2. $\Psi(\boldsymbol{k}_1,x_{d1};\boldsymbol{k}_2,x_{d2})$ can be written as

$$\begin{aligned}&\Psi(\boldsymbol{k}_1,x_{d1};\boldsymbol{k}_2,x_{d2}) = \\ &\frac{1}{\sqrt{2}}\sum_{x_1,x_2}\mathrm{e}^{\mathrm{i}\phi_0(x_1)+\mathrm{i}\phi_0(x_2)}\times \\ &\mathrm{e}^{\mathrm{i}\overline{\phi}_s(x_1)+\mathrm{i}\overline{\phi}_s(x_2)}\Big\{A(\boldsymbol{\kappa}_1(x_1),x_1)A(\boldsymbol{\kappa}_2(x_2),x_2)\times \\ &\exp\left\{-\mathrm{i}\int_{x_1}^{x_{f1}}\boldsymbol{\kappa}_1(x')\cdot\mathrm{d}x' - \mathrm{i}\boldsymbol{k}_1\cdot(x_{d1}-x_{f1})\right\}\times \\ &\exp\left\{-\mathrm{i}\int_{x_2}^{x_{f2}}\boldsymbol{\kappa}_2(x')\cdot\mathrm{d}x' - \mathrm{i}\boldsymbol{k}_2\cdot(x_{d2}-x_{f2})\right\} + \\ &A(\boldsymbol{\kappa}_1(x_2),x_2)A(\boldsymbol{\kappa}_2(x_1),x_1)\times \\ &\exp\left\{-\mathrm{i}\int_{x_2}^{x_{f2}}\boldsymbol{\kappa}_1(x')\cdot\mathrm{d}x' - \mathrm{i}\boldsymbol{k}_1\cdot(x_{d1}-x_{f2})\right\}\times \\ &\exp\left\{-\mathrm{i}\int_{x_1}^{x_{f1}}\boldsymbol{\kappa}_2(x')\cdot\mathrm{d}x' - \mathrm{i}\boldsymbol{k}_2\cdot(x_{d2}-x_{f1})\right\}\Big\}. \quad (12)\end{aligned}$$

Dividing the summation in Eq. (12) into chaotic part and coherent part, and letting the production phases associated with the coherent source be zero, we obtain

$$\begin{aligned}&\Psi(\boldsymbol{k}_1,\, x_{d1};\boldsymbol{k}_2,x_{d2}) = \frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\boldsymbol{k}_1\cdot x_{d1}}\mathrm{e}^{-\mathrm{i}\boldsymbol{k}_2\cdot x_{d2}}\times \\ &\Big\{2(1-\chi)^2\widetilde{\rho}(\boldsymbol{k}_1)\widetilde{\rho}(\boldsymbol{k}_2) + 2(1-\chi)\widetilde{\rho}(\boldsymbol{k}_1)\times \\ &\sum_{x_2}^{\chi}\mathrm{e}^{\mathrm{i}\phi_0^{\chi}(x_2)}\mathrm{e}^{\mathrm{i}\overline{\phi}_s(x_2)}A(\boldsymbol{\kappa}_2(x_2),x_2)M_{22} + \\ &2(1-\chi)\widetilde{\rho}(\boldsymbol{k}_2)\sum_{x_1}^{\chi}\mathrm{e}^{\mathrm{i}\phi_0^{\chi}(x_1)}\mathrm{e}^{\mathrm{i}\overline{\phi}_s(x_1)}\times \\ &A(\boldsymbol{\kappa}_1(x_1),x_1)M_{11} + \\ &\sum_{x_1,x_2}^{\chi}\mathrm{e}^{\mathrm{i}\phi_0^{\chi}(x_1)}\mathrm{e}^{\mathrm{i}\overline{\phi}_s(x_1)}\mathrm{e}^{\mathrm{i}\phi_0^{\chi}(x_2)}\mathrm{e}^{\mathrm{i}\overline{\phi}_s(x_2)}\times \\ &\Big\{A(\boldsymbol{\kappa}_1(x_1),x_1)A(\boldsymbol{\kappa}_2(x_2),x_2)M_{11}M_{22} + \\ &A(\boldsymbol{\kappa}_2(x_1),x_1)A(\boldsymbol{\kappa}_1(x_2),x_2)M_{12}M_{21}\Big\}\Big\}, \quad (13)\end{aligned}$$

where

$$M_{11} = \exp\left\{-\mathrm{i}\int_{x_1}^{x_{f1}}\boldsymbol{\kappa}_1(x')\cdot\mathrm{d}x' + \mathrm{i}\boldsymbol{k}_1\cdot x_{f1}\right\}, \quad (14)$$

$$M_{22} = \exp\left\{-\mathrm{i}\int_{x_2}^{x_{f2}}\boldsymbol{\kappa}_2(x')\cdot\mathrm{d}x' + \mathrm{i}\boldsymbol{k}_2\cdot x_{f2}\right\}, \quad (15)$$

$$M_{12} = \exp\left\{-\mathrm{i}\int_{x_2}^{x_{f2}}\boldsymbol{\kappa}_1(x')\cdot\mathrm{d}x' + \mathrm{i}\boldsymbol{k}_1\cdot x_{f2}\right\}, \quad (16)$$

$$M_{21} = \exp\left\{-\mathrm{i}\int_{x_1}^{x_{f1}}\boldsymbol{\kappa}_2(x')\cdot\mathrm{d}x' + \mathrm{i}\boldsymbol{k}_2\cdot x_{f1}\right\}. \quad (17)$$

Because the randomness of the production phases which associated with the chaotic source, we have

$$\begin{aligned}P(\boldsymbol{k}_1,\,\boldsymbol{k}_2) &= \frac{1}{2}\left|\Psi(\boldsymbol{k}_1,x_{d1};\boldsymbol{k}_2,x_{d2})\right|^2 = (1-\chi)^4\times \\ &\left|\widetilde{\rho}(\boldsymbol{k}_1)\widetilde{\rho}(\boldsymbol{k}_2)\right|^2 + (1-\chi)^2\Big\{\left|\widetilde{\rho}(\boldsymbol{k}_1)\right|^2P_{\chi}(\boldsymbol{k}_2) + \\ &\left|\widetilde{\rho}(\boldsymbol{k}_2)\right|^2P_{\chi}(\boldsymbol{k}_1)\Big\} + (1-\chi)^2\widetilde{\rho}(\boldsymbol{k}_1)\widetilde{\rho}^*(\boldsymbol{k}_2)\times \\ &\int\mathrm{d}^4x_2\,\rho_{\chi}(x_2)A(\boldsymbol{\kappa}_2(x_2),x_2)A(\boldsymbol{\kappa}_1(x_2),x_2)\times \\ &\mathrm{e}^{-2\mathrm{Im}\overline{\phi}_s(x_2)}M_{22}M_{12}^* + (1-\chi)^2\widetilde{\rho}(\boldsymbol{k}_2)\widetilde{\rho}^*(\boldsymbol{k}_1)\times \\ &\int\mathrm{d}^4x_1\,\rho_{\chi}(x_1)A(\boldsymbol{\kappa}_1(x_1),x_1)A(\boldsymbol{\kappa}_2(x_1),x_1)\times \\ &\mathrm{e}^{-2\mathrm{Im}\overline{\phi}_s(x_1)}M_{11}M_{21}^* + P_{\chi}(\boldsymbol{k}_1)P_{\chi}(\boldsymbol{k}_2) + \\ &\left|\int\mathrm{d}^4x\,\mathrm{e}^{\mathrm{i}(\boldsymbol{k}_1-\boldsymbol{k}_2)\cdot x + \mathrm{i}\phi_{\mathrm{mf}}(x,\boldsymbol{k}_1\boldsymbol{k}_2) - 2\mathrm{Im}\overline{\phi}_s(x)}\times\right. \\ &\left.\rho_{\chi}(x)A(\boldsymbol{\kappa}_1(x),x)A(\boldsymbol{\kappa}_2(x),x)\right|^2, \quad (18)\end{aligned}$$

where

$$P_{\chi}(\boldsymbol{k}) = \chi\int\mathrm{d}^4x\,\mathrm{e}^{-2\mathrm{Im}\overline{\phi}_s(x)}\rho(x)A^2(\boldsymbol{\kappa}(x),x)\,, \quad (19)$$

and

$$\phi_{\mathrm{mf}}(x,\boldsymbol{k}_1\boldsymbol{k}_2) = -\int_x^{x_f}\left\{[\boldsymbol{\kappa}_1(x') - \boldsymbol{\kappa}_2(x')] - [\boldsymbol{k}_1 - \boldsymbol{k}_2]\right\}\cdot\mathrm{d}x'\,. \quad (20)$$

From Eqs. (1), (11), and (18), we can obtain the two-pion correlation function $C(\boldsymbol{k}_1,\boldsymbol{k}_2)$ for the partially coherent source, which includes the effects of multiple scattering and source collective motion. For a completely coherent source, $\chi=0$, the correlation function will be equal to unit. While for a completely chaotic source, $\chi=1$, $C(\boldsymbol{k}_1,\boldsymbol{k}_2)$ will become the two-pion correlation function as in Ref. [4].

3 Model calculation

We consider a pion-emitting source of hadronic gas with a finite baryon density produced in the high-energy heavy-ion collisions at AGS energies. As in Ref. [7], we assume that the hadronic gas consist of only nucleons, $\Delta(1232)$, and pions, and the source has a spherical geometry for simplicity[7]. We use relativistic hydrodynamics to describe the source evolution. The energy-momentum tensor of a thermalized fluid element in the center-of-mass frame of the source is[9, 10]

$$T^{\mu\nu}(x)=\left[\epsilon(x)+p(x)\right]u^{\mu}(x)u^{\nu}(x)-p(x)g^{\mu\nu}\,, \quad (21)$$

where ϵ, p, and $u^{\mu}=\gamma(1,v)$ are respectively the energy density, pressure, and 4-velocity of the element, and $g^{\mu\nu}$ is the metric tensor. The local conservation of energy and momentum can be expressed by

$$\partial_{\mu}T^{\mu\nu}(x)=0, \qquad (\nu=0,1,2,3). \quad (22)$$

The conservation of baryon number gives

$$\partial_{\mu}j_{\rm b}^{\mu}(x)=0, \qquad (j_{\rm b}^{\mu}=n_{\rm b}u^{\mu}), \quad (23)$$

where $j_{\rm b}^{\mu}$ is the 4-current-density of baryon and $n_{\rm b}$ is baryon density. In the thermalized element, the number density n_i, the energy density ϵ_i, and the pressure p_i of the particle species i can be expressed in terms of local temperature $T(x)$ and local chemical potencial $\mu_i(x)$ by

$$n_i=\frac{4\pi g_i}{(2\pi)^3}\int_{m_i}^{\infty}f_iE\sqrt{E^2-m_i^2}\,{\rm d}E\,, \quad (24)$$

$$\epsilon_i=\frac{4\pi g_i}{(2\pi)^3}\int_{m_i}^{\infty}f_iE^2\sqrt{E^2-m_i^2}\,{\rm d}E\,, \quad (25)$$

$$p_i=\frac{1}{3}\frac{4\pi g_i}{(2\pi)^3}\int_{m_i}^{\infty}f_i(E^2-m_i^2)^{3/2}\,{\rm d}E\,, \quad (26)$$

where

$$f_i=\frac{1}{\exp[(E-\mu_i)/T]\pm 1}\,, \quad (27)$$

g_i and m_i are the internal freedom and mass of particle species i, and the sign (+) or (−) is for fermions or bosons. The fluid energy density ϵ and pressure p are the sum

$$\epsilon=\sum_i\epsilon_i, \qquad p=\sum_i p_i\,. \quad (28)$$

For a given set of variable (T,μ_i) $(i=1,2,\cdots)$, we can obtain the equation of state $p=p(\epsilon,n_{\rm b})$ of the hadronic gas from Eqs. (24)—(28). With the equation of state and initial conditions of the source, one can solve the equations of motion (22) and (23), and finally obtain the space-time variations of density, velocity, and thermodynamical functions of the fluid elements[7, 10]. In our model calculations, the initial source expanding velocity is zero and the initial energy density of the source is taken as a Gaussian distribution, $\epsilon(0,r)=\epsilon_0\,{\rm e}^{r^2/2R_0^2}$, with ϵ_0=0.5 GeV/fm^3 and R_0=4.0 fm[7].

Knowing the hydrodynamical solution as the space-time variations of density, velocity, and thermodynamical functions, we can calculate the $P(\boldsymbol{k})$ and $P(\boldsymbol{k}_1,\boldsymbol{k}_2)$ according to Eqs. (11) and (18). In our calculations, the freeze-out temperature T_f is taken as $0.5T(t=0,r=0)\approx 70$ MeV. The pion production amplitude $A(\boldsymbol{\kappa}(x),x)$ is proportional to the Bose-Einstein distribution characterized by the local temperature $T(x)$ and the local chemical potential $\mu_\pi(x)$ at the produced point x, in the local frame of the fluid element. The absorption factor due to multiple scattering is[4—7]

$${\rm e}^{-2{\rm Im}\overline{\phi}_s(x)}=\exp\left(-\int_x^{x_f}\sigma_{\rm abs}(\sqrt{s_{\pi N}})\,n_N(x)\,{\rm d}l\right), \quad (29)$$

where $\sigma_{\rm abs}(\sqrt{s_{\pi N}})$ is the absorption cross section of $\pi+N\to\varDelta$ at the center-of-mass energy $\sqrt{s_{\pi N}}$ and dl is the spatial line element along the classical path of particle propagation.

Using the relative momentum of the two pions, $q=|\boldsymbol{k}_1-\boldsymbol{k}_2|$, as variable, we can construct the two-pion correlation function $C(q)$ from $P(\boldsymbol{k}_1,\boldsymbol{k}_2)$ and $P(\boldsymbol{k}_1)P(\boldsymbol{k}_2)$ by summing over $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ for each q bin,

$$C(q)=\frac{{\rm Cor}\,(q)}{{\rm Uncor}\,(q)}\,, \quad (30)$$

where

$${\rm Cor}\,(q)=\int{\rm d}\,\boldsymbol{k}_1{\rm d}\,\boldsymbol{k}_2P(\boldsymbol{k}_1,\boldsymbol{k}_2)\,\delta(|\boldsymbol{k}_1-\boldsymbol{k}_2|-q)\,, \quad (31)$$

$${\rm Uncor}\,(q)=\int{\rm d}\,\boldsymbol{k}_1{\rm d}\,\boldsymbol{k}_2P(\boldsymbol{k}_1)P(\boldsymbol{k}_2)\,\delta(|\boldsymbol{k}_1-\boldsymbol{k}_2|-q)\,. \quad (32)$$

The HBT radius R and the chaotic parameter λ of the source can be extracted by fitting the calculated two-pion correlation function with the parameterized formula

$$C(q)=1+\lambda\,{\rm e}^{-q^2R^2}. \quad (33)$$

In Figs. 1(a), (b), and (c), the symbols • give the calculated two-pion correlation functions with the effects of source expansion and multiple scattering, for the partially coherent sources with $\chi=1.0$, 0.5, and

0.3, respectively. The symbols △ represent the two-pion correlation functions for the case that the detected pions are produced at the freeze-out configuration of the source. Because the freeze-out is the last stage of the expanding source, the corresponding geometry of the source is the largest and the pions produced at this stage will not be subjected to multiple scattering. The corresponding HBT radius and chaotic parameter results for the two cases are also shown in the figures. The HBT radii for the freeze-out stage are larger than those corresponding results for the case symbolized with •, because the freeze-out geometry of the source is larger. However, it can be seen that for each χ value the HBT chaotic parameters for the two cases are equal within the statistical errors.

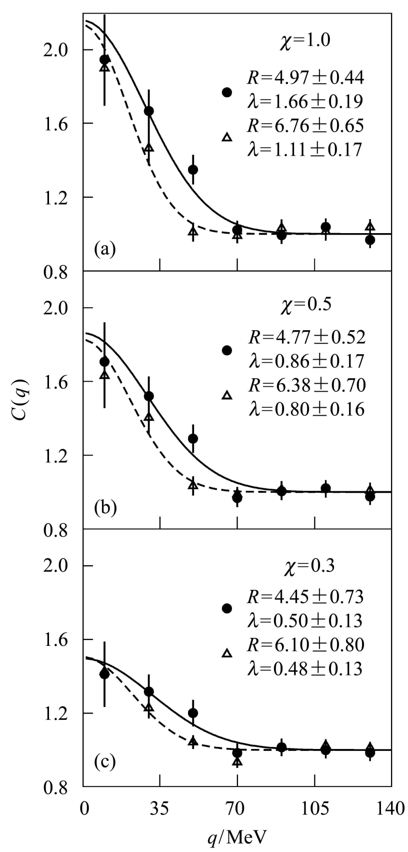

Fig. 1. Two-pion correlation functions for the partially coherent sources with (a) $\chi = 1.0$, (b) $\chi = 0.5$, and (c) $\chi = 0.3$ respectively.

For a static partially coherent source which has the same coherent and chaotic components in density distribution as described in Eqs. (3) and (4), the HBT chaotic parameter $\lambda^{\rm static}$ (without the effects of source expansion and multiple scattering) can be expressed as[11, 12]

$$\lambda^{\rm static} = \frac{1+2\gamma}{(1+\gamma)^2}, \qquad \gamma = \frac{\langle N_{\rm c}\rangle}{\langle N_{\chi}\rangle}, \tag{34}$$

where $\langle N_{\rm c}\rangle$ and $\langle N_{\chi}\rangle$ are the average pion number emitted from the coherent and chaotic components of the source, which are proportional to average coherent and chaotic source densities respectively. From Eqs. (3), (4), and (34), we have

$$\lambda^{\rm static} = \chi(2-\chi)\,. \tag{35}$$

For $\chi = 1.0$, 0.5, and 0.5, we have $\lambda^{\rm static} = 1.0$, 0.75, and 0.51. They are equal to the corresponding HBT chaotic parameters presented in Fig. 1 within statistical errors. Because the extracted HBT chaotic parameters for the expanding sources considered are equal to the corresponding $\lambda^{\rm static}$ results for the static sources with statistical errors and the HBT results extracted from the source freeze-out configuration do not include the effect of multiple scattering, we conclude that neither the source expansion nor the multiple scattering may affect the HBT chaotic parameter, although they may affect the HBT radius[7].

4 Summary and discussion

We derive the two-pion HBT correlation function for a partially coherent evolution pion-emitting source, in which the coherent and chaotic source components have the same density distribution, using quantum probability amplitudes in a path-integral formalism. We use relativistic hydrodynamics to describe the source evolution and treat the multiple scattering of the particles in the source with the Glauber scattering theory. As an example, we examine two-pion interferometry for a partially coherent source of hadronic gas with a finite baryon density. The influence of multiple scattering and source collective expansion on HBT results is investigated. We do not find observable effect of either the multiple scattering or the source collective expansion on HBT chaotic parameter based on our model calculations.

In high-energy heavy-ion collisions, multiple scattering is thought to be the reason that a coherent source tends to chaotic one during its evolution. However, based on the Glauber scattering theory the interactions of the test particle with the medium particles in the source are coherent scattering, which just adds a phase shift factor to the propagation probability amplitude. As it can not change the randomness of the phase of the probability amplitude, the source coherent degree will keep unchanged after the multiple scattering. On the other hand, there may exist incoherent scatterings in the source produced in high-energy heavy-ion collisions, which can lead to a chaotic source after the multiple scattering of this kind[13]. Therefore, further studies on the particle interactions in the source and their influence on HBT results using more detailed microscopic model than the Glauber model will be of great interest.

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