Study of ψ(4415) in the QPC model*

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Abstract Some authors consider the ψ(4415) to be the $4S$ or $5S$ excited state of a $c\bar{c}$ pair. Starting from this assumption, we study the decays of the ψ(4415) to $D\bar{D}$, $D^*\bar{D}^*$, $D_S\bar{D}_S$, $D_S^*\bar{D}_S^*$, and get the corresponding branching ratios in terms of the Quark-Pair-Creation (QPC) model. Compared with the experimental data, we find that the results of $4S$ state agree much better than those of the $5S$ state. Therefore, it is more reasonable to assume the ψ(4415) to be a $4S$ state.

Key words QPC model, strong decay, wave function, branching ratio

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1 Introduction

Considered as a $c\bar{c}$ excited state, the ψ(4415) has attracted the attention of both, theorists and experimentalists of high energy physics during the last few years. However, there is a dispute about the state of ψ(4415). Some people thought it should be a $4S$ state[1, 2], while others pointed out a $5S$ state is more reasonable[3, 4]. So, it is very important to ascertain the state of the ψ(4415). To clarify the mist, we study the strong decays of ψ(4415), with the QPC model and the harmonic oscillator wave functions of both, the $4S$ state and the $5S$ state.

Based on the assumption that ψ(4415) can be the $4S$ or the $5S$ state, we study the decays $\psi(4415) \rightarrow (D\bar{D}, D^*\bar{D}^*, D_S\bar{D}_S, D_S^*\bar{D}_S^*)$, and calculate the corresponding decay widths in the QPC model. We compare the obtained branching ratios with the experimental data in the Particle Data Book, and find that the results of the $4S$ state are in better agreement than those of the $5S$ state.

The paper is organized as follows. After this introduction, in Section 2, we introduce the QPC model, and in Section 3, we formulate the decays of $\psi(4415) \rightarrow (D\bar{D}, D^*\bar{D}^*, D_S\bar{D}_S, D_S^*\bar{D}_S^*)$. The numerical results together with all the input parameters are presented in Section 4. The last section is devoted to a simple discussion and conclusion.

2 The QPC model

The QPC model was first proposed by MICU[5] in 1969, and used to calculate the strong decays of mesons. The idea is that the decay occurs through a quark-antiquark pair created from the vacuum with its quantum numbers $J^{PC} = 0^{++}$. In the 1970s, the QPC model was developed by Yaouanc et al.[2, 6—8], and extensively applied to hadron decays. Recently some studies of the QPC model[9—11] and its applications in calculating hadron decays have been done[12—16]. The meson decay is graphically shown in Fig. 1. The quark-antiquark ($q\bar{q}$) pair (3 and 4) is created from the vacuum, with which the quark-antiquark pair (1 and 2 in A) from the initial particle constitutes the new particles (B and C). In the process of the $q\bar{q}$ pair creation, the other quarks remain unaffected.

In Fig. 1 the $q\bar{q}$ pair (3 and 4) created from the vacuum, has the quantum numbers of the vacuum, such that it is a color- and flavor-singlet, of zero momentum and zero total angular momentum. Due to parity conservation in the process, it must be of positive parity. As a fermion-antifermion pair, the $q\bar{q}$ has $P = (-1)^{L+1}$, $C = (-1)^{L+S}$, and, since it is created

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from the vacuum , it has the vacuum quantum numbers $J^{PC}=0^{++}$, from which we get $L=1$, $S=1$ of the $q\bar{q}$ pair.

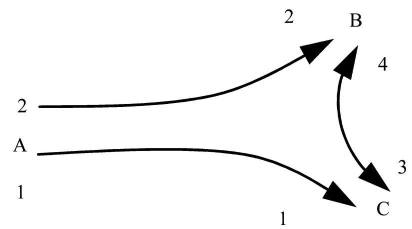

Fig. 1. The decay $A\rightarrow B+C$ in the QPC model.

In terms of the QPC model[2, 6, 7], we calculate the Feynman amplitudes $\mathcal{M}(l,s)$ for the process $A\rightarrow B+C$ and get the branching ratios of the decays. We use the notation J_X, L_X, S_X and I_X (X = A, B, C, P) for the spin, internal quark orbital angular momentum, total quark spin, and isospin of the mesons (A, B, C) or of the pair (P). The Feynman amplitudes $\mathcal{M}(l,s)$ for the decay $A\rightarrow B+C$ can be expressed by the reduced matrix elements:

$$\begin{aligned}\mathcal{M}(l,s) = & f\mathcal{I}^{I_AI_BI_C}\sum_{L_i,L_f,S}\mathcal{L}^{L_AL_BL_C}_{L_iL_fl}\mathcal{S}^{S_AS_BS_C}_{S}\times \\ & (-1)^{1+L_f+L_A+L_P}(2S+1)(2L_i+1)\times \\ & \left[\frac{1}{3}(2J_B+1)(2L_f+1)(2J_C+1)\times\right. \\ & \left.(2s+1)\right]^{1/2}\begin{Bmatrix} L_A & S_A & J_A \\ S & L_i & L_P\end{Bmatrix}\times \\ & \begin{Bmatrix} L_f & l & L_i \\ J_A & S & s\end{Bmatrix}\begin{Bmatrix} L_B & S_B & J_B \\ L_C & S_C & J_C \\ L_f & S & s\end{Bmatrix}.\end{aligned} \tag{1}$$

The constant f takes on the value $\sqrt{\frac{2}{3}}$ or $-\sqrt{\frac{1}{3}}$ according to the isospin $\frac{1}{2}$ or 0 of the created quarks, and the subscripts i and f denote the initial state and the final state, respectively. There are some conventions for adding angular momenta and isospins as follows:

$$J_B+J_C=s,\ l+s=J_A,\ I_B+I_C=I=I_A,$$

l is the orbital angular momentum between the final mesons B and C.

In Eq. (1) $\mathcal{I}^{I_AI_BI_C}$ and $\mathcal{S}^{S_AS_BS_C}_{S}$ are isospin and spin reduced matrix elements given by:

$$\begin{aligned}\mathcal{I}^{I_AI_BI_C} = & (-1)^{I_1+I_C+I_A+I_2}\left[\frac{1}{2}(2I_C+1)\times\right. \\ & \left.(2I_B+1)\right]^{\frac{1}{2}}\begin{Bmatrix} I_1 & I_B & I_3 \\ I_C & I_2 & I_A\end{Bmatrix},\end{aligned} \tag{2}$$

$$\begin{aligned}\mathcal{S}^{S_AS_BS_C}_{S} = & [(2S_B+1)(2S_C+1)(2S_A+1)\times \\ & (2S_P+1)]^{\frac{1}{2}}\begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & S_B \\ \frac{1}{2} & \frac{1}{2} & S_C \\ S_A & S_P & S\end{Bmatrix},\end{aligned} \tag{3}$$

where $S_A+S_P=S$, $S_B+S_C=S$.

We calculate the decay widths and obtain the branching ratios with the formulae as follows:

$$\Gamma^{l,s}_{A\rightarrow B+C}=2\pi\frac{E_BE_C}{m_A}|P_B||\mathcal{M}(l,s)|^2, \tag{4}$$

$$\Gamma_{A\rightarrow B+C}=\sum\Gamma^{l,s}_{A\rightarrow B+C}. \tag{5}$$

3 Formulation of the meson decay

In the decay $\psi(4415)\rightarrow D\bar{D}$, the spatial integral can be expressed as:

$$\begin{aligned}I^{L_AL_BL_C}_{M_AM_PM_BM_C}(\boldsymbol{P}_B) = & \sum_{L_i,L_f,l}\mathcal{L}^{L_AL_BL_C}_{L_iL_fl}(|\boldsymbol{P}_B|)\times \\ & \sum_{M_i,M_f,m}\langle L_AL_PM_AM_P|L_iM_i\rangle\times \\ & \langle L_BL_CM_BM_C|L_fM_f\rangle\times \\ & \langle lL_fmM_f|L_iM_i\rangle Y^{M_P}_1(\boldsymbol{P}_B),\end{aligned} \tag{6}$$

where

$$L_A+L_P=L_i,\quad L_B+L_C=L_f,\quad l+L_f=L_i.$$

With quantum numbers of $\psi(4415)$, $D\bar{D}$ and $q\bar{q}$,

$$\begin{aligned}& J^P_A=1^-,\ L_A=0,\ S_A=1,\ I_1=I_2=0, \\ & J^P_B=0^-,\ L_B=0,\ S_B=0,\ I_B=\frac{1}{2}, \\ & J^P_C=0^-,\ L_C=0,\ S_C=0,\ I_C=\frac{1}{2}, \\ & J^P_P=0^+,\ L_P=1,\ S_P=1,\ I_3=I_4=\frac{1}{2},\end{aligned} \tag{7}$$

we compute the Clebsch-Gordan coefficients and obtain the reduced I:

$$I^{0,0,0}_{0,M_P,0,0}(\boldsymbol{P}_B)=\mathcal{L}^{0,0,0}_{1,0,1}(|\boldsymbol{P}_B|)Y^{M_P}_1(\boldsymbol{P}_B). \tag{8}$$

After we eliminating the total-momentum-conserving δ-factor function, the spatial integral reads as:

$$\begin{aligned}I_{M_P}(A,B,C) = & \gamma\frac{1}{8}\int d\boldsymbol{P}\mathcal{Y}^{M_P}_1(\boldsymbol{P}_B-\boldsymbol{P})\psi^*_B(-\boldsymbol{P})\times \\ & \psi^*_C(\boldsymbol{P})\psi_A(\boldsymbol{P}_B+\boldsymbol{P}),\end{aligned} \tag{9}$$

where γ is the vacuum creation rate, which increases with energy[17].

D is in a $1S$ state and $\psi(4415)$ is supposed to be in a $4S$ state or a $5S$ state, respectively in our calculation. For the reader's convenience, we present below the harmonic oscillator wave functions in momentum space:

$$\psi_{1S} = \left(\frac{R^2}{\pi}\right)^{3/4} \exp\left(-\frac{1}{2}R^2\boldsymbol{P}^2\right), \tag{10}$$

$$\psi_{4S} = \left(\frac{R^2}{\pi}\right)^{3/4} \exp\left(-\frac{1}{2}R^2\boldsymbol{P}^2\right)\frac{1}{12\sqrt{35}}\times (-105+210R^2\boldsymbol{P}^2-84R^4\boldsymbol{P}^4+ 8R^6\boldsymbol{P}^6), \tag{11}$$

$$\psi_{5S} = \left(\frac{R^2}{\pi}\right)^{3/4} \exp\left(-\frac{1}{2}R^2\boldsymbol{P}^2\right)\frac{1}{72\sqrt{70}}\times (945-2520R^2\boldsymbol{P}^2+1512R^4\boldsymbol{P}^4- 288R^6\boldsymbol{P}^6+16R^8\boldsymbol{P}^8). \tag{12}$$

The momentum ($\boldsymbol{P}$) is half the relative momentum between two quarks that constitute the particle. Then we can obtain the expressions for $\mathcal{L}^{0,0,0}_{1,0,1}(|\boldsymbol{P}_{\rm B}|)$ corresponding to the $4S$ state and the $5S$ state, respectively.

$$\mathcal{L}^{0,0,0}_{1,0,1}(4S) = \gamma\sqrt{\frac{2}{35}}\mathrm{e}^{\frac{P_{\rm B}^2R_{\rm A}^2R_{\rm B}^2}{-4R_{\rm A}^2-8R_{\rm B}^2}}P_{\rm B}(R_{\rm A}^2)^{\frac{3}{4}}(R_{\rm B}^2)^{\frac{3}{2}}\big[105R_{\rm A}^{14}-6720R_{\rm B}^{14}+3360R_{\rm A}^2R_{\rm B}^{12}(-6+P_{\rm B}^2R_{\rm B}^2)+ 105R_{\rm A}^{12}R_{\rm B}^2(-3+2P_{\rm B}^2R_{\rm B}^2)-336R_{\rm A}^4R_{\rm B}^{10}(5-18P_{\rm B}^2R_{\rm B}^2+P_{\rm B}^4R_{\rm B}^4)+42R_{\rm A}^{10}R_{\rm B}^4(-50-3P_{\rm B}^2R_{\rm B}^2+2P_{\rm B}^4R_{\rm B}^4)+ 8R_{\rm A}^6R_{\rm B}^8(1470-42P_{\rm B}^2R_{\rm B}^2-54P_{\rm B}^4R_{\rm B}^4+P_{\rm B}^6R_{\rm B}^6)+4R_{\rm A}^8R_{\rm B}^6(525-588P_{\rm B}^2R_{\rm B}^2+9P_{\rm B}^4R_{\rm B}^4+2P_{\rm B}^6R_{\rm B}^6)\big]\times \frac{1}{3\pi^{\frac{3}{4}}(R_{\rm A}^2+2R_{\rm B}^2)^{\frac{17}{2}}}; \tag{13}$$

$$\mathcal{L}^{0,0,0}_{1,0,1}(5S) = \gamma\frac{1}{18\sqrt{35}}\mathrm{e}^{\frac{P_{\rm B}^2R_{\rm A}^2R_{\rm B}^2}{-4R_{\rm A}^2-8R_{\rm B}^2}}P_{\rm B}(R_{\rm A}^2)^{\frac{3}{4}}(R_{\rm B}^2)^{\frac{3}{2}}\big[945R_{\rm A}^{18}+241920R_{\rm B}^{18}-80640R_{\rm A}^2R_{\rm B}^{16}(-11+2P_{\rm B}^2R_{\rm B}^2)+ 315R_{\rm A}^{16}R_{\rm B}^2(-13+8P_{\rm B}^2R_{\rm B}^2)+8064R_{\rm A}^4R_{\rm B}^{14}(10-44P_{\rm B}^2R_{\rm B}^2+3P_{\rm B}^4R_{\rm B}^4)+ 504R_{\rm A}^{14}R_{\rm B}^4(-50-7P_{\rm B}^2R_{\rm B}^2+3P_{\rm B}^4R_{\rm B}^4)-1152R_{\rm A}^6R_{\rm B}^{12}(630-21P_{\rm B}^2R_{\rm B}^2-33P_{\rm B}^4R_{\rm B}^4+P_{\rm B}^6R_{\rm B}^6)+ 72R_{\rm A}^{12}R_{\rm B}^6(630-588P_{\rm B}^2R_{\rm B}^2-3P_{\rm B}^4R_{\rm B}^4+4P_{\rm B}^6R_{\rm B}^6)+16R_{\rm A}^8R_{\rm B}^{10}(-9450+13608P_{\rm B}^2R_{\rm B}^2- 324P_{\rm B}^4R_{\rm B}^4-88P_{\rm B}^6R_{\rm B}^6+P_{\rm B}^8R_{\rm B}^8)+16R_{\rm A}^{10}R_{\rm B}^8(13230+1134P_{\rm B}^2R_{\rm B}^2-972P_{\rm B}^4R_{\rm B}^4+10P_{\rm B}^6R_{\rm B}^6+P_{\rm B}^8R_{\rm B}^8)\big]\times \frac{1}{\pi^{\frac{3}{4}}(R_{\rm A}^2+2R_{\rm B}^2)^{\frac{21}{2}}}. \tag{14}$$

Table 1. Feynman amplitudes of the decays.

orbit	Feynman amplitude
$\rm D\bar{D}$	$\mathcal{M}_{1,0}=-\mathrm{i}\frac{\sqrt{2}}{6}\mathcal{L}^{0,0,0}_{1,0,1}(\lvert\boldsymbol{P}_{\rm B}\rvert)$
$\rm D^*\bar{D}^*$	$\mathcal{M}_{1,0}=-\mathrm{i}\frac{1}{3\sqrt{6}}\mathcal{L}^{0,0,0}_{1,0,1}(\lvert\boldsymbol{P}_{\rm B}\rvert)$, $\mathcal{M}_{1,1}=0$,
	$\mathcal{M}_{1,2}=-\mathrm{i}\frac{\sqrt{10}}{3\sqrt{3}}\mathcal{L}^{0,0,0}_{1,0,1}(\lvert\boldsymbol{P}_{\rm B}\rvert)$
$\rm D_S\bar{D}_S$	$\mathcal{M}_{1,0}=-\frac{\sqrt{2}}{12}\mathcal{L}^{0,0,0}_{1,0,1}(\lvert\boldsymbol{P}_{\rm B}\rvert)$
$\rm D_S^*\bar{D}_S^*$	$\mathcal{M}_{1,0}=-\frac{1}{6\sqrt{6}}\mathcal{L}^{0,0,0}_{1,0,1}(\lvert\boldsymbol{P}_{\rm B}\rvert)$, $\mathcal{M}_{1,1}=0$,
	$\mathcal{M}_{1,2}=-\frac{\sqrt{10}}{6\sqrt{3}}\mathcal{L}^{0,0,0}_{1,0,1}(\lvert\boldsymbol{P}_{\rm B}\rvert)$

With the quantum numbers of Eq. (7) as input and the Wigner (6-j and 9-j) coefficients, we calculate $\mathcal{M}(l,s)$ for the decays $\psi(4415)\to\rm D\bar{D}$ taking ψ(4415) as a $4S$ state and $5S$ state, respectively. In $\mathcal{M}(l,s)$ only the expressions $\mathcal{L}^{0,0,0}_{1,0,1}(|\boldsymbol{P}_{\rm B}|)$ corresponding to the two states are different:

$$\mathcal{M}_{1,0}=-\mathrm{i}\,\frac{\sqrt{2}}{6}\mathcal{L}^{0,0,0}_{1,0,1}(|\boldsymbol{P}_{\rm B}|). \tag{15}$$

In the same way, we calculate the decays $\psi(4415)\to(\rm D^*\bar{D}^*, D_S\bar{D}_S, D_S^*\bar{D}_S^*)$, and obtain the corresponding Feynman amplitudes $\mathcal{M}(l,s)$. The results are given in the following Table 1.

Thus, supposing ψ(4415) is the $4S$ state or $5S$ state, we obtain the widths and branching ratios of the decays, from Eqs. (4) and (5).

4 Numerical results

In order to obtain the relevant parameters we use the experimental results of $\psi(4040)\to\rm D\bar{D}, D^*\bar{D}^*, D_S\bar{D}_S$[15, 17, 18], and after some numerical calculation, we obtain the values of $R^2_{\rm S}$ and $\gamma_{\rm u,d}$ in the harmonic oscillator wave functions: $R^2_{\rm A}=6.00\ \mathrm{GeV}^{-2}$, $R^2_{\rm D}=5.25\ \mathrm{GeV}^{-2}$, $R^2_{\rm D^*}=6.70\ \mathrm{GeV}^{-2}$, $R^2_{\rm D_s}=5.20\ \mathrm{GeV}^{-2}$, $\gamma_{\rm u,d}=3$. There are not enough data to determine $R^2_{\rm D_s^*}$ and $\gamma_{\rm s}$, so we invoke the theoretical assumption[2, 15], $\gamma_{\rm s}=\frac{\gamma_{\rm u,d}}{\sqrt{3}}$ and $R^2_{\rm D_s}=R^2_{\rm D_s^*}$. So we can calculate the widths and branching ratios with the above parameters and the masses of the initial and final state particles: $m_{\rm A}=4.415$ GeV, $m_{\rm D}=1.869$ GeV, $m_{\rm D^*}=2.01$ GeV, $m_{\rm D_S}=1.968$ GeV, $m_{\rm D_S^*}=2.112$ GeV. With the above parameters we obtain the numerical results shown in Table 2.

Table 2. The first set of numerical results.

	4S		5S	
	Γ/MeV	$\mathcal{B}$(%)	Γ/MeV	$\mathcal{B}$(%)
$\mathrm{D\bar{D}}$	0.95	2.2	0.00022	0.0005
$\mathrm{D^*\bar{D}^*}$	13.3	31.0	5.08	11.8
$\mathrm{D_S\bar{D}_S}$	0.088	0.21	0.0058	0.014
$\mathrm{D_S^*\bar{D}_S^*}$	0.17	0.39	0.0014	0.0033

In Table 2, we find that the results are tiny. As indicated by Yaouanc[17], the vacuum creation rate γ can be larger at higher energies, so we adjust the parameters as: $\gamma_{\rm s}=\gamma_{\rm u,d}=3.5$, $R^2_{\rm D_S}=R^2_{\rm D_S^*}=6.65\ \mathrm{GeV}^{-2}$. Then we repeat the calculation, the results being shown in Table 3.

Table 3. The second set of numerical results.

	4S		5S	
	Γ/MeV	$\mathcal{B}$(%)	Γ/MeV	$\mathcal{B}$(%)
$\mathrm{D\bar{D}}$	1.30	3.0	0.00030	0.0007
$\mathrm{D^*\bar{D}^*}$	18.2	42.2	6.91	16.1
$\mathrm{D_S\bar{D}_S}$	1.50	3.5	0.28	0.65
$\mathrm{D_S^*\bar{D}_S^*}$	6.04	14.0	0.29	0.67

Other parameters are also suitable. For example with $R^2_{\rm A}=6.20\ \mathrm{GeV}^{-2}$, $R^2_{\rm D}=5.55\ \mathrm{GeV}^{-2}$, $R^2_{\rm D^*}=7.10\ \mathrm{GeV}^{-2}$, $R^2_{\rm D_s}=R^2_{\rm D_s^*}=5.35\ \mathrm{GeV}^{-2}$, $\gamma_{\rm s}=\dfrac{\gamma_{\rm u,d}}{\sqrt{3}}$, $\gamma_{\rm u,d}=3$, we obtain the results of Table 4.

Table 4. The third set of numerical results.

	4S		5S	
	Γ/MeV	$\mathcal{B}$(%)	Γ/MeV	$\mathcal{B}$(%)
$\mathrm{D\bar{D}}$	1.03	2.4	0.00011	0.0003
$\mathrm{D^*\bar{D}^*}$	18.4	42.7	6.52	15.2
$\mathrm{D_S\bar{D}_S}$	0.091	0.21	0.0050	0.012
$\mathrm{D_S^*\bar{D}_S^*}$	0.13	0.31	0.00053	0.0012

If we adjust[17] the parameters on the basis of the third set of parameters, $\gamma_{\rm s}=\gamma_{\rm u,d}=3.5$, $R^2_{\rm D_S}=R^2_{\rm D_S^*}=6.90\ \mathrm{GeV}^{-2}$, and we obtain the results of Table 5.

Table 5. The fourth set of numerical results.

	4S		5S	
	Γ/MeV	$\mathcal{B}$(%)	Γ/MeV	$\mathcal{B}$(%)
$\mathrm{D\bar{D}}$	1.41	3.3	0.00016	0.0004
$\mathrm{D^*\bar{D}^*}$	25.0	58.1	8.87	20.6
$\mathrm{D_S\bar{D}_S}$	1.68	3.9	0.28	0.66
$\mathrm{D_S^*\bar{D}_S^*}$	5.49	12.8	0.22	0.51

5 Discussion and conclusion

In this work, we study the decays of $\psi(4415)$ considering it to be in a 4S state or in a 5S state, respectively and obtain the corresponding branching ratios in the QPC model. In Table 2 and Table 4, the all numerical values are tiny, so we have to adjust some parameters by a recipe given by Yaouanc[17]. This way we obtain the results given in Table 3 and Table 5, which are more reasonable.

From all four tables we find that, the numerical results for $\psi(4415)$ being a 5S state are small, especially in the decay channel $\psi(4415)\rightarrow\mathrm{D\bar{D}}$. It is obvious that, the numerical results for $\psi(4415)$ being the 5S state can not match the experimental data[18] well. The numerical results for the 4S state are about three times those of the 5S state. Compared with the experimental data, the numerical results of the 4S state are more reasonable than those of the 5S state. Our result is consistent with Yaouanc's point of view[17]. The conclusion of our work is then that the $\psi(4415)$ is most likely a 4S state.

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