Measurement of Strong Coupling Constant at Low Energy Range*

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Abstract The strong coupling $\alpha_s(s)$ is an important free parameter of Quantum Chromodynamics. Based on R values measured at BES, the values of the strong running coupling constant $\alpha_s(s)$ at 2.0—3.7GeV are determined using the $\mathcal{O}(\alpha_s^3)$ and $\mathcal{O}(\alpha_s^4)$ order expressions calculated by pQCD, then $\alpha_s(s)$ is deduced to the M_z scale. The numerical prediction on the improvement of the uncertainty of $\alpha_s(s)$ with the decrease of the experimental error of R value in the future experiment is also given.

Key words R value, strong coupling constant, least squares fit

1 Introduction

The strong coupling α_s is a basic parameter in Quantum Chromodynamics (QCD). The precise determination of α_s and its evolution with energy have significant effects on all the hadronic theories and experiments. QCD predicts the energy dependence of α_s and the asymptotic freedom property. But the actual value of α_s cannot be predicted by QCD, it must be determined from experiments, such as deep inelastic scattering[1], τ decay[2, 3] and e^+e^- annihilation[4] processes.

The cross section of $e^+e^- \rightarrow$ hadrons is often expressed as R value, $\sigma_{had}(s) = R\cdot\sigma_{\mu\mu}(s)$, where s is the squared center-of-mass energy in e^+e^- annihilation. The value of α_s can be obtained by solving the equation $R_{QCD}(\alpha_s) = R_{exp}(s)$ with the conventional method[4], where $R_{QCD}(\alpha_s)$ is the expression calculated by perturbative QCD, and R_{exp} is the experimental R value. Fig. 1 shows the values of α_s and their uncertainties determined from experiments.

In this work, R values measured at BES[5, 6] between 2.0—3.7GeV are used to determine α_s by both means of solving the equation and the least squares fitting respectively, and give its evolution to M_z scale. Through the latter method we can obtain the dimensional parameter Λ of QCD, and may predict the value of $\alpha_s(s)$ at any energy in the fitting region, instead of only getting the separate values of $\alpha_s(s_i)$ at the experimental energy points s_i, like the issue in the scheme of solving the equation. In the last section,

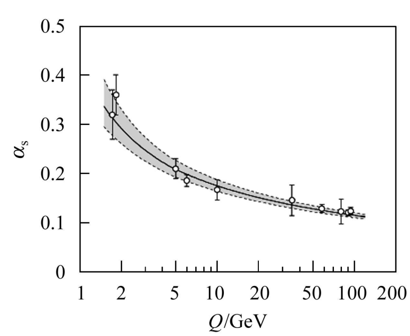

Fig. 1. The energy dependence of α_s. The hollow dots with error bars are experimental results. The dash line is the experimental average, and the shadow indicates the region within $\pm 1\sigma$.

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the numerical prediction on the improvement of the uncertainty of $\alpha_s(s)$ with the decrease of the experimental error of R value is also given.

2 QCD predictions on $\alpha_s(s)$ and R

In QCD, $\alpha_s(s)$ actually depends on the energy scale Q^2. If the renormalized coupling α_s is fixed at a certain given scale μ^2, QCD can precisely derive the value of α_s at any other energy scale Q^2 through the renormalization group equation[7]

$$Q^2\frac{\partial\alpha_s(Q^2)}{\partial Q^2}=\beta(\alpha_s(Q^2)). \tag{1}$$

In complete 4-loop approximation and using the Λ-parametrization, the running coupling is given by

$$\begin{aligned}\alpha_s(Q^2) = {} & \frac{1}{\beta_0 L}-\frac{\beta_1\ln(L)}{\beta_0^3L^2}+\\ & \frac{1}{\beta_0^3L^3}\left(\frac{\beta_1^2}{\beta_0^2}(\ln^2(L)-\ln(L)-1)+\frac{\beta_2}{\beta_0}\right)+\\ & \frac{1}{\beta^4L^4}\left[\frac{\beta_1^3}{\beta_0^3}\left(-\ln^3(L)+\frac{5}{2}\ln^2L+2\ln L-\frac{1}{2}\right)-\right.\\ & \left.3\frac{\beta_1\beta_2}{\beta_0^2}\ln(L)+\frac{\beta_3}{2\beta_0}\right],\end{aligned} \tag{2}$$

where $L=\ln(Q^2/\Lambda^2_{\overline{\rm MS}})$, $\overline{\rm MS}$ indicates the modified minimal subtraction scheme, and

$$\begin{aligned}\beta_0 &= \frac{33-2N_f}{12\pi},\\ \beta_1 &= \frac{153-19N_f}{24\pi^2},\\ \beta_2 &= \frac{77139-15099N_f+325N_f^2}{3456\pi^3},\\ \beta_3 &\approx \frac{29243-6946.3N_f+405.089N_f^2+1.49931N_f^3}{256\pi^4},\end{aligned}$$

with the number of active flavor N_f.

The strong coupling α_s is not a direct observable quantity by itself, it should be determined by the experimental observable. R value can be expressed by a perturbation series in powers of the coupling parameter $\alpha_s(s)$. Up to the $\mathscr{O}(\alpha_s^3)$ order, it may be written as[8]:

$$\begin{aligned}R_{\rm QCD}(s) = {} & 3\sum_f Q_f^2\left[1+\left(\frac{\alpha_s(s)}{\pi}\right)+r_1\left(\frac{\alpha_s(s)}{\pi}\right)^2+\right.\\ & \left. r_2\left(\frac{\alpha_s(s)}{\pi}\right)^3\right]+\mathscr{O}(\alpha_s^4),\end{aligned} \tag{3}$$

where

$$\begin{aligned}r_1 &= 1.9857-0.1153N_f,\\ r_2 &= -6.6368-1.2001N_f-0.0052N_f^2-\\ & \quad 1.2395\frac{\left(\sum_f Q_f\right)^2}{3\sum_f Q_f^2},\end{aligned}$$

with quark electric charge Q_f. In this work, N_f=3 and Q_f is the electric charge of u, d, s quark.

If considering the higher order QCD correction, i.e. up to the 4-loop approximation ($\mathscr{O}(\alpha_s^4)$)[9],

$$\begin{aligned}R_{\rm QCD}(s) = {} & 3\sum_f Q_f^2\left[1+\left(\frac{\alpha_s(s)}{\pi}\right)+(1.98571-\right.\\ & 0.115295N_f)\left(\frac{\alpha_s(s)}{\pi}\right)^2+(-6.63694-\\ & 1.20013N_f-0.00517836N_f^2)\left(\frac{\alpha_s(s)}{\pi}\right)^3+\\ & \left.\left(\frac{\alpha_s(s)}{\pi}\right)^4 r_0^{V,4}\right]+\mathscr{O}(\alpha_s^5).\end{aligned} \tag{4}$$

For the coefficient of $\left(\frac{\alpha_s(s)}{\pi}\right)^4$, it can be further decomposed as a polynomial in N_f , namely

$$r_0^{V,4}=r_{0,0}^{V,4}+r_{0,1}^{V,4}N_f+r_{0,2}^{V,4}N_f^2+r_{0,3}^{V,4}N_f^3, \tag{5}$$

with[10]

$$r_{0,0}^{V,4}=-186,\qquad r_{0,1}^{V,4}=21.3,$$
$$r_{0,2}^{V,4}=-0.797,\qquad r_{0,3}^{V,4}=2.15\times10^{-2}.$$

In the above calculations, the massless approximation for the u, d and s quarks are adopted.

3 The determination of $\alpha_s(s)$

The energy range 2.0—3.7GeV belongs to the continuous region (except for the narrow resonances J/ψ and ψ′), and it is below the open charm threshold. The interactive energy is far larger than the mass of the active quarks (u, d and s), so the prediction of the perturbative QCD is reliable in this energy region. In the following, two methods are used to determine α_s from the measured R values, one is to solve the equation, and the other is to adopt the method of least squares. In QCD, the energy dependence of the running α_s is a smooth curve, so using the least squares fitting guarantees the consistency and smoothness of the energy dependence of α_s, and avoids the discrepancy of the value of α_s brought by the experimental errors of R value in the method of solving equation.

It is noticed that QCD indicates the strict restriction on the value of R_{QCD}. Fig. 2 shows the variation of R_{QCD} with α_s varying from 0.0 to 1.0. It is found that the maximum R_{QCD} predicted by QCD theory is 2.385 for 3-loop approximation, and 2.1985 for 4-loop approximation. Therefore, some experimental values R_{exp} between 2.0—3.7GeV measured with BEPC/BES and other groups surpass the upper limit permitted by QCD, hence only the R values at four energy points $\sqrt{s}$=2.8, 2.9, 3.0 and 3.7GeV are used in the method of solving equation, and the R values at eleven energy points $\sqrt{s}$=2.0, 2.2, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.0, 3.7 and 3.73GeV are adopted in the least squares fitting.

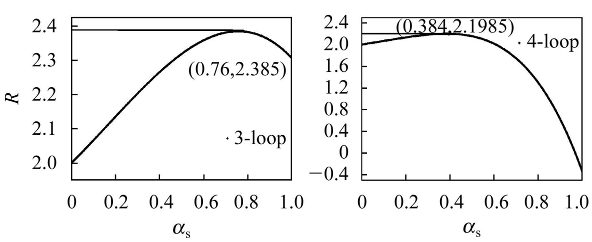

Fig. 2. The variation of theoretical R_{QCD} with α_s, the left one is for Eq. (3), and the right one is for Eq. (4).

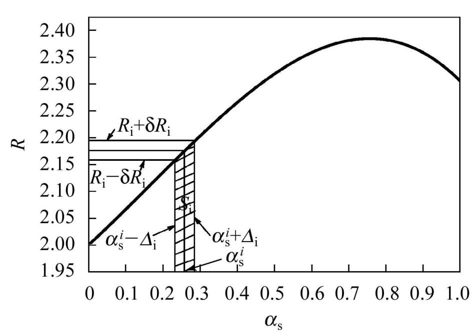

Fig. 3. The error calculation of $\overline{\alpha_s}$(5GeV).

In the method of solving the algebraic equation, it is supposed that the theoretical $R_{QCD}(s)$ is equal to the experimental value within one standard deviation: $R_{QCD}=R_{exp}\pm\Delta R_{exp}$, then $\alpha_s(s)\pm\Delta\alpha_s(s)$ at energy s is obtained by solving Eq. (3) or Eq. (4). The value of $\alpha_s(s)$ at other energy scale (usually to the standard reference scale M_Z) can be derived from Eq. (2)[11]. The error calculation of $\alpha_s(s)$ at each energy point is direct: the experimental values are taken as $R_{ex}+\Delta R_{exp}$ and $R_{ex}-\Delta R_{exp}$ respectively, and $\alpha_{s-\Delta_i'}^{+\Delta_i}$ are obtained, in which Δ_i and Δ_i' are unsymmetrical errors. All $\alpha_s(s_i)$ may evolve to 5GeV using Eq. (2). And for the calculation of the $\overline{\alpha_s}$(5GeV), the area shown in Fig. 3 is used. The R_{QCD} curve and the two vertical lines cross α_s axis at $\alpha_{s_i}+\Delta_i$ and $\alpha_{s_i}-\Delta_i'$ construct a region with area S_i. The weighted average of α_s(5GeV) is

$$\overline{\alpha_s}(5\text{GeV})=\frac{\sum\limits_i \dfrac{\alpha_s(s_i)}{S_i}}{\sum\limits_i \dfrac{1}{S_i}}\ , \tag{6}$$

where the area is

$$S_i=\int_{\alpha_s(s_i)-\Delta_i'}^{\alpha_s(s_i)+\Delta_i} R(\alpha_s)\mathrm{d}\alpha_s\ . \tag{7}$$

The upper and the lower errors $\overline{\Delta}_{up}$ and $\overline{\Delta}_{down}$ of $\overline{\alpha_s}$(5GeV) are calculated respectively through

$$\overline{\Delta}_{up}=\sqrt{\frac{1}{\sum\limits_i \dfrac{1}{\Delta_i^2}}}\ ,\qquad \overline{\Delta}_{down}=\sqrt{\frac{1}{\sum\limits_i \dfrac{1}{\Delta_i'^2}}}\ . \tag{8}$$

Evolving $\overline{\alpha_s}$(5GeV) up to M_Z, we have

$$\alpha_s(M_z)=0.129^{+0.014}_{-0.021}\ , \tag{9}$$

which agrees with the world average value within error[12], $\alpha_s(M_z)=0.1176\pm0.002$. The results are summarized in Table 1 and Table 2.

The second method is the least squares fitting. The object function of fitting is

$$\chi^2=\sum_i\frac{\left(f\cdot R_{exp}(s_i)-R_{QCD}(s_i)\right)^2}{\left(f\cdot\Delta\tilde{R}_{exp}^{(i)}\right)^2}+\frac{(f-1)^2}{\sigma_f^2}, \tag{10}$$

Table 1. The values of α_s determined by R values measured with BES at 2.8, 2.9, 3.0 and 3.7GeV. The values of α_s evolving to 5GeV are also shown. The first term is statistical error, and the second term is systematical error.

$\sqrt{s}$/GeV	R_{exp}	$\alpha_s(s)$	α_s/5GeV
2.80	$2.17\pm0.06\pm0.14$	$0.251^{+0.091+0.233}_{-0.087-0.215}$	$0.207^{+0.056+0.126}_{-0.063-0.162}$
2.90	$2.22\pm0.07\pm0.13$	$0.326^{+0.118+0.249}_{-0.105-0.192}$	$0.257^{+0.064+0.121}_{-0.069-0.135}$
3.00	$2.21\pm0.05\pm0.11$	$0.311^{+0.080+0.215}_{-0.075-0.162}$	$0.251^{+0.047+0.114}_{-0.050-0.116}$
3.70	$2.23\pm0.08\pm0.08$	$0.342^{+0.141+0.141}_{-0.121-0.121}$	$0.296^{+0.095+0.095}_{-0.098-0.094}$

Table 2. The evolution of α_s from 5GeV to M_z scale.

$\sqrt{s}$/GeV	α_s/5GeV	area S	$\overline{\alpha_s}$/5GeV	$\alpha_s(M_z)$
2.80	$0.207^{+0.138}_{-0.174}$	0.6628		
2.90	$0.257^{+0.137}_{-0.151}$	0.6245	$0.254^{+0.066}_{-0.071}$	$0.129^{+0.014}_{-0.021}$
3.00	$0.251^{+0.123}_{-0.127}$	0.5423		
3.70	$0.296^{+0.135}_{-0.133}$	0.5888		

where $R_{\rm exp}(s_{\rm i})$ and $\Delta\tilde{R}^{(\rm i)}_{\rm exp}$ are the R value measured at the energy $s_{\rm i}$ and its error (not includes the common error) respectively, $R_{\rm QCD}(s_{\rm i})$ is the corresponding theoretical expressions in Eq. (3) or Eq. (4); f is the scale factor corresponding to the influence of the common error $\sigma_{\rm f}$, and the sum runs over the measured energy points included in the fitting. The fitted parameters are Λ and f. The dimensional parameter Λ[13] is directly obtained through fit, and $\alpha_s(s)$ is gotten with Eq. (2). The fitted results are

$$\alpha_s(M_z)=0.141^{+0.020}_{-0.025},\quad \Lambda_{\overline{\rm MS}}=0.79\pm0.48{\rm GeV},\quad (11)$$

for 3-loop, and

$$\alpha_s(M_z)=0.131^{+0.011}_{-0.014},\quad \Lambda_{\overline{\rm MS}}=0.58\pm0.25{\rm GeV},\quad (12)$$

for 4-loop approximations respectively, see Fig. 4 and Fig. 5. The fit curve shown in Fig. 5 based on 4-loop approximation is constrained by the model predicted maximum $R_{\rm QCD}$ value 2.1985. The theoretical error may be estimated by comparing the difference between the results of 3-loop and 4-loop approximations. Therefore, the result may be reported as $\alpha_s(M_z)=0.141^{+0.020}_{-0.025}\pm0.010$, the second term represents the theoretical uncertainty.

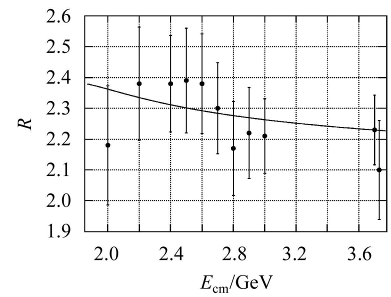

Fig. 4. The fit results for 3-loop approximation, dots with error bars are the experimental data. In the fitting, $\chi^2/n_{\rm d.o.f}$=3.03/9.

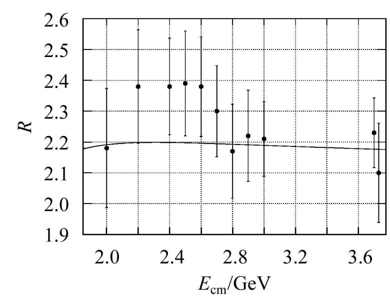

Fig. 5. The fit results for 4-loop approximation, dots with error bars are the experimental data. In the fitting, $\chi^2/n_{\rm d.o.f}$=4.23/9.

4 Determination of $\alpha_s(s)$ in the future

To measure R value with smaller error at the fu-

Table 3. The improvement of the uncertainty of $\alpha_s(s)$ with the decrease of the experimental error of R value.

R error / α_s error	3.0%		2.5%		2.0%		1.5%		1.0%	
$E_{\rm cm}$/GeV	Up(%)	Dw(%)	Up(%)	Dw(%)	Up(%)	Dw(%)	Up(%)	Dw(%)	Up(%)	Dw(%)
2.00	37.7	35.4	31.1	29.6	24.7	23.7	18.4	17.8	12.2	11.9
2.10	38.1	35.9	31.4	29.9	25.0	24.0	18.6	18.1	12.3	12.1
2.20	38.4	36.3	31.8	30.3	25.3	24.3	18.8	18.3	12.5	12.2
2.30	38.8	36.8	32.0	30.7	25.5	24.6	19.0	18.5	12.6	12.4
2.40	39.2	37.2	32.4	31.0	25.8	24.9	19.2	18.7	12.8	12.5
2.50	39.6	37.6	32.8	31.4	26.0	25.2	19.4	18.9	12.9	12.6
2.60	40.0	38.1	33.0	31.8	26.3	25.4	19.6	19.1	13.0	12.7
2.70	40.2	38.5	33.3	32.1	26.5	25.8	19.8	19.3	13.1	12.9
2.80	40.6	38.9	33.6	32.4	26.7	26.0	20.0	19.5	13.2	13.0
2.90	41.0	39.3	33.9	32.7	27.0	26.2	20.2	19.7	13.3	13.2
3.00	41.4	39.7	34.3	33.1	27.3	26.5	20.4	19.9	13.5	13.3
3.10	41.6	40.1	34.4	33.4	27.4	26.7	20.4	20.1	13.5	13.4
3.20	42.0	40.4	34.8	33.7	27.7	27.0	20.7	20.2	13.7	13.5
3.30	42.3	40.8	35.0	34.0	27.8	27.2	20.8	20.4	13.8	13.7
3.40	42.6	41.1	35.3	34.2	28.1	27.4	21.0	20.6	14.0	13.7
3.50	42.9	41.5	35.6	34.6	28.3	27.6	21.1	20.8	14.1	13.8
3.60	43.1	41.8	35.8	34.8	28.3	27.9	21.3	20.9	14.1	14.0
3.70	43.4	42.1	36.0	35.1	28.7	28.1	21.4	21.0	14.2	14.1

ture BEPCⅡ/BESⅢ is one of the important experimental subjects, which will decrease the uncertainty of α_s. Using the similar method, the numerical predictions on the improvement of the uncertainty of $\alpha_s(s)$ with the decrease of the experimental error of R value are given in Table 3. It shows that the uncertainty of α_s is about 12—15 times larger than the error of R value, so to determine α_s with R value is not an economic way.

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低能区强耦合常数的测量*

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摘要 强耦合常数$\alpha_s(s)$是量子色动力学最重要的参数. 基于BES的R值测量结果, 分别利用精确到3圈和4圈的微扰QCD的计算, 测定了$\alpha_s(s)$在2.0—3.7GeV能量范围的数值, 并推断了$\alpha_s(s)$演化到Z^0能标下的值$\alpha_s(M_z)$. 同时对在未来实验中R值测量精度的提高对$\alpha_s(s)$的不确定性的减小作了定量的预言.

关键词 R值 强耦合常数 最小二乘法拟合

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