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# Form Factor Fit for $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ *

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**Abstract** Based on the experimental data of the cross section $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ measured by the BABAR collaboration, we apply the theoretical cross section deduced from the extended VMD (Vector Meson Dominance) model to fit these experimental results. It is the first time that the isovector form factor and the relevant parameters are obtained through one decay mode.

**Key words** form factor, cross section, fit

## 1 Introduction

Since 1988, ρ(1600) was replaced by two new resonances: ρ(1450) and ρ(1700) in PDG. It is suggested that this possibility can be explained by a theoretical analysis on the consistency of 2π and 4π electromagnetic form factors. Furthermore, detailed experimental data on the cross section of $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ make possible the accurate determination of the parameters of ρ-meson and its radial recurrencies. Therefore it is meaningful to fit these parameters simultaneously and compare current theoretical model with the experimental data.

Recently, VMD model has been developed, which includes not only the contribution of the lowest-lying vector-mesons, but also those of their high-mass recurrencies[1, 2]. Pervious analysis[1] used different decay channels to obtain the relevant parameters. And for some parameters, it only gave the upper limits, so the form factor cannot be fixed numerically. This paper aims to adopt the cross section of $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ derived from the extended VMD model to fit the experimental data from BABAR, and find the definite values of all the parameters directly.

## 2 Expression of the cross section

The cross section of $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ can be described generally in terms of the extended VMD model[1] which takes into account the mixing of the resonances ρ(770), $\rho'_1$, $\rho'_2$ in the frame work of the field theory-inspired approach based on the summation of the loop corrections to the propagators of the unmixed states. The cross section can be written as follows[1, 2]

$$\sigma(s)=\frac{(4\pi\alpha)^2}{s^{3/2}}\left|F_{\rho^0\pi^+\pi^-}(s)\right|^2 W_{\pi^+\pi^-\pi^+\pi^-}(s), \tag{1}$$

in which $s$ is the total center-of-mass energy squared, $\alpha \approx 1/137$ is the fine structure factor, and $W_{\pi^+\pi^-\pi^+\pi^-}$ is the final state factor.

According to the vector current conservation, the relation $g_{\rho^0\rho^0\pi^+\pi^-}=2g^2_{\rho\pi\pi}$ [1] can be considered as a guide for the corresponding coupling constant. So the expression of the isovector form factor can be written as

$$F_{\rho^0\pi^+\pi^-}(s) = \left(\frac{m^2_\rho}{f_\rho}, \frac{m^2_{\rho'_1}}{f_{\rho'_1}}, \frac{m^2_{\rho'_2}}{f_{\rho'_2}}\right) G^{-1}(s) \times \begin{pmatrix} 2g^2_{\rho^0\pi\pi} \\ g_{\rho'_1\rho^0\pi^+\pi^-} \\ g_{\rho'_2\rho^0\pi^+\pi^+} \end{pmatrix}, \tag{2}$$

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in which the leptonic coupling constants $f_{\rho_i}$ is

$$\Gamma_{\rho_i e^+e^-} = \frac{4\pi\alpha^2}{3f^2_{\rho_i}} m_{\rho_i}. \tag{3}$$

The matrix of inverse propagators is

$$G(s) = \begin{pmatrix} D_\rho & -\Pi_{\rho\rho'_1} & -\Pi_{\rho\rho'_2} \\ -\Pi_{\rho\rho'_1} & D_{\rho'_1} & -\Pi_{\rho'_1\rho'_2} \\ -\Pi_{\rho\rho'_2} & -\Pi_{\rho'_1\rho'_2} & D_{\rho'_2} \end{pmatrix}. \tag{4}$$

It consists of the inverse propagators of the unmixed states $\rho_i = \rho(770)$, $\rho'_1$ and $\rho'_2$,

$$D_{\rho_i} \equiv D_{\rho_i}(s) = m^2_{\rho_i} - s - i\sqrt{s}\Gamma_{\rho_i}(s), \tag{5}$$

where

$$\begin{aligned}\Gamma_{\rho_i}(s) \ = \ & \frac{g^2_{\rho_i\pi\pi}}{6\pi s} q^3_{\pi\pi} + \\ & \frac{g^2_{\rho_i\omega\pi}}{12\pi}\left(q^3_{\omega\pi} + q^3_{K*K} + \frac{2}{3}\langle q^3_{\rho\eta}\rangle\right) + \\ & \frac{3}{2} g^2_{\rho_i\rho^0\pi^+\pi^-} W_{\pi^+\pi^-\pi^+\pi^-}(s) + \\ & g^2_{\rho_i\rho^+\rho^-} W_{\pi^+\pi^-\pi^0\pi^0}(s),\end{aligned} \tag{6}$$

in which,

$$\begin{aligned} q_{ij} \ \equiv \ & q(M, m_i, m_j) = \\ & \frac{\sqrt{[M^2-(m_i-m_j)^2][M^2-(m_i+m_j)^2]}}{2M} \end{aligned} \tag{7}$$

is the magnitude of the momentum of either particle i or j, in the rest frame of the decaying particle, and the nondiagonal polarization operators $\Pi_{\rho_i\rho_j} = \mathrm{Re}\Pi_{\rho_i\rho_j} + i\,\mathrm{Im}\Pi_{\rho_i\rho_j}$. The real parts are still unknown and may be supposed to be some constants, at the same time, the imaginary parts could be deduced from the unitarity relatoin as

$$\begin{aligned}\mathrm{Im}\Pi_{\rho_i\rho_j} \ = \ \sqrt{s}\Bigg[ & \frac{g_{\rho_i\pi\pi}g_{\rho_j\pi\pi}}{6\pi s} q^3_{\pi\pi} + \\ & \frac{g_{\rho_i\omega\pi}g_{\rho_j\omega\pi}}{12\pi}(q^3_{\omega\pi} + q^3_{K^*K} + \frac{2}{3}\langle q^3_{\rho\eta}\rangle) + \\ & \frac{3}{2} g_{\rho_i\rho^0\pi^+\pi^-} g_{\rho_j\rho^0\pi^+\pi^-} W_{\pi^+\pi^-\pi^+\pi^-}(s) + \\ & g_{\rho_i\rho^+\rho^-} g_{\rho_j\rho^+\rho^-} W_{\pi^+\pi^-\pi^0\pi^0}(s)\Bigg].\end{aligned} \tag{8}$$

## 3 The procedure of form factor fit

The cross section of $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ has been studied and measured by many experiments. BABAR collaboration has measured this cross section in the center-of-mass energies from 0.6 to 4.5GeV, considering a hard photon radiated from the initial state. Because of the wide energy range and the relatively small errors, we use the expression described in Eq. (1) to fit the undressed (without vacuum polarization) cross sections[3].

The fit is carried out through minimum $\chi$-square

$$\chi^2(\alpha) = \sum_{i=1}^{n}\left(\frac{\sigma_i - \sigma_{\mathrm{the}}(x_i,\alpha)}{\Delta\sigma_i}\right)^2, \tag{9}$$

where $\sigma_i$ is the experimental cross section, $\sigma_{\mathrm{the}}(x_i,\alpha)$ is the corresponding theoretical expressions, $\alpha$ is the vector of free parameters which are required to be fitted, and $\Delta\sigma_i$ is the measurement error of the individual cross section.

Then Minuit package[4] has been chosen as the tool to find the minimum value of the multi-parameter function Eq. (9) and analyzes the shape of this function around the minimum. The fit results are successful for describing the shape of the cross section and the form factor of $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$. A minimum has been found, and the parameter errors, taking into account the parameter correlations, have been calculated by MIGRAD processor. But it seems difficult to use MINOS, another Minuit processor, to calculate the errors taking into account both parameter correlations and non-linearities. As we know, Eq. (1) includes a 3-dimensional integral (taking into account initial state radiative correction) and fit for $\chi^2$ is sensitive to some parameters. As a result, the possibility of calculating the parameter errors by MINOS is reduced by these effects.

The fit parameters are $m_{\rho_i}$, $g_{\rho_i\pi^+\pi^-}$, $g_{\rho_i\rho^0\pi^+\pi^-}$, $g_{\rho_i\rho^+\rho^-}$, $f_{\rho_i}$, $\mathrm{Re}\Pi_{\rho_i\rho_j}$. The masses of $\rho_i$ have been measured relatively well, so in the fit they are fixed to the values in the PDG[5]: $m_{\rho_0}$=0.7755GeV, $m_{\rho'_1}$=1.459GeV, $m_{\rho'_2}$=1.720GeV. Meanwhile, according to the analysis in Ref. [1], the real parts of nondiagonal polarization operators $\mathrm{Re}\Pi_{\rho\rho'_1}$ and $\mathrm{Re}\Pi_{\rho\rho'_2}$ are set to zero. The parameter $g_{\rho^0\omega\pi}$ has been obtained from the measurement of $e^+e^- \to \omega\pi$ and its value is fixed to 14.3[6]. The physical strong coupling constant $g_{\rho^0\rho^+\rho^-}$ is set to 6.05 according to Ref. [7]. $f_{\rho^0}$ is calculated from Eq. (3), and it is fixed to 5.1. Considering $g_{\rho^0\rho^0\pi^+\pi^-} = 2g^2_{\rho\pi\pi}$ and $g_{\rho^0\rho^+\rho^-} = g_{\rho^0\pi^+\pi^-}$[1], we set $g_{\rho^0\rho^0\pi^+\pi^-}$ and $g_{\rho^0\rho^+\rho^-}$ to be 73.205 and 6.05 respectively.

## 4 The fit results and the discussion

The values of the parameters and their errors in the fit are presented in Table 1. The cross section and the form factor of $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ measured by BABAR and the curves predicted by the extended VMD model are shown in Fig. 1 and Fig. 2.

Table 1. The values of the parameters result from fitting with data from BABAR collaboration. Parameters without errors are the fixed parameters.

| parameter | BABAR |
|---|---|
| $m_{\rho^0}$/GeV | 0.7755 |
| $m'_{\rho_1}$/GeV | 1.4590 |
| $m'_{\rho_2}$/GeV | 1.7200 |
| $f_{\rho^0}$ | 5.1 |
| $f_{\rho'_1}$ | 0.442±0.005 |
| $f_{\rho'_2}$ | 0.334±0.003 |
| $g_{\rho^0\pi^+\pi^-}$ | 6.05 |
| $g_{\rho^0\omega\pi}$/GeV$^{-1}$ | 14.3 |
| $g_{\rho^0\rho^0\pi^+\pi^-}$ | 73.205 |
| $g_{\rho^0\rho^+\rho^-}$ | 6.05 |
| $g_{\rho'_1\pi^+\pi^-}$ | −3.783±1.219 |
| $g_{\rho'_1\omega\pi}$/GeV$^{-1}$ | −4.553±2.315 |
| $g_{\rho'_1\rho^0\pi^+\pi^-}$ | 229.80±4.680 |
| $g_{\rho'_1\rho^+\rho^-}$ | −14.383±0.859 |
| $g_{\rho'_2\pi^+\pi^-}$ | 19.302±1.263 |
| $g_{\rho'_2\omega\pi}$/GeV$^{-1}$ | 30.113±1.555 |
| $g_{\rho'_2\rho^0\pi^+\pi^-}$ | −411.56±5.261 |
| $g_{\rho'_2\rho^+\rho^-}$ | −0.390±1.585 |
| $\mathrm{Re}\Pi_{\rho\rho'_1}$ | 0 |
| $\mathrm{Re}\Pi_{\rho\rho'_2}$ | 0 |
| $\mathrm{Re}\Pi_{\rho'_1\rho'_2}$/GeV$^2$ | −6.054±0.356 |
| $\chi^2/n_{\mathrm{d.o.f}}$ | 131.30/130 |

Compared with the preceding work in Ref. [1], which used diverse experimental data from different channels to carry out the fit respectively, we focus on the possibility to fit all the parameters simultaneously. Table 1 shows the final results in our work. In Figs. 1 and 2, the fitted curve and the data which have small errors agree well in the whole measured region, and the $\chi^2/n_{\mathrm{d.o.f}}$ is near 1. Meanwhile most values of the parameters are in the limit range in Ref. [1]. However, there are some differences between the values from our work and those from the previous work.

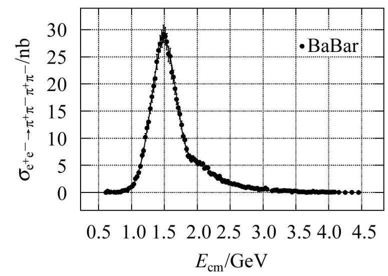


Fig. 1. The Born cross section of $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$, dots with error-bars are experimental results given by BABAR, the line shows the prediction of the model[1,2] with the fitted parameters in Table 1.

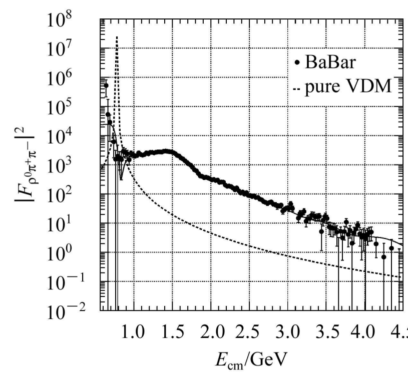


Fig. 2. The $\rho^0\pi^+\pi^-$ form factor squared. Dots are recalculated from the results given by BABAR. The solid line is the fitted curve and the dash line represents the contribution of $\rho(770)$ only.

In Fig. 2, the dash line only takes into account the contribution of $\rho(770)$. The VMD model estimate in this case is[2]

$$F_{\rho_0\pi^+\pi^-}(s) = \frac{2g_{\rho\pi\pi}m^2_\rho}{m^2_\rho - s} \ . \tag{10}$$

The fitted lines agree with the experimental data relatively well above 0.98GeV.

## 5 Conclusion

From the fit, values and errors of all the parameters are obtained. Furthermore, the shape and values of the isovector form factor are also similar to the previous work. Considering that some parameters may have a strong correlation and the fit is sensitive to some parameters, the results of our work may differ from those which are gained by respectively fitting different experimental data from different channels.

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# $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ 形状因子拟合*

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**摘要** 基于BABAR实验组对$e^+e^- \to \pi^+\pi^-\pi^+\pi^-$的反应截面的测量结果, 用VMD模型给出的理论截面拟合实验数据, 首次从单一反应道得到$\pi^+\pi^-\pi^+\pi^-$末态形状因子的所有参数值.

**关键词** 形状因子 截面 拟合

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