Giant Monopole Resonance and Symmetry Energy*

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Abstract Nuclear matter incompressibility is discussed by the monopole compression modes in nuclei in the framework of a fully consistent relativistic random phase approximation, based on effective Lagrangians with a mixed isoscalar-isovector nonlinear coupling term. A predicted value of the matter incompressibility coefficient is given by comparison between experimental and calculated energies of the isoscalar giant monopole resonance (ISGMR) in nuclei ^{208}Pb, ^{144}Sm, ^{116}Sn and ^{90}Zr. The new isoscalar-isovector nonlinear coupling softens the nuclear matter symmetry energy without ruining the agreement with experimentally existing ground state properties. The effect of the softening of the symmetry energy on the ISGMR is discussed.

Key words giant monopole resonances, relativistic random phase approximation, symmetry energy, isoscalar and isovector non-linear coupling

1 Introduction

The question of determining the nuclear matter incompressibility coefficient $K_{\rm nm}$ is one of the important issues in present day physics. An accurate determination of compression modulus places important constraints on theoretical models of nuclear structure and a variety of essential phenomena, such as heavy-ion collisions, neutron stars, supernova explosions and so on[1].

The measurement of the centroid energy of the ISGMR provides a very sensitive method to determine the value of $K_{\rm nm}$. Theoretical investigations in various models with assorted values of the nuclear matter incompressibility $K_{\rm nm}$ predict different ISGMR energies. In comparison with the experimental data, one could give the constraint on the nuclear matter incompressibility. However, relativistic and non-relativistic random phase approximation predict different $K_{\rm nm}$, 250—270MeV[2—4] and 220—235MeV[5] respectively. The difference in the values of $K_{\rm nm}$ predicted by relativistic and non-relativistic models is in part attributed to the density dependence of the symmetry energy[6, 7]. The nuclear matter symmetry energy has been extensively studied[8—12], the purpose of this paper is to investigate the effect of the density dependence of the symmetry energy on the ISGMR by introducing a nonlinear coupling of isoscalar and isovector mesons in the relativistic mean field (RMF) approach. The additional nonlinear coupling of isoscalar and isovector mesons softens the symmetry energy without changing the properties of symmetric nuclear matter due to the characteristic of isovector mesons. The collective monopole compression modes are depicted in a fully consistent relativistic random phase approximation(RRPA) built on the RMF ground state. The centroid energies of the ISGMR in nuclei ^{208}Pb, ^{144}Sm, ^{116}Sn, and ^{90}Zr are systematically studied in the RRPA with various parameter sets. Theoretically predicted ISGMR ener-

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gies are compared with the experimental data. Then we investigate the effect of the density dependence of the nuclear matter symmetry energy on the ISGMR.

The paper is arranged as follows. In Sec. 2 the fully consistent RRPA built on the RMF ground state is briefly presented. An additional nonlinear coupling of isoscalar and isovector mesons is introduced in the effective Lagrangian. The monopole compression modes are investigated in Sec. 3. Finally we give the summary in Sec. 4.

2 Fully consistent relativistic random phase approximation

We start from an effective Lagrangian of the form:

$$\begin{aligned}\mathscr{L} = & \bar{\psi}[\gamma^\mu(\mathrm{i}\partial_\mu - g_\omega\omega_\mu - g_\rho\boldsymbol{\tau}\cdot\boldsymbol{b}_\mu - \frac{1}{2}e(1+\tau_3)A_\mu) - \\ & (M+g_\sigma\sigma)]\psi + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \\ & \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - \frac{1}{4}\boldsymbol{b}^{\mu\nu}\cdot\boldsymbol{b}_{\mu\nu} + \\ & \frac{1}{2}m_\rho^2\boldsymbol{b}^\mu\cdot\boldsymbol{b}_\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - U_{\mathrm{eff}}(\sigma,\omega^\mu,\boldsymbol{b}^\mu)\ ,\end{aligned} \tag{1}$$

where M, m_σ, m_ω, m_ρ are the nucleon-, the σ-, the ω- and the ρ- masses, respectively, while g_σ, g_ω, g_ρ and $e^2/4\pi = 1/137$ are the corresponding coupling constants for mesons and the photon; various field tensors have been defined as follows:

$$\omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu\,, \tag{2}$$

$$\boldsymbol{b}_{\mu\nu} = \partial_\mu\boldsymbol{b}_\nu - \partial_\nu\boldsymbol{b}_\mu\,, \tag{3}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,, \tag{4}$$

$$\begin{aligned}U_{\mathrm{eff}}(\sigma,\omega^\mu,\boldsymbol{b}^\mu) = & \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 - \frac{1}{4}c_3(\omega^\mu\omega_\mu)^2 - \\ & 4\Lambda_v g_\rho^2\boldsymbol{b}_\mu\cdot\boldsymbol{b}^\mu g_\omega^2\omega_\mu\omega^\mu\,,\end{aligned} \tag{5}$$

where g_2, g_3 and c_3 are the nonlinear coupling parameters for the self-interactions of the scalar and vector fields. The last term is a mixed nonlinear isoscalar-isovector coupling with a strength Λ_v. The density dependence of the symmetry energy can be changed by tuning the nonlinear coupling strength Λ_v. Increasing Λ_v, the density dependence of the symmetry energy becomes soft (see Fig. 1).

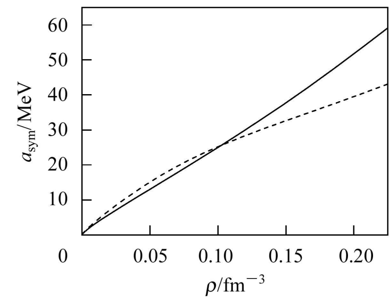

Fig. 1. The density dependence of symmetry energy in nuclear matter for the NL3 parameter set. The cases Λ_v=0 and Λ_v=0.025 are denoted by solid and dash curves, respectively.

Since the symmetry energy at saturation density is not well constrained experimentally, yet some average between the symmetry energy at the saturation density and the surface energy may be constrained by binding energies. As a simple approximation, we keep the symmetry energies fixed at the average density $\rho = 0.1\mathrm{fm}^{-3}$, which is at the nuclear surface. This simple prescription produces a nearly constant proton radius and binding energy, only the neutron radius is changed[13, 14], as can be seen in Table 1.

Table 1. Results for NL3 parameter set. The binding energy per nucleon, E/A, the proton root-mean-square (rms) radius R_p and the neutron skin thickness $R_\mathrm{n}-R_\mathrm{p}$ in ^{208}Pb are listed (center of mass corrections are not included)

Λ_v	g_ρ	E/A	R_p	$R_\mathrm{n}-R_\mathrm{p}$
0.0	4.474	−7.853	5.459	0.281
0.005	4.621	−7.862	5.459	0.266
0.01	4.784	−7.870	5.461	0.252
0.015	4.965	−7.877	5.462	0.238
0.02	5.168	−7.883	5.465	0.224
0.025	5.399	−7.888	5.468	0.209

The fully consistent RRPA is built on the RMF ground state. The details of the RRPA method used in the present study are described in Refs. [15, 16]. The linear response of a system to an external field is given by the imaginary part of the retarded polarization operator,

$$R(Q,Q;\boldsymbol{k},E) = \frac{1}{\pi}\mathrm{Im}\Pi(Q,Q;\boldsymbol{k},\boldsymbol{k};E), \tag{6}$$

where Q is a one-body operator represented by a 4×4 matrix. The retarded polarization operator Π can be obtained as a solution of the Bethe-Salpeter equation[17]:

$$\Pi(Q,Q;\boldsymbol{k},\boldsymbol{k}',E)=\Pi_0(Q,Q;\boldsymbol{k},\boldsymbol{k}',E)-\sum_i g_i^2\int \mathrm{d}^3k_1\mathrm{d}^3k_2\Pi_0(Q,\Gamma^i;\boldsymbol{k},\boldsymbol{k}_1,E)\times D_i(\boldsymbol{k}_1-\boldsymbol{k}_2,E)\Pi(\Gamma_i,Q;\boldsymbol{k}_2,\boldsymbol{k}',E)\ , \tag{7}$$

where Π_0 is the unperturbed (Hartree) polarization operator. The residual $p-h$ interactions are just meson exchanges, described by corresponding propagators D_i. In this equation the index i runs over σ, ω and ρ mesons with g_i being the corresponding coupling constants. The detailed expressions for the $D_i(\boldsymbol{k}_1-\boldsymbol{k}_2,E)$ can be found in Refs. [15,18]. $\Gamma^i=1$ for σ and $\Gamma^i=\gamma^\mu,\gamma^\mu\tau$ for ω and ρ, respectively.

In the field theory, equations of motion for fermion and boson fields are obtained by variations of the action with respect to the corresponding fields. The first order variation of the action with respect to a given meson field ϕ gives the field equation (Klein-Gordon equation) satisfied by this field. The second order variation of the action at the classical value ϕ_0 of the meson field ϕ will lead to the equation of the meson propagator[19]

$$\left(\partial^\mu\partial_\mu+\left.\frac{\partial^2 U_{\rm eff}(\phi)}{\partial\phi^2}\right|_{\phi_0}\right)D_\phi(x,y)=-\delta^4(x,y). \tag{8}$$

In practice, it is more convenient to calculate $D_\phi(x,y)$ by solving the above equation in the momentum space[15]. Taking the Fourier transform of Eq. (8), we obtain the expression of the meson propagator in the momentum space

$$(E^2-\boldsymbol{k}^2)D_\phi(\boldsymbol{k},\boldsymbol{k}',E)-\frac{1}{2\pi^3}\int S_\phi(\boldsymbol{k}-\boldsymbol{k}_1)D_\phi(\boldsymbol{k},\boldsymbol{k}',E)\mathrm{d}^3k_1=(2\pi)^3\delta(\boldsymbol{k}-\boldsymbol{k}')\,, \tag{9}$$

where $S_\phi(\boldsymbol{k}-\boldsymbol{k}_1)$ is the Fourier transform of $\left.\frac{\partial^2 U_\phi(\phi)}{\partial\phi^2}\right|_{\phi_0}$,

$$S_\phi(\boldsymbol{k}-\boldsymbol{k}_1)\ =\ \int \mathrm{e}^{-\mathrm{i}(\boldsymbol{k}-\boldsymbol{k}_1)\cdot\boldsymbol{r}}\left.\frac{\partial^2 U_\phi(\phi)}{\partial\phi^2}\right|_{\phi_0}\mathrm{d}^3r\equiv \int \mathrm{e}^{-\mathrm{i}(\boldsymbol{k}-\boldsymbol{k}_1)\cdot\boldsymbol{r}}V_\phi\mathrm{d}^3r. \tag{10}$$

Therefore, the function V_ϕ for σ, ω, and ρ mesons can be expressed as

$$V_\sigma=m_\sigma^2+2g_2\sigma(r)+3g_3\sigma^2(r), \tag{11}$$

$$V_\omega=m_\omega^2+3c_3\omega^2(r)+8\Lambda_v g_\rho^2 g_\omega^2 b_0^2(r), \tag{12}$$

and

$$V_\rho=m_\rho^2+8\Lambda_v g_\rho^2 g_\omega^2\omega_0^2(r), \tag{13}$$

respectively. $\sigma(r)$, $\omega_0(r)$ and $b_0(r)$ are the classical values of σ, ω and ρ fields. They can be obtained by a self-consistent calculation in the RMF[19].

3 Monopole compression mode

The ISGMR in some stable double magic or semi magic nuclei are studied in the RRPA by varying the nonlinear coupling constant of Λ_v with various parameter sets. We have selected some of the commonly used parameter sets in the literatures spanning a wide range of values of $K_{\rm nm}$, which are NL3[20], NLSH[21], TM1[22], NLC[23], NLVT, NLE[24], NLZ2, NLVT1[25] and NLBA[26]. In order to obtain the centroid energies of ISGMR strengths, we first calculate various moments of the response function in a given interval,

$$m_k=\int_0^{E_{\max}}R^{\rm L}(E')E'^k\mathrm{d}E'\ , \tag{14}$$

$E_{\max}$ is the maximum excitation energy, which is carried out until 60MeV in the present calculations. From those moments we can obtain the centroid energy of the ISGMR,

$$\overline{E}=m_1/m_0\ . \tag{15}$$

The centroid energies of the ISGMR in ^{208}Pb, ^{144}Sm, ^{116}Sn, and ^{90}Zr are calculated in the RRPA. In Fig. 2, we display the centroid energies as a function of the corresponding nuclear matter compression modulus $K_{\rm nm}$. The dependence of the excitation energy on $K_{\rm nm}$ is approximately linear, and lines in the figure are obtained by a linear fit. The areas delimited by two horizontal lines correspond to the experimental values of $E_{\rm ISGMR}$ with error bars. The compression moduli are predicted in comparison between experimental and theoretical energies, especially in ^{208}Pb. The left part of the figure corresponds to the case without the mixed isoscalar-isovector coupling term.

In comparison with the experimental ISGMR energy in ^{208}Pb one could give a constraint on $K_{\rm nm}$ around 260—280MeV.

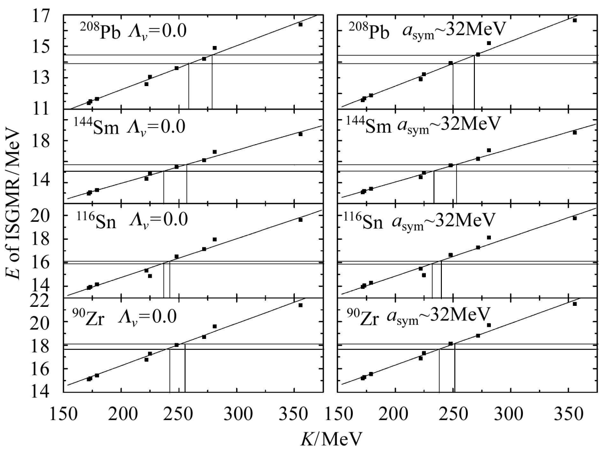

Fig. 2. Centroid energies of the ISGMR as a function of $K_{\rm nm}$. The left corresponds to the case without adding the mixed isoscalar-isovector coupling term; the right corresponds to the case adding the mixed isoscalar-isovector coupling term.

It is well known that RMF calculations with various parameterizations depict not only a wide range of values of the nuclear matter incompressibility, but also dissimilar symmetry energies at the saturation density. In order to study the effect of the symmetry energy on the ISGMR energies it is desirable to tune the nonlinear isoscalar and isovector coupling constant. The additional nonlinear isoscalar-isovector coupling softens the nuclear mater symmetry energy and therefore enlarges the neutron rms radius in finite nuclei. The symmetry energy is softened, which is approximately equal to 32MeV at the saturation density for various parameter sets by tuning the coupling constant Λ_v and without changing the agreement with the existing experimental data, such as the bounding energy per nucleon and the charge rms radius, as has been mentioned above.

Then the ISGMR centroid energies are calculated with various parameter sets at a fixed symmetry energy, which are plotted at the right panel in Fig. 2. It is found the calculated centroid ISGMR energies become larger due to the softened symmetry energy. With the same stratagem described above, and only ^{208}Pb is considered, one could give a soften constraint on $K_{\rm nm}$ at 250—270MeV. (If ^{208}Pb, ^{144}Sm, ^{116}Sn, and ^{90}Zr are all considered, the case without adding the mixed coupling term predicts the range 230—280MeV; and the case adding the mixed coupling term predicts the range 230—270MeV, respectively.) Varying the coupling constant Λ_v, we calculate the RMF ground state as well as the ISGMR in ^{208}Pb with the parameter sets NLC, NLBA, NL3 and TM1. A strong linear correlation between the centroid energy of the ISGMR and the symmetry energy at the saturation density is observed. Centroid energies of the ISGMR versus the symmetry energy are plotted in Fig. 3.

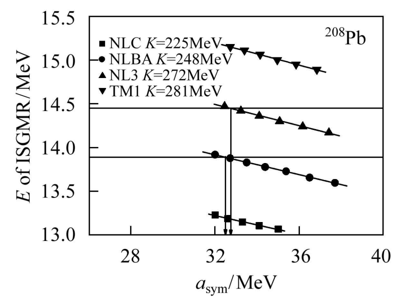

Fig. 3. ISGMR centroid energy for ^{208}Pb as a function of the symmetry energy of symmetric nuclear matter.

It is clearly shown that in comparison with the experiment ISGMR energy the RRPA calculation by using parameter sets TM1 and NLC with too large or too small incompressibility can not reproduce the experimental ISGMR energy at a reasonable symmetry energy. Only those parameter sets NL3 and NLBA with the incompressibility $K_{\rm nm}$ around 240—270MeV could be adopted to describe the ISGMR energies with a reasonable symmetry energy.

4 Summary

In summary, we discuss the nuclear matter incompressibility by monopole compression modes in the nuclei ^{208}Pb, ^{144}Sm, ^{116}Sn, and ^{90}Zr in the framework of a fully consistent relativistic random phase approximation, based on the effective Lagrangians with a mixed isoscalar-isovector nolinear coupling term. The RRPA in this work predicts the range of values of $K_{\rm nm}$ is 260—280MeV, if ^{208}Pb is considered, and without adding the mixed isoscalar-isovector term. As the isoscalar-isovector term is included and the symmetry energy is adjusted to equal 32MeV approximately at the saturation density, a slight different range 250—270MeV is predicted. The modification of the symmetry energy has slight impact on predicting the nuclear matter incompressibility. A strong linear correlation between the centroid energy of the ISGMR in ^{208}Pb and the symmetry energy is observed. To produce a reasonable symmetry energy a consistent result of the constraint on $K_{\rm nm}$ is obtained.

References

1 Giai V N et al. Nucl.Phys. , 2001, **A687**: 44c
2 col ò G et al. Phys. Lett., 1992, **B276**: 279
3 Blaizot J P et al. Nucl. Phys., 1995, **A591**: 435
4 Hamamoto I et al. Phys. Rev., 1997, **C56**: 3121
5 MA Z Y et al. Nucl. Phys., 2001, **A686**: 173
6 Pickarewicz J. Phys. Rev., 2002, **C66**: 034305
7 Agrawal B K et al. Phys. Rev., 1995, **C56**: 031304(R)
8 ZUO W et al. HEP & NP,2002, **26**(12): 1238 (in Chinese) (左维等. 高能物理与核物理, 2002, **26**(12): 1238)
9 LIU B et al. Phys. Rev., 2002, **C65**: 045201
10 Furnstahl R J et al. Nucl. Phys., 2002, **A85**: 706
11 Dieperink A E L et al. Phys. Rev., 2003, **C68**: 064307
12 Todd B G et al. Phys. Rev., 2003, **C67**: 044317
13 Horowitz C J et al. Phys. Rev. Lett., 2001, **86**: 5647
14 LIANG J et al. Chin. Phys. Lett., 2006, **23**: 1719
15 MA Z Y et al. Phys. Rev., 1997, **C55**: 2385
16 L'Huillier M et al. Phys. Rev., 1989, **C39**: 2022
17 Fetter A L et al. Quantum Theory of Many-Particle Systems. New York: Mc Graw-Hill, 1977
18 MA Z Y et al. Nucl. Phys., 1997, **A627**: 1
19 Serot B D et al. Adv. Nucl. Phys., 1986, **16**: 1
20 Lalazissis G A et al. Phys. Rev., 1997, **C55**: 540
21 Sharma M M et al. Phys. Lett., 1993, **B312**: 377
22 Sugahara Y et al. Nucl. Phys., 1994, **A579**: 557
23 Serot B D et al. Int. Jour. Mod. Phys., 1997, **E6**: 515
24 Rufa M et al. Phys. Rev., 1988, **C35**: 390
25 Bender M et al. Phys. Rev., 1999, **C60**: 034304
26 Gmuca S. Proc. 2nd Int. Conf. Fission and Properties of Neutron-Rich Nuclei. Singapore: World Sci, 2000. 75

巨单极共振和对称能*

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摘要 基于带有混合同位旋标量–矢量非线性耦合的有效拉格朗日量, 在完全自洽的相对论无规位相近似的框架内, 通过单极压缩模式讨论了核物质的不可压缩性. 比较核 ^{208}Pb, ^{144}Sm, ^{116}Sn 和 ^{90}Zr 实验和计算的巨单极共振能量, 给出了核物质不可压缩系数的取值范围. 新的同位旋标量–矢量非线性耦合软化核物质的对称能, 但无损于基态性质与实验的一致性. 讨论了对称能的软化对巨单极共振的影响.

关键词 巨单极共振 相对论无规位相近似 对称能 同位旋标量–同位旋矢量介子的非线性耦合

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