Entropy and equilibrium property of QCD-instanton induced final state in deep-inelastic scattering*

DENG Wei-Bing(邓为炳)[1] LU Ding(陆丁)[1] ZHANG Ji-Ying(张纪英)[1] XU Ming-Mei(许明梅)[2;1)]
Boris Levchenko[3;2)] LIU Lian-Shou(刘连寿)[2;3)]

1 (Training Base for Research and Teaching in Fundamental Sciences (Physics)
College of Physical Science and Technology, Huazhong Normal University, Wuhan 430079, China)
2 (Institute of Particle Physics, Huazhong Normal University, Wuhan 430079, China)
3 (Skobeltsyn Institute of Nuclear Physics, Moscow State University, 119992 Moscow, Russia)

Abstract The scaling and additivity properties of Rényi entropy in rapidity space of the instanton final state (IFS) and current jet identified by the r-sorting method from the QCDINS Monte Carlo event sample are studied. Asymptotic scaling of the Rényi entropy H_2 is observed for the IFS while H_2 for the current jet tends to saturation with decreasing phase space scale. Furthermore, it is found that the additivity of H_2 holds well for the IFS in narrow rapidity windows at different positions. These results indicate that the IFS produced in the instanton-induced process of deep inelastic scattering has reached local equilibrium.

Key words instanton, quark-gluon fusion, deep-inelastic scattering, entropy, local equilibrium

PACS 13.85.Hd, 13.60.-r

1 Introduction

The basic theory of strong interaction — quantum chromo-dynamics (QCD) — is a non-Abelian gauge theory and as such has a complicated vacuum structure. There are degenerate vacua of different topologies, leading to a special effect — tunneling transition between degenerate vacua —, referred to as instanton[1—4]. Instantons are non-perturbative fluctuations of the gluon field, which are expected to contribute to deep-inelastic scattering (DIS) with a sizable rate[5—7].

Deep-inelastic scattering (DIS) accompanied by instanton-induced hard processes, cannot be described by conventional perturbation theory. Therefore, theoretical and experimental studies of the processes induced by instantons are of fundamental significance for a thorough understanding of the non-perturbative sector of QCD[5, 6].

The two experimental groups, H1 and ZEUS, of DESY-HERA in Germany have been searching for instanton induced events in e^+p deep inelastic scattering experiments, and only upper limits on the cross-section of such processes were set[8, 9]. At present, beside the experimental searches for the instanton induced events, a Monte Carlo study of the physical properties of IFS is also important.

Figure 1 shows the leading graph of QCD-instanton induced e-p collisions. The incident lepton emits a photon, with 4-momentum q, which in turn transforms into a quark-antiquark pair. One of these quarks with 4-momentum q'' hadronizes to form the current jet. The other quark, with 4-momentum q', fuses with a gluon (with 4-momentum g) from the proton in the presence of an instanton (I). The hadron system produced from the fusion of quark and gluon in the presence of an instanton is referred to as instanton final state, or IFS in short.

The quark-gluon fusion process with the instanton as background gives rise to a high multiplicity final state. The particles produced from this process, i.e. the IFS, are expected to be isotropically distributed

Received 2 February 2007, Revised 1 June 2007
* Supported by NSFC-RFBR Cooperative Research Project (10475030, 05-02-39028-NNSF-a), NSFC (10375025, 90503001) and Ministry of Education of China NO CFKSTIP-704035

1) E-mail: xumm@iopp.ccnu.edu.cn
2) E-mail: levtchen@mail.desy.de
3) E-mail: liuls@iopp.ccnu.edu.cn

in their center of mass frame[10].

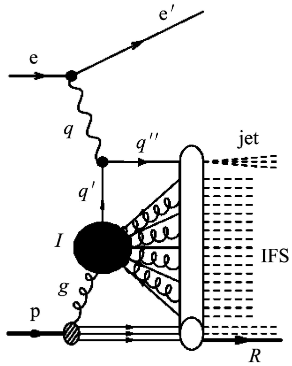

Fig. 1. The leading QCD-instanton induced process in the DIS regime of e-p scattering.

A problem of high interest is whether the IFS has reached equilibrium. Refs. [11, 12] have studied the final-state momentum distribution of the IFS using a Monte Carlo simulation. An approximate isotropic distribution is found for the IFS in contrast to the high anisotropy of the current jet. In the present paper the equilibrium of IFS will be studied by using the Rényi-entropy analysis, based on Ma's coincidence probability.

2 Method of Rényi-entropy analysis

Let us first discuss shortly the concept of equilibrium. For a closed macroscopic system, arriving at (thermal) equilibrium, the corresponding ensemble is the micro-canonical one, where the microscopic states of the system are uniformly distributed in the allowed region of phase space of the system (Γ-space). Starting from this ensemble all the thermodynamical properties of the system, including the thermodynamical quantities — entropy, temperature, etc. —, are derived.

The physical origin of thermal equilibrium of a macroscopic system are random collisions among the molecules constituting the system. In our case, each parton produced in the instanton-induced quark-gluon-fusion process develops a parton shower. Hadrons only experience resonance decay, if any. There is no rescattering and no random thermal motion neither at parton level nor at hadron level. Therefore, no thermal equilibrium in the strict sense is possible. However, the distribution of the final states in the allowed region of phase space may be uniform. Such a distribution mimics the micro-canonical ensemble, and the "thermodynamical" properties of the system in consideration can be derived therefrom. We will refer to such a uniform distribution in phase space also as equilibrium. The "equilibrium", if any, of the IFS and current jet that we will check in the following should be understood in this sense.

To check the equilibrium of IFS and jet we will make use of the Rényi-entropy analysis based on Ma's coincidence probability[13—15]. According to Ma[16] an event is characterized by a set of integer numbers $s\equiv\{m_i\}$, $i=1,\cdots,M$, where a certain phase space region Δ is partitioned equally into M bins with the size of each bin $\delta=\dfrac{\Delta}{M}$, m_i is the number of particles in the ith bin. When two events have the same set of numbers $\{m_i\}$, we say that they are coincident. If there are n_s events in the whole sample having the same set $s\equiv\{m_i\}$, then we define the number of coincidences of k configurations as

$$N_k=\sum_s n_s(n_s-1)\ \cdots\ (n_s-k+1). \tag{1}$$

The coincidence probability of k configurations is given by

$$C_k=\frac{N_k}{N(N-1)\cdots(N-k+1)}\,, \tag{2}$$

where N is the total number of events in the sample, and the Rényi entropies are defined as[17]

$$H_k\equiv-\frac{\ln C_k}{k-1}\,. \tag{3}$$

The Rényi entropies are closely related to the Shannon entropy (S), which is formally equal to the limit of H_k as $k\to1$ and can be obtained through extrapolation[14, 15]. It has been shown that the extrapolation method for determining S is in general not unique and will produce additional uncertainty[14, 15], while the Rényi entropies are of interest by themselves and can provide information on the equilibrium of the system, cf. Eqs. (5), (6) below. So our study will concentrate on the behavior of Rényi entropies, in particular on H_2.

For a system close to equilibrium the Rényi entropies obey the scaling property:

$$H_k(M)=H_k(M_0)+d\ln\frac{M}{M_0} \tag{4}$$

for a sufficiently fine-grained subdivision of phase space[13, 14], where d is a constant related to the dimension of space, M is bin number with the size of each bin $\delta=\Delta/M$, M_0 is another bin number with the size of each bin $\delta_0=\Delta/M_0$. Substituting $M=\dfrac{\Delta}{\delta}$, we can rewrite Eq. (4) as

$$H_k(M)=A-d\ln\delta\quad\text{for}\quad M\to\infty\,, \tag{5}$$

where $A=H_k(M_0)+d\ln\delta_0$ is a constant.

Furthermore, the additivity property[15]:

$$H_k^{(R)}(2M)=H_k^{(R_1)}(M)+H_k^{(R_2)}(M)\,, \tag{6}$$

should hold for two non-overlapping and independent phase space regions R_1 and R_2 with R the union.

Note that the equality of the two sides of Eq. (6) is examined at the same scale $\delta=\dfrac{\Delta}{M}=\dfrac{2\Delta}{2M}$.

3 Results and discussions

The above-mentioned method has been applied to hadron-hadron (h-h) collision data from both PYTHIA-JETSET Monte Carlo generator[18] and NA22 experiment[19]. It turns out that in both cases the Rényi entropy H_2 plotted versus $-\ln\delta y$ tends to saturate instead of approaching a straight line to be expected by the scaling law Eq. (5) and the additivity Eq. (6) does not hold. This shows that there is no equilibrium in the final state system of hadron-hadron collisions.

In the present paper we apply the method to the hadron system produced in the quark-gluon fusion process $q'+g \xrightarrow{(\mathrm{I})} X$ in the background of an instanton. Our study is based on the Monte Carlo code QCDINS[20, 21], which is a package for instanton induced events embedded in the HERWIG[22, 23] event generator. The default parameters of the QCDINS 2.0 version are used in our study, i.e. $x'>0.35$, $Q'^2>113\ \mathrm{GeV}^2$ and the number of quark flavors is set to be $n_{\mathrm{f}}=3$.

In total 493400 instanton-induced DIS events are generated with the energies of electron and proton being equal to 27.5 and 820 GeV, respectively. The IFS and current jet are identified using the r-sorting method proposed in Ref. [11]. The method is to define a distance in the θ-ϕ-plane in the hadronic center-of-mass frame, i.e.

$$r(\theta,\phi)=\sqrt{\frac{\left(\dfrac{\theta-\theta_0}{\pi}\right)^2+\left(\dfrac{\phi-\phi_0}{\pi}\right)^2}{2}}\,, \qquad (7)$$

where $(\theta_0,\phi_0=0)$ is the position of the current quark. This variable measures how far is every final state particle from the jet axis. Choosing an appropriate value for r_0, the particles with $r(\theta,\phi)<r_0$ are attributed to the jet and those with $r(\theta,\phi)>r_0$ to the IFS. First discard the proton remnant by an 1D cut. Renumber the left n particles by their r values, let $r_1<r_2<\cdots<r_n$. Choose the value of r_0 in between r_k and r_{k+1} $(r_k<r_0<r_{k+1})$, i.e. assign $1,\cdots,k$ to the particles of the current jet and $k{+}1,\cdots,n$ to the IFS. Accumulating the energies from particle 1 to particle k gives $E_k=\sum_{i=1}^{k}\varepsilon_i$; simultaneously accumulating the energies from $k{+}1$ to n gives $E'_k=\sum_{i=k+1}^{n}\varepsilon_i$. The energy reconstruction errors for jet and instanton are $\Delta E_{\mathrm{jet}}=\dfrac{E_k-E_{\mathrm{C}}}{E_{\mathrm{C}}}\times100\%$ and $\Delta E_{\mathrm{I}}=\dfrac{E'_k-E_{\mathrm{I}}}{E_{\mathrm{I}}}\times100\%$, respectively, where E_{C} is the energy of the current parton, E_{I} is that of the quark and gluon included in the IFS at partonic level. Then choose the value of k to optimize the energy reconstruction, i.e. make $\Delta E=0.4\times|\Delta E_{\mathrm{jet}}|+0.6\times|\Delta E_{\mathrm{I}}|$ minimal. For the entropy analysis the particles in the IFS and those in the current jet are then transformed into their own rest frame and therefore the rapidity we are dealing with in the following also refer to these rest frames. Fig. 2 shows the reconstruction error ΔE_{jet} for the jet energy and ΔE_{I} for the instanton energy. It can be seen that the reconstruction is good and the resulting IFS and jet are reliable.

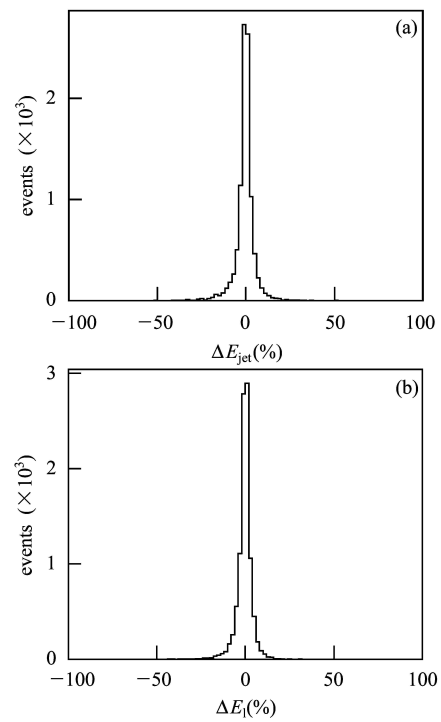

Fig. 2. Distributions of the reconstruction error ΔE_{jet} for the jet energy (a) and ΔE_{I} for the instanton energy (b) in the r-sorting method.

Figure 3(a) and 3(b) show the Rényi entropy H_2 versus $-\ln\delta y$ in the central rapidity regions $|y|<2$ (circles) and $|y|<0.5$ (triangles) for the IFS and current jet, respectively. A striking feature is that for the IFS the behavior of H_2 versus $-\ln\delta y$ differs qualitatively from that of the DIS-current jet and the h-h collision final states. It bends upwards and tends to a straight line for large $-\ln\delta y$, cf. Fig. 3(a), in contrast to the downward bending and saturation tendency for the current jet (Fig. 3(b)) and h-h collision final-state systems[18, 19]. This is a strong indication that, unlike the violation of the scaling law in the case of the DIS-current jet and h-h collision final-states, the scaling law Eq. (5) is satisfied for the IFS for a sufficiently fine-grained subdivision of the phase space.

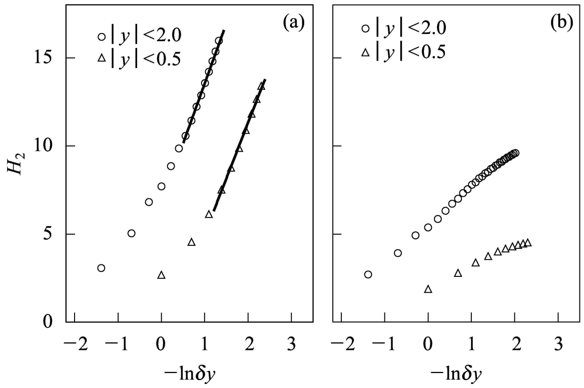

Fig. 3. The Rényi entropy H_2 versus $-\ln\delta y$ in central rapidity regions $|y| < 2$ (circles) and $|y| < 0.5$ (triangles) for (a) IFS and (b) current jet. The straight lines in (a) are linear fits to the high $-\ln\delta y$ part of H_2 as a function of $-\ln\delta y$. The slopes of the two lines are 6.820 ± 0.013 (upper line) and 6.304 ± 0.005 (lower line), respectively.

Let us now check the additivity property Eq. (6) for the instanton final state IFS. In Fig. 4 are shown the Rényi entropies H_2 of the IFS obtained from taking the sum of the results calculated from two regions R_1 and R_2 in comparison with those directly calculated from the union $R = R_1 + R_2$. The legend in the figure shows the rapidity regions corresponding to each set of points. For example, the solid circle in Fig. 4(a) is the sum of the H_2's from the regions $-2 < y < 0$ (R_1) and $0 < y < 2$ (R_2), while the open circle is the H_2 from the union $R = R_1 + R_2$ ($-2 < y < 2$). It can be seen from Fig. 4(a) that for regions of narrow widths R_1 and R_2 (~ 0.5) the additivity holds excellently, while for wider widths (~ 2) the additivity holds only approximately, especially for large M. In Fig. 4(b) the additivity is investigated for R_1 and R_2 of the same width (=1) but located at different places. It can be seen that for adjacent R_1 and R_2 the additivity holds well, however, if they are separated by a gap it holds only approximately.

The asymptotic scaling and additivity of the IFS are special and impressive. These properties have never been observed in other systems, e.g. hadronic jets and h-h collision final states, studied up to now. They strongly indicate that the system produced in quark-gluon fusion in the background of an instanton, i.e. the IFS, has reached local equilibrium.

Our Monte Carlo study is based on the QCDINS code, where the 4-momenta of the $n = 2n_{\rm f} - 1 + n_{\rm g}$ produced partons in the instanton rest frame (for convenience we will refer in the following to the partons produced in the instanton-induced quark-gluon-fusion process as IQGF) are uniformly generated in energy-weighted phase space[20] according to the leading-order matrix element[24] with different energy weights for gluons and quarks. Every event of IQGF is a realization of the instanton-induced fusion process and can be regarded as a microscopic state of the system. The fact that the 4-momenta of the partons in IQGF are uniformly generated in energy-weighted phase space means that the microscopic states of IQGF are uniformly distributed in phase space, mimicking the micro-canonical ensemble of a macroscopic system in thermal equilibrium. In this sense the local equilibrium in parton level is built-in into the model due to theoretical considerations of the property of the instanton.

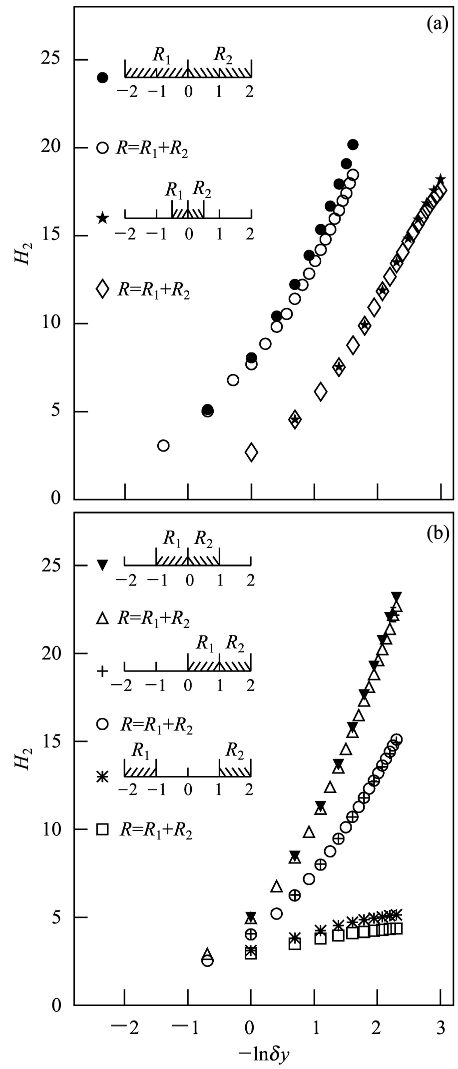

Fig. 4. The sum of the Rényi entropies H_2 from IFS versus $-\ln\delta y$ for two regions R_1 and R_2 and that directly calculated from the whole region $R = R_1 + R_2$. The legend shows the rapidity regions corresponding to each set of points.

In this letter we have investigated the equilibrium properties of the IFS, i.e. the hadron system after the hadronization of IQGF. It is unclear whether and how the equilibrium property of IQGF is preserved in the hadronization process. Our work shows that the local equilibrium is present in the IFS and can be represented by the scaling and additivity of Rényi entropy.

Since the IQGF and/or IFS are in local equilibrium we can derive the "thermodynamical" quantities for them according to the usual thermodynamic formalism. However, the meaning of the obtained values of these quantities are different from those of a macroscopic system. For example, the temperature of a macroscopic system measures the average kinetic energy of molecules, but the "temperature" in our case does not have this kind of meaning. This explains why the hadron system IFS can have a temperature much higher than the critical temperature (∼160— 170 MeV) of the deconfining phase transition, as shown in Ref. [11].

4 Conclusion

We have studied the scaling and additivity properties of Rényi entropy H_2 for the IFS and current jet in the instanton-induced process of deep inelastic scattering produced from the QCDINS Monte Carlo generator, identified by the r-sorting method.

The Rényi entropy H_2 for IFS is found to obey the scaling law Eq. (5) for a sufficiently fine-grained phase space region and the additivity property Eq. (6) holds in narrow phase space windows at different positions, especially in adjacent intervals. These results provide a strong indication that the IFS has reached local equilibrium.

In order to get a better understanding of the equilibrium of the IFS, further investigations along this line are suggested, e.g. to study the entropy properties in 3 dimensional momentum space, transforming the variables to the corresponding cumulant ones[13], or to use other characteristics of equilibrium, e.g. $|K_l(E,n)|^{2/l} \ll K_2(E,n),\ l=3,4,\cdots,$[25—27] which guarantees the smallness of the higher-order energy correlations $K_l(E,n)=\langle\prod_{k=1}^{l}(\epsilon_k-\langle\epsilon\rangle)\rangle$ with ϵ_k being the energy of the kth particle.

The authors thank Li Zhiming for helpful discussions.

References

1 Belavin A, Polyakov A, Schwartz A et al. Phys. Lett., 1975, **B59**: 85
2 't Hooft G. Phys. Rev. Lett., 1976, **37**: 8
3 't Hooft G. Phys. Rev., 1976, **D14**: 3432; Phys. Rev., 1978, **D18**: 2199
4 't Hooft G. Phys. Rep., 1986, **142**: 357
5 Moch S, Ringwald A, Schrempp F. Nucl. Phys., 1997, **B507**: 134
6 Ringwald A, Schrempp F. Phys. Lett., 1998, **B438**: 217
7 Ringwald A, Schrempp F. Phys. Lett., 1999, **B459**: 249
8 The H1 Coll. (Adloff C et al.). Eur. Phys. J., 2002, **C25**: 495—509
9 The ZEUS Coll. (Chekanov S et al.). Eur. Phys. J., 2004, **C34**: 255—265
10 Ringwald A, Schrempp F. Proc. 8th Int. Seminar. Vladimir, Russia, 1994, D. Grigoriev et al. eds., Singapore: World Scientic, 1995. 170 hep-ph/9411217
11 XU Ming-Mei et al. in preparation
12 ZHANG Kun-Shi et al. A Monte Carlo Study on the Dynamical Fluctuations inside Quark-Gluon Fusion Products in QCD-Instanton Induced Processes in Deep-Inelastic Scattering. Intern. J Modern Phys. E, to be published
13 Białas A, Czyż W, Wosiek J. Acta Phys. Pol., 1999, **B30**: 107
14 Białas A, Czyż W. Phys. Rev., 2000, **D61**: 074021
15 Białas A, Czyż W. Acta Phys. Pol., 2000, **B31**: 687
16 Ma S K. Statistical Mechanics. Singapore: World Scientific, 1985. 425; Ma S K. J. Stat. Phys., 1981, **26**: 221; Ma S K, Payne M. Phys. Rev., 1981, **B24**: 3984
17 Rényi A. Acta Math. Acad. Sci. Hung., 1959, **10**: 193
18 Fialkowski K, Wit R. Phys. Rev., 2000, **D62**: 114016
19 NA22 Coll. (Atayan M R et al.). Acta Phys. Pol., 2005, **B36**: 2969
20 Ringwald A, Schrempp F. Comput. Phys. Commun., 2000, **132**: 267. hep-ph/9911516
21 Gibbs M, Ringwald A, Schrempp F. in: Proc. DIS 1995. Paris, France, 1995. Laporte J F, Sirois Y, eds. 341. hep-ph/9506392
22 Marchesini G, Webber B. Nucl. Phys., 1988, **B310**: 461
23 Marchesini G et al. Comput. Phys. Commun., 1992, **67**: 465
24 Moch S, Ringwald A, Schrempp F. Nucl. Phys., 1997, **B507**: 134
25 Manjavidze J, Sissakian A. hep-ph/0204281
26 Sissakian A N. Physics Uspekhi, 2003, **46**: 3. http://www.ufn.ru/archive/Index03.html#rel_2_2003_303
27 Manjavidze J, Sissakian A. Phys. Rept., 2001, **346**: 1