# Field Theory on Non-commutative Space-Time and Some Related Phenomenology

LIAO Yi

(Department of Physics, Nankai University, Tianjin 300071, China)

**Abstract** I introduce first the basic ideas of quantum field theory defined on non-commutative space-time. Some phenomenological investigations at linear colliders are very briefly reviewed. Then, I give some details on detecting non-commutative signals by pair production of neutral Higgs bosons in $e^+e^-$ collisions, and discuss how to use Lorentz symmetry violation to isolate signals from standard model backgrounds. Finally, very recent developments on realistic model building are also briefly mentioned.

**Key words** non-commutative space-time, linear colliders, Higgs particles

Noncommutative (NC) spacetime was first suggested by Heisenberg as a possible means to regularize the ultraviolet divergences appearing in quantum field theory. Its first version was formulated by Snyder in 1947[1]. But it had been largely forgotten afterwards due mainly to the successful program of renormalization in quantum field theory. The basic idea is that when coordinates become noncommutative, there arises a minimal length scale from the commutation relations of coordinates which could offer a natural ultraviolet cutoff of momentum according to the Heisenberg uncertainty relations. The interest in NC spacetime and quantum field theory built on it has recently been revived mainly because of its connection to the string theory[2]. Nevertheless, the noncommutative field theory (NCQFT) is interesting in its own right. Due to noncommutativity, physics at extremely short distance becomes highly nonlocal and NCQFT provides at least a model to it. Furthermore, it also offers a test-ground for violation of basic concepts that we feel familiar with, like Lorentz symmetry, causality and unitarity, etc.

The canonical NC spacetime is defined by

$$[x^\mu, x^\nu] = \mathrm{i}\theta^{\mu\nu}, \tag{1}$$

where $\theta^{\mu\nu}$ is a real antisymmetric constant matrix that parameterizes the noncommutativity of spacetime, and has dimensions of length squared. The physics scale that governs the NC effects is given by $\Lambda \sim \theta^{-\frac{1}{2}}$ where $\theta$ is a typical value of the matrix. A convenient way to formulate NCQFT is to replace the ordinary point-wise product of field functions by the Moyal-Weyl $\star$-product:

$$(\phi_1 \star \phi_2)(x) = \left[\exp\left(\frac{\mathrm{i}}{2}\theta^{\mu\nu}\partial^x_\mu \partial^y_\nu\right)\phi_1(x)\phi_2(y)\right]_{y=x}. \tag{2}$$

There are also other approaches based on the $\theta$ expanded Seiberg-Witten mapping[3], the twisted Poincare symmetry[4], etc.

Since the $\star$-product involves spacetime derivatives to an infinite order, NCQFT thus obtained is highly nonlocal and analytically nontrivial. This is the origin of all exotic properties. When time does not commute with space, i.e., $\theta_{0\mathrm{i}} \neq 0$, the conventional formulation of perturbation theory breaks the unitarity of $S$ matrix[5]. Possible cures to it have been suggested[6] but it is difficult to generalize to gauge theories. The appearance of ultraviolet-infrared mixing[7] makes the standard renormalization program difficult to imple-

Received 30 March 2007

ment if not possible at all. Furthermore, causality has also been found to be violated[8], and the micro-causality condition is modified from light cone to less restrictive light wedge[9] so the analyticity properties of Green functions are significantly altered[10].

The appearance of $\star$-product highly constrains the structure of interactions. A realistic model must involve gauge interactions which are however severely restricted[11—14]. The gauge group must be $U(N)$ or the product of them since $SU(N)$ is not close under starred gauge transformations. The allowed representations are fundamental, anti-fundamental, bi-fundamental and adjoint ones, and a matter field cannot be charged under more than two groups. We'll restrict ourselves in this talk to the simplest possible case of $U(1)$ gauge theory, i.e., NCQED. In this case, the only allowed charges are $+1,-1,0$[15]. But neutral particles can interact with photons, and photons also interact among themselves.

Because of restriction on allowed charges, the NC-QED phenomenology has mainly been done for linear colliders. The standard QED processes were studied in Ref. [16], and the charged Higgs pair production at photon colliders was discussed in Ref. [17]. Based on the observation that a neutral particle can have non-trivial interactions with photons, we calculated in Ref. [18] the neutral Higgs pair production at a linear collider $\mathrm{e^+e^-}\to\mathrm{HH}$, which we discuss below in more detail.

The constant matrix $\theta_{\mu\nu}$ breaks Lorentz invariance by defining a preferred direction. There are two important consequences from this. First, the result by standard computation is not directly applicable to a practical experiment because the collider beam rotates around the preferred direction as the Earth rotates. Second, ignoring this also causes unnecessary loss of information specific to NC signals. To utilize the advantage of dependence on the Earth rotation, we consider two types of angular distributions. The first one is over the local angles $\theta$, $\varphi$ upon averaging over the Earth rotation:

$$\frac{4\pi}{\sigma_0}\overline{\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right]}=f(\theta,\varphi), \tag{3}$$

where $\sigma_0$ is the normalized cross section as shown in Fig. 1, and $f(\theta,\varphi)$ is the distribution function. This type of distributions corresponds to analyzing data as is usually done, namely by collecting data for a period of time. The second type of distributions is over the Earth's rotation angle, which can be best presented by the day-night asymmetry:

$$A_{\mathrm{DN}}(\omega_{\mathrm{a}},\omega_{\mathrm{b}})=\frac{\left[\int_{\omega_{\mathrm{a}}}^{\omega_{\mathrm{b}}}\mathrm{d}\omega-\int_{\omega_{\mathrm{a}}+\pi}^{\omega_{\mathrm{b}}+\pi}\mathrm{d}\omega\right]\sigma(\omega)}{\left[\int_{\omega_{\mathrm{a}}}^{\omega_{\mathrm{b}}}\mathrm{d}\omega+\int_{\omega_{\mathrm{a}}+\pi}^{\omega_{\mathrm{b}}+\pi}\mathrm{d}\omega\right]\sigma(\omega)}\ , \tag{4}$$

where $\omega$ measures the rotation. Then $A_{\mathrm{DN}}(0,\pi)$ gives the integrated asymmetry over 24 hours. These distributions are shown in Figs. 2—4.

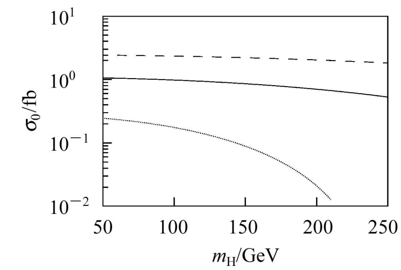


Fig. 1. $\sigma_0$ as a function of the H mass, $m_{\mathrm{H}}$ at c.m. energy $\sqrt{s}=0.5$TeV (dotted), 1.0TeV (solid) and 1.5TeV (dashed) for $\Lambda_{\mathrm{NC}}=1$TeV. The standard model (SM) result is about $0.1\sim0.2$fb[19].

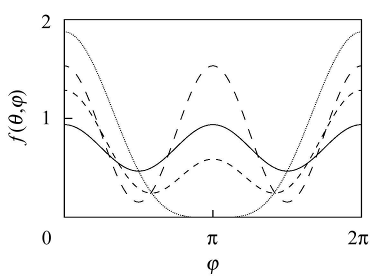


Fig. 2. $f(\theta,\varphi)$ as a function of $\varphi$ at $\theta=\frac{\pi}{4}$ for three representative sets of parameters specifying the beam and preferred directions. The long-dashed curve is the result upon further averaging over $\theta$. The SM result is independent of $\varphi$.

For $\sqrt{s}\geqslant 1$TeV and $\Lambda_{\mathrm{NC}}\approx 1$TeV, the NC signal dominates over the SM background for an intermediate Higgs mass. For $\Lambda_{\mathrm{NC}}\gg s$, however, the NC signal drops fast: $\sigma\sim\alpha^2 s\Lambda_{\mathrm{NC}}^{-4}$. Since the NCQED process conserves helicity, $\sigma_{\mathrm{NC}}(\mathrm{e^-_{RH}e^+_{LH}})=\sigma_{\mathrm{NC}}(\mathrm{e^-_{LH}e^+_{RH}})=\frac{1}{2}\sigma_{\mathrm{NC}}^{\mathrm{unpol}}$. For comparison, the SM background is dominated by $\mathrm{W}^{\pm}$ box which implies that $\sigma_{\mathrm{SM}}(\mathrm{e^-_{RH}e^+_{LH}})\ll$

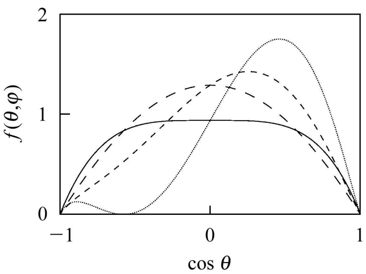


Fig. 3. $f(\theta,\varphi)$ as a function of $\theta$ at $\varphi=\frac{\pi}{4}$ for three representative sets of parameters specifying the beam and preferred directions. The long-dashed curve is the result upon further averaging over $\varphi$. The SM result is roughly proportional to $\sin^2\theta$[19].

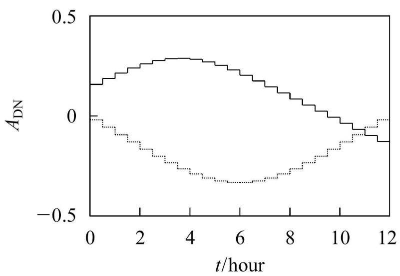


Fig. 4. Day-night asymmetry as a function of time for two representative sets of orientation parameters. The integrated day-night asymmetry is respectively $A_{\rm DN}(0,\pi)=+0.133$ (solid), $-0.196$ (dotted). There is no such asymmetry in SM.

$\sigma_{\rm SM}(e^-_{\rm LH}e^+_{\rm RH})$. There is thus a very different polarization dependence between the two. The sharp difference also appears in the $\varphi$ distributions, and the Lorentz violating feature is further strengthened by the day-night asymmetry. To summarize, we see that (1) there is a good signal over background ratio for unpolarized beams, and excellent signal over background ratio can be reached with suitably polarized beams; (2) the Earth's rotation can be used as an advantage in discriminating NC signals from ordinary new physics.

Finally we mention very briefly some recent developments concerning realistic model building on NC spacetime. The Durham group realizes that it is possible to employ the UV/IR mixing to separate out the excessive factors of $U(1)$. The basic idea is that while trace $U(1)$ of $U(N)$ runs as $SU(N)$ in the ultraviolet, it runs oppositely in the infrared[20]. This means that trace $U(1)$ decouples in the infrared. Thus at low energies explored in experiments, $U(N)$ resembles $SU(N)$. This offers some kind of dynamical splitting of $U(N)\rightarrow SU(N)\times U(1)$ while product of $SU(N)$ factors breaks down spontaneously by the Higgs mechanism. Very rich phenomenology is expected at both linear and hadronic colliders in this approach[21].

*I would like to thank H. Grosse and K. Sibold for collaboration over the years on which the talk is based.*

## References


1 Snyder H S. Phys. Rev., 1974, **71**: 38

2 Connes A, Douglas M R, Schwarz A. JHEP, 1998, **9802**: 003; Douglas M R, Hull C. JHEP, 1998, **9802**: 008; Seiberg N, Witten E. JHEP, 1999, **9909**: 032

3 Seiberg N, Witten E. J. High Energy Phys., 1999, **09**: 032. hep-th/9908142

4 Chaichian M, Kulish P P, Nishijima K. Phys. Lett., 2004, **B604**: 98. hep-th/0408069

5 Gomis J, Mehen T. Nucl. Phys., 2000, **B591**: 265. hep-th/0005129; Seiberg N, Susskind L, Toumbas N. J. High Energy Phys., 2000, **06**: 044. hep-th/0005015

6 Bahns D, Doplicher S, Fredenhagen K et al. Phys. Lett., 2002, **B533**: 178. hep-th/0201222; LIAO Y, Sibold K. Eur. Phys. J., 2002, **C25**: 469. hep-th/0205269; LIAO Y, Sibold K. Eur. Phys. J., 2002, **C25**: 479. hep-th/0206011

7 Minwalla S, Raamsdonk M V, Seiberg N. J. High Energy Phys., 2000, **02**: 020. hep-th/9912072; Aref'eva I Ya, Belov D M, Koshelev A S. Phys. Lett., 2000, **B476**: 431. hep-th/9912075; Raamsdonk M V, Seiberg N. J. High Energy Phys., 2000, **03**: 035. hep-th/0002186; Matusis A, Susskind L, Toumbas N. J. High Energy Phys., 2000, **12**: 002. hep-th/0002075

8 Seiberg N, Susskind L, Toumbas N. J. High Energy Phys., 2000, **06**: 044. hep-th/0005015; Alvarez-Gaume L, Barbon J L F. Int. J. Modern Phys., 2001, **A16**: 1123. hep-th/0006209

9 Alvarez-Gaume L, Barbon J L F, Zwicky R. J. High Energy Phys., 2001, **05**: 057. hep-th/0103069

10 LIAO Y, Sibold K. Phys. Lett., 2002, **B549**: 352. hep-th/0209221

11 Terashima S. Phys. Lett., 2000, **B482**: 276. hep-th/0002119; Matsubara K. ibid, 2000, **B482**: 417. hep-th/0003294

12 Armoni A. Nucl. Phys., 2001, **B593**: 229. hep-th/0005208

13 Gracia-Bondia J M, Martin C P. Phys. Lett., 2000, **B479**: 321. hep-th/0002171

14 Chaichian M, Presnajder P, Sheikh-Jabbari M M et al. Phys. Lett., 2002, **B526**: 132—136. hep-th/0107037

15 Hayakawa M. Phys. Lett., 2000, **B478**: 394. hep-th/9912094, hep-th/9912167

16 Arfaei H, Yavartanoo M H. hep-ph/0010244; Hewett J L, Petriello F J, Rizzo T G. hep-ph/0010354; Mathews P. Phys. Rev., 2001, **D63**: 075007; Godfrey S, Doncheski M A. Phys. Rev., 2002, **D65**: 015005. hep-ph/0108268
17 Baek S, Ghosh D K, HE X G et al. Phys. Rev., 2001, **D64**: 056001
18 Grosse H, LIAO Y. Phys. Rev., 2001, **D64**: 115007
19 Gaemers K J F, Hoogeveen F. Z. Phys., 1984, **C26**: 249; Djouadi A, Driesen V, Junger C. Phys. Rev., 1996, **D54**: 759
20 Khoze V V, Travaglini G. J. High Energy Phys., 2001, **01**: 026. hep-th/0011218
21 Khoze V V, Levell J. J. High Energy Phys., 2004, **09**: 019. hep-th/0406178

# 非对易时空中的量子场论及其唯象学

廖益

(南开大学物理系　天津　300071)

**摘要**　首先介绍非对易时空量子场论的基本思想, 并简短地回顾直线对撞机上的唯象学研究. 然后, 较详细地讨论通过$e^+e^-$碰撞的中性Higgs粒子对产生来探测非对易信号, 及如何利用洛仑兹对称性破坏从标准模型背景中分离出信号. 最后, 简要地提及构造现实模型方面的近期进展.

**关键词**　非对易时空　直线对撞机　Higgs粒子

2007 – 03 – 30 收稿