Probing Noncommutative Space-Time Scale Using $\gamma\gamma \to$ Z at ILC

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Abstract In this talk we report our work on testing Noncommutative Space-Time Scale Using $\gamma\gamma \to$ Z at ILC. In ordinary space-time theory, decay of a spin-1 particle into two photons is strictly forbidden due to the Yang's Theorem. With noncommutative space-time this process can occur. This process thus provides an important probe for noncommutative space-time. The $\gamma\gamma$ collision mode at the ILC provides an ideal place to carry out such a study. Assuming an integrated luminosity of 500fb^{-1}, we show that the constraint which can be achieved on $\Gamma(\mathrm{Z}\to\gamma\gamma)$ is three to four orders of magnitude better than the current bound of 5.2×10^{-5}GeV. The noncommutative scale can be probed up to a few TeVs.

Key words noncommutative space-time, Yang's theorem, photon collider

In this talk we report our work[1] on testing Noncommutative Space-Time Scale Using $\gamma\gamma \to$ Z at ILC. In ordinary space-time field theory, decay of a spin-1 particle into two photons is strictly forbidden due to the Yang's Theorem[2]. Therefore $\gamma\gamma \to$ Z cannot occur in the Standard Model (SM). With noncommutative space-time this process can occur. This process thus provides an important probe for noncommutative space-time. The $\gamma\gamma$ collision at the ILC by laser backscattering of the electron and positron beams provides an ideal place to carry out such a study.

To start with, let us briefly review why Z $\to \gamma\gamma$ cannot occur in ordinary space-time field theory by constructing Z-γ-γ interaction from Z_μ, $F_{\mu\nu}$.

The Lagrangian must be symmetric in the two photons F_1 and F_2 due to the Bose-Einstein statistics. Using $\partial_\mu F^{\mu\nu}=0$, the independent terms with even parity that can be constructed are

$$\partial_\nu Z_\mu(F_1^{\mu\alpha}F_{2\alpha}^{\nu}+F_2^{\mu\alpha}F_{1\alpha}^{\nu}),$$
$$\partial_\mu Z_\nu(F_1^{\mu\alpha}F_{2\alpha}^{\nu}+F_2^{\mu\alpha}F_{1\alpha}^{\nu}),$$
$$Z_\mu(\partial_\nu F_1^{\mu\alpha}F_{2\alpha}^{\nu}+\partial_\nu F_2^{\mu\alpha}F_{1\alpha}^{\nu}).$$

In momentum space, the first term is given by

$$(k_1+k_2)\cdot\epsilon^Z(k_1\cdot\epsilon_2 k_2\cdot\epsilon_1-k_1\cdot k_2\epsilon_1\cdot\epsilon_2),$$

which is zero for on-shell Z. Similarly, one can show that the other terms are also zero when particles are on-shell.

Another type of terms involves $\tilde{F}^{\mu\nu}=(\mathrm{i}/2)\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ which has odd parity. Using $\partial_\mu\tilde{F}^{\mu\nu}=0$, we find the independent terms to be given by

$$\partial_\nu Z_\mu(\tilde{F}_1^{\mu\alpha}F_{2\alpha}^{\nu}+\tilde{F}_2^{\mu\alpha}F_{1\alpha}^{\nu}),\ \partial_\mu Z_\nu(\tilde{F}_1^{\mu\alpha}F_{2\alpha}^{\nu}+\tilde{F}_2^{\mu\alpha}F_{1\alpha}^{\nu}),$$
$$\epsilon_{\mu\nu\sigma\rho}\partial^\sigma Z^\rho(F_1^{\mu\alpha}F_{2\alpha}^{\nu}+F_2^{\mu\alpha}F_{1\alpha}^{\nu}),$$
$$\epsilon_{\mu\nu\sigma\rho}Z^\rho(\partial^\sigma F_1^{\mu\alpha}F_{2\alpha}^{\nu}+\partial^\sigma F_2^{\mu\alpha}F_{1\alpha}^{\nu}).$$

The first term in momentum space is given by

$$\epsilon_{\mu\nu\sigma\rho}Z^\rho[k_1^\sigma k_2^\nu(\epsilon_1^\mu\epsilon\cdot k_1-\epsilon_2^\mu\epsilon_1\cdot k_2)-(k_1^\sigma-k_2^\sigma)\epsilon_1^\mu\epsilon_2^\nu k_1\cdot k_2]. \tag{1}$$

In this frame the momenta and polarizations of the prticles are given by

$$P_z=(m_z,0,0),\ Z^\rho=(0,\boldsymbol{\epsilon}_Z),$$
$$k_1=(k_z,0,k_z),\ k_2=(k_z,0,-k_z),$$
$$\epsilon_1^L\sim(k_z,0,k_z),\ \epsilon_2^L\sim(k_z,0,-k_z),$$
$$\epsilon_1^T=(0,\boldsymbol{a},0),\ \epsilon_2^T=(0,\boldsymbol{b},0).$$

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Inserting the above into Eq. (1), one can easily check that the contribution is zero. Similarly, one can show that the other terms are also zero for on-shell particles. Z$\rightarrow\gamma\gamma$ and $\gamma\gamma\rightarrow$Z are forbidden.

In noncommutative (NC) space-time[3], the processes Z $\rightarrow\gamma\gamma$ and $\gamma\gamma\rightarrow$ Z are not forbidden. We now describe how this can happen by using a simple and commonly studied noncommutative quantum field theory based on the following commutation relation of space-time[4],

$$[\hat{x}_\mu,\hat{x}_\nu]=\mathrm{i}\Theta_{\mu\nu}, \tag{2}$$

as an example. In the above expression, $\hat{x}_\mu$ is the noncommutative space-time coordinates. $\Theta_{\mu\nu}$ is a constant, real, anti-symmetric matrix, and has mass^{-2} dimension. The size of $1/\sqrt{|\Theta_{\mu\nu}|}$ represents the noncommutative scale $\Lambda_{\rm NC}$. There have been extensive studies on related phenomenology[5].

Quantum field theory based on the commutation relation in Eq. (2) can be easily studied using the Weyl-Moyal correspondence replacing the product of two fields $A(\hat{x})$ and $B(\hat{x})$ with NC coordinates by the star "*" product[6]

$$\begin{aligned}&A(\hat{x})B(\hat{x})\rightarrow\hat{A}(x)*\hat{B}(x)=\\&\exp\left[\mathrm{i}\frac{1}{2}\Theta_{\mu\nu}\partial_x^\mu\partial_y^\nu\right]A(x)B(y)|_{x=y}\ .\end{aligned} \tag{3}$$

Here the fields with and without 'hat' indicate the fields in the noncommutative space-time and the ordinary space-time, respectively.

The promotion of the usual space-time coordinates x_μ to the noncommutative space-time coordinates $\hat{x}_\mu$ has very interesting consequences[7]. We denote the noncommutative gauge field to be $\hat{A}_\mu=\hat{A}^a_\mu T^a$ of a group with generators normalized as $\mathrm{Tr}(T^aT^b)=\delta^{ab}/2$. In noncommutative space-time two consecutive local gauge transformations $\hat{\alpha}$ and $\hat{\beta}$ of a gauge field $\hat{A}_\mu$ of the type $\delta_\alpha\hat{\Psi}=\mathrm{i}\hat{\alpha}*\hat{\Psi}$ on matter field $\hat{\Psi}$, transforming as a fundamental representation of the gauge group, is given by $(\delta_\alpha\delta_\beta-\delta_\beta\delta_\alpha)=(\hat{\alpha}*\hat{\beta}-\hat{\beta}*\hat{\alpha})$. This commutation relation is consistent with $U(N)$ Lie algebra, but not consistent with $SU(N)$ Lie algebra since it cannot be reduced to the matrix commutator of the $SU(N)$ generators. Also note that even with $U(1)$ group the above consecutive transformation does not commute implying that the charge for a $U(1)$ gauge theory is fixed to only three possible charges which can be normalized to 1, 0, −1.

The above properties pose difficulties in constructing noncommutative standard model for the strong and electroweak interactions because the standard gauge group contains $SU(3)_{\rm C}$ and $SU(2)_{\rm L}$ which cannot be naively gauged with noncommutative space-time. Also the charges of $U(1)_{\rm Y}$ are not just 1, 0, −1, some of them are fractionally charged after normalizing the right-handed electron to have −1 hypercharge, such as, 1/6, 1/2, 2/3, −1/3 for left-handed quarks, left-handed leptons, right-handed up and down quarks, respectively. This is the so called charge quantization problem. However, all these difficulties can be overcome with the use of the Seiberg-Witten (SW)[6] map which maps noncommutative gauge field to ordinary commutative gauge field. A consistent noncommutative $SU(N)$ gauge theory can be constructed by expanding $\hat{\alpha}$ to powers of Θ with $\hat{\alpha}=\alpha+\alpha^{(1)}_{ab}:T^aT^b:+...+\alpha^{(n-1)}_{a_1...a_n}:T^{a_1}...T^{a_n}:...$ to form a closed envelop algebra. Here '$:T^{a_1}...T^{a_n}:$' is totally symmetric in exchanging a_i. Detailed description of the method can be found in Ref. [8]. One can then expand gauge and mater fields in powers of Θ to have a consistent $SU(N)$ gauge theory order by order in Θ. To the first order in Θ, one has for the gauge field[8]

$$\hat{A}_\mu=A_\mu-\frac{1}{4}g_N\Theta^{\alpha\beta}\{A_\alpha,\partial_\beta A_\mu+F_{\beta\mu}\}. \tag{4}$$

Using the above gauge field new terms in the interaction Lagrangian compared with the ordinary $SU(N)$ gauge theory will be generated. For example the term $-(1/2)\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$ in the Lagrangian for a $SU(N)$ gauge field will become, to the first order in Θ[8],

$$\begin{aligned}L=&-\frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}+\\&g_N\Theta^{\mu\nu}\frac{1}{4}\mathrm{Tr}[F_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}-4F_{\mu\rho}F_{\nu\sigma}F^{\rho\sigma}].\end{aligned} \tag{5}$$

The SW map can also cure the charge quantization problem by associating a gauge field $\hat{A}^{(n)}_\mu$ for the a matter field $\psi^{(n)}$ with $U(1)$ charge $gQ^{(n)}$, $\hat{A}^{(n)}_\mu=A_\mu-(gQ^{(n)}/4)\Theta^{\alpha\beta}\{A_\alpha,\partial_\beta A_\mu+F_{\beta\mu}\}$, where A_μ is the gauge field of $U(1)$ in ordinary space-time. With the help of SW map specific method to construct NCSM and grand unified theories have been developed[8—11].

The new terms in Eq. (5) when applied to the NCSM will generate terms inducing Z-γ-γ interaction. These terms can be parameterized as

$$L_{Z\gamma\gamma} = eg_{Z\gamma\gamma}\Theta^{\alpha\beta}(8Z_{\mu\alpha}A_{\nu\beta}A^{\mu\nu}+4A_{\mu\alpha}A_{\nu\beta}Z^{\mu\nu}- 2A_{\alpha\beta}A_{\mu\nu}Z^{\mu\nu}-Z_{\alpha\beta}A_{\mu\nu}A^{\mu\nu}). \tag{6}$$

In NCSM, $g_{Z\gamma\gamma}$ is not uniquely determined due to the need of introducing for each matter field with different $U(1)_Y$ charge a gauge field to solve the charge quantization problem[8]. This is because that when summing over different $U(1)_Y$ gauge fields for all matter fields to give the kinetic energy term, even when the first term in Eq. (5) is fixed with the right normalization, the triple gauge field terms are not fixed. This problem may be solved by obtaining low energy NCSM from grand unified theories such as noncommutative $SO(10)$[9], $SU(5)$[10] grand unification and $SU(3)^3 = SU(3)_C\times SU(3)_L\times SU(3)_R$ trinification[11] theories where there is no $U(1)$ charge quantization problem to start with. In noncommutative $SO(10)$ grand unification, due to the same reason as for anomaly free in this theory, the triple gauge coupling is automatically zero, and therefore in this model $\gamma\gamma\to Z$ cannot occur. Naively, noncommutative $SU(5)$ grand unification can fix the triple gauge boson couplings[10]. However, in this model, there are several different multiplets for fermion and Higgs representations, $\bar{5}$, 10, 24, etc., one needs to associate different gauge fields with them which lead to a similar problem of non-uniqueness of triple gauge boson couplings for different $U(1)_Y$ gauge field in the NCSM[9]. $SU(5)$ is not truly a unified model in noncommutative space-time. Unique non-trivial triplet gauge boson couplings can be generated in noncommutative trinification model[11]. In this model, the fermion and Higgs representations are all in the 27 representation of the gauge group resulting in fixed triple gauge boson couplings. The coupling $eg_{Z\gamma\gamma}$ in $SU(3)^3$ is given, at the unification scale, by[11]

$$eg_{Z\gamma\gamma} = -\frac{g_U}{16\sqrt{15}}\,\frac{4}{5}\sin\theta_W\left(1+\frac{19}{4}\cos2\theta_W\right). \tag{7}$$

Using the normalization $g_Y=\sqrt{3/5}g_U$, and running down to energy scale $\mu=m_Z$, we have $eg_{Z\gamma\gamma} = -5.58\times10^{-3}$. In the rest of the discussions we will use noncommutative $SU(3)^3$ as an illustration to show how the limit on the noncommutative scale can be determined using $\gamma\gamma\to Z$ at the photon collision mode of ILC.

The matrix element for on-shell $\gamma(k)\gamma(k')\to Z(p)$ in momentum space after symmetrizing the two photons is given by

$$M = -ieg_{Z\gamma\gamma}16[k\cdot k'(k'\cdot\epsilon_Z^*\epsilon\cdot\Theta\cdot\epsilon'+ \epsilon'\cdot\epsilon_Z^*k\cdot\Theta\epsilon'+\epsilon'\cdot\epsilon_Z^*k'\cdot\Theta\cdot\epsilon)+ k\cdot\Theta\cdot k'(k'\cdot\epsilon\epsilon'\cdot\epsilon_Z^*-k\cdot\epsilon'\epsilon\cdot\epsilon_Z^*+\epsilon\cdot\epsilon'k\cdot\epsilon_Z^*)], \tag{8}$$

where $a\cdot\Theta\cdot b=a_\alpha\Theta^{\alpha\beta}b_\beta$.

With the above expression, we obtain

$$\sigma(\gamma\gamma\to Z,s)=6\pi^2\frac{m_Z\Gamma(Z\to\gamma\gamma)}{m_Z^2}\delta(s-m_Z^2),$$
$$\Gamma(Z\to\gamma\gamma)=\frac{4}{3}\alpha_{em}g_{Z\gamma\gamma}^2m_Z^5\left(\Theta_S^2+\frac{7}{3}\Theta_T^2\right), \tag{9}$$

where $\Theta_T^2=\theta_{01}^2+\theta_{02}^2+\theta_{03}^2$ and $\Theta_S^2=\theta_{12}^2+\theta_{13}^2+\theta_{23}^2$. The expression for $\Gamma(Z\to\gamma\gamma)$ agrees with that obtained in Ref. [12]. It is clear that the on-shell processes $Z\to\gamma\gamma$ and $\gamma\gamma\to Z$ can occur in noncommutative space-time.

At ILC using $\gamma\gamma$ collision mode one can test the space-time nature by studying $\gamma\gamma\to Z$. Convoluting the energies of the two photon beams produced by using the laser backscattering technique[13] on the electron and positron beams in an e^+e^- collider with the center of mass frame energy $\sqrt{s}$, we have

$$\sigma_c = \int_{x_{\min}}^{x_{\max}}dx_1\int_{x_{\min}}^{x_{\max}}dx_2\sigma(\gamma\gamma\to Z,x_1x_2s)\times F(x_1)F(x_2)= I(m_Z^2/s)6\pi^2\frac{m_Z\Gamma(Z\to\gamma\gamma)}{m_Z^4}, \tag{10}$$

where

$$I(y)=\int_{y/x_{\max}}^{x_{\max}}dx\frac{y}{x}F(x)F\left(\frac{y}{x}\right), \tag{11}$$

with $y=m_Z^2/s$, and $x_{\max}=\xi/(1+\xi)$ with $\xi=2(1+\sqrt{2})$. The $F(x)$ function is given by

$$F(x)=\frac{1}{D(\xi)}\left(1-x+\frac{1}{1-x}-\frac{4x}{\xi(1-x)}+\frac{4x^2}{\xi^2(1-x)^2}\right),$$
$$D(\xi)=\left(1-\frac{4}{\xi}-\frac{8}{\xi^2}\right)\ln(1+\xi)+\frac{1}{2}+\frac{8}{\xi}-\frac{1}{2(1+\xi)^2}. \tag{12}$$

Note that the function $I(y)$ is a function of m_Z^2/s only. The model-dependent part is purely in the expression for $\Gamma(Z\to\gamma\gamma)$. In Fig. 1 we show $I(y)$ as a function of y. We see that for a large range of m_Z^2/s, $I(y)$ is sizeable. An ILC of energy between 120 to 250GeV can be very useful for studying $\gamma\gamma\to Z$. When the energy becomes higher the cross section goes down. If there is a Z′ particle with a mass of a few hundred GeV, an ILC of energy around several hundred GeV to one TeV would be an excellent place to look for Z′ using $\gamma\gamma\to Z'$.

The proposed ILC energy will be in the range from several hundred GeV to TeV, and therefore can be an ideal place to study $\gamma\gamma\to Z$. We list the upper bounds reachable (using current bound 5.2×10^{-5}GeV[14] on the decay rate $\Gamma(Z\to\gamma\gamma)$) on the signal event number and the decay rate in Table 1, assuming an integrated luminosity of 500fb^{-1}. We see that if a theory gives Γ for $Z\to\gamma\gamma$ close to the current upper limit of 5.2×10^{-5}GeV, one would see more than 10^5 events. If no events are seen, this would translate into a bound on the rate Γ of $Z\to\gamma\gamma$ to be less than a few times 10^{-10}GeV. This is much better than the constraint obtained before[12]. Even assuming an efficiency as low as 1%, one can still set an upper bound of $\Gamma<10^{-8}$GeV which is still more than three orders of magnitude better than the current bound.

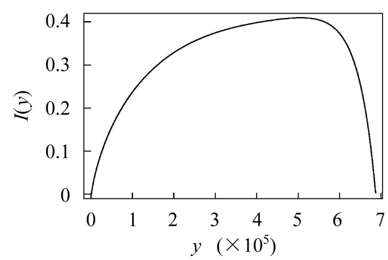

Fig. 1. The function $I(y)$.

Table 1. The upper limit on event number for $e^+e^-\to\gamma\gamma\to Z$ (with current bound 5.2×10^{-5}GeV on $\Gamma(Z\to\gamma\gamma)$, and upper bound on $\Gamma(Z\to\gamma\gamma)$). In obtaining these bounds the integrated luminosity is assumed to be 500fb^{-1}.

$\sqrt{s}$/GeV	120	200	250	500	1000
$I/(m_Z^2/s)$	0.397	0.333	0.275	0.120	0.043
upper limit on event number	3.13×10^5	2.65×10^5	2.19×10^5	0.95×10^5	0.34×10^5
upper bound on Γ/GeV	1.66×10^{-10}	1.96×10^{-10}	2.38×10^{-10}	5.45×10^{-10}	1.51×10^{-9}
$SU(3)^3$: Λ_S/TeV	4.72	4.53	4.31	3.51	2.72
$SU(3)^3$: Λ_T/TeV	5.83	5.59	5.33	4.33	3.36

One can obtain the bound on the noncommutative scale Λ_{NC} from the bound on the event rate for $\gamma\gamma\to Z$. We list the upper limits on the scales $\Lambda_S=1/\sqrt{\Theta_S^2}$ and $\Lambda_T=1/\sqrt{\Theta_T^2}$ in Table 1 in the last two rows. We see that the noncommutative scale can be probed up to a few TeV. If the efficiency is lowered to 1%, the noncommutative scale can still be probed up to 1.5TeV.

To summarize, we have studied a strictly forbidden process $\gamma\gamma\to Z$ in the standard model of strong and electroweak interactions, but allowed in the noncommutative space-time theories. We have shown that the $\gamma\gamma$ collision mode at the ILC by laser backscattering of the electron and positron beams with an integrated luminosity of 500fb^{-1} can obtain a constraint on $\Gamma(Z\to\gamma\gamma)$ three to four orders of magnitude better than the current bound of 5.2×10^{-5}GeV. The noncommutative scale can be probed up to a few TeV. The process $\gamma\gamma\to Z$ at ILC can be a very powerful test for NCSM.

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在ILC上用$\gamma\gamma \to Z$过程检验非对易时空标度

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摘要 讨论关于在ILC用gamma gamma到Z过程检验非对易时空能标(原文发在hep-ph/0604115). 在通常时空量子场论中, 由杨氏定理可知一个自旋为1的粒子不可能衰变为两个光子. 但在非对易时空中此过程是允许的. 因此这个过程能作为检验非对易时空的工具. ILC的光子对撞模式能实现这个过程. 如果总亮度能达到500fb^{-1}, 我们证明对Gamma (Z to gamma gamma)宽度的测量精度将比现有限制($<5.2\times10^{-5}$GeV)好3—4个数量级. 对非对易时空能标的检测可高达几个TeV.

关键词 非对易时空 杨氏定理 光子对撞机

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