第 30 卷 增刊 II
2006 年 12 月
高 能 物 理 与 核 物 理
HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS
Vol. 30, Supp. II
Dec., 2006

Splitting of Pseudospin and Spin Partners in the Relativistic Square Well*

XU Qiang[1)] ZHU Sheng-Jiang

(Department of Physics,Tsinghua University, Beijing 100084, China)

Abstract Pseudospin symmetry and spin symmetry in the relativistic square well are investigated systematically by solving the Dirac equation with scalar and vector potentials and are found to be a good approximation in realistic nuclei such as ^{208}Pb. The pseudospin breaking and spin breaking are shown in correlation with nuclear mean field. The square well radius plays an important role in the splittings of energy and wavefunction. The dependence of splittings with quantum numbers is also analyzed.

Key words pseudospin symmetry, spin symmetry, relativistic square well, Dirac equation

1 Introduction

About 30 years ago, a quasidegeneracy was observed in heavy nuclei between single-nucleon doublets with quantum numbers $(n, l, j = l+1/2)$ and $(n-1, l+2, j = l+3/2)$ where n, l, and j are the radial, the orbital, and the total angular momentum quantum numbers, respectively[1, 2]. The quasidegenerative states are suggested as the pseudospin doublets $j = \tilde{l} \pm \tilde{s}$ with the pseudo orbital angular momentum $\tilde{l}$ and the pseudospin angular momentum $\tilde{s}$.

However, as stated in some papers[3], the conditions $\Sigma = 0$ or $\mathrm{d}\Sigma/\mathrm{d}r = 0$ can not be realized in nuclei. Therefore, it is necessary to study the pseudospin symmetry for relativistic nuclear potential. Our purpose here is to investigate Pseudospin symmetry and spin symmetry in the relativistc square well shaped by the square well radius and the square well depth. In the following, we first present the theoretical formalism, then analyze systemically the pseudospin breaking and spin breaking for the relativistic square well.

2 Formalism

The Dirac equation of a nucleon with mass M moving in an attractive scalar potential $S(\boldsymbol{r})$ and a repulsive vector potential $V(\boldsymbol{r})$ can be written as

$$[\boldsymbol{a}\cdot\boldsymbol{p}+\beta(M+S)+V]\Psi = E\Psi. \tag{1}$$

For spherical nuclei, the wavefunctions can be classified according to there angular momentum j and κ,

$$\Psi_{n\kappa}(\boldsymbol{r}) = \begin{pmatrix} f_{n\kappa} \\ g_{n\kappa} \end{pmatrix} = \frac{1}{r}\begin{pmatrix} F_{n\kappa}(r)Y^{l}_{jm}(\theta,\phi) \\ \mathrm{i}G_{n\kappa}(r)Y^{\tilde{l}}_{jm}(\theta,\phi) \end{pmatrix}, \tag{2}$$

when the angular part was split off, the radial wavefunction satisfy the following equations

$$\left(\frac{\mathrm{d}}{\mathrm{d}r}+\frac{\kappa}{r}\right)F_{n\kappa}(r) = (M+E_{n\kappa}-\Delta)G_{n\kappa}(\kappa), \tag{3}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}r}-\frac{\kappa}{r}\right)G_{n\kappa}(r) = (M-E_{n\kappa}+\Sigma)F_{n\kappa}(\kappa). \tag{4}$$

where Δ and Σ are assumed to be radial functions, i.e., $\Delta = V(r)-S(r)$ and $\Sigma = V(r)+S(r)$.We perform a calculation using a relativistic potential of square

* Supported by National Natural Science Foundation of China (10375032) and Doctoral Fund of Ministry of Education of China (20030003090)

1) E-mail: xuqiang05@mails.tsinghua.edu.cn

well type in the Dirac equation. The corresponding scalar and vector components are the mean field central nuclear potential, given as

$$U(r)=\begin{cases} U_0, & r\leqslant r_0, \\ 0, & r>r_0, \end{cases} \tag{5}$$

where $U(r)$ stands either for the vector or for the scalar potential.

Table 1. The parameters in square well potential determined by fitting the scalar and vector potentials from the RMF calculations with the NL3 for ^{208}Pb. Listed are two central depths $U_0(\Sigma_0$ and $\Delta_0)$ and two radius parameters r_0.

	U_0	r_0
$V+S$	−68.0	8.0
$V-S$	710.0	8.0

3 Results and discussion

Using square well potential for Σ and Δ , we solved numerically the Dirac equations for the radial fields $F(r)$ and $G(r)$. In order to make the square well potential closer to nuclear mean field, the parameters in square well potential are determined by fitting the scalar and vector potentials derived from the RMF calculations where ^{208}Pb is chosen as a reference. The parameters determined are listed in the Table 1. By using these parameters, the radial wavefunctions of relativistic square well are obtained with the upper components and lower components are plotted in Fig. 1, in which are found to be quite agreeable for the spin partners and pseudospin partners, as noted previously in Refs. [4,5]. Although such a near agreement of wavefunction has been obtained for the relativistic square well, the splitting between the doublets cannot be neglected, especially on the surface. The extent of splitting is connected with the single particle orbital. For example, when l or $\tilde{l}$ increases, the pseudospin wavefunction splittings increase and the maximum splittings of all the wavefunctions move towards the outside of the nucleus for the spin doublets n=1 in Fig. 1(a) and psudospin doublets $\tilde{n}$=1 in Fig. 1(b), as was noted for the spherical square well potential in Ref. [3]. The similar case also appears in other pseudospin partners for the doublets $\tilde{l}$=1 and $\tilde{l}$=2, as was shown in Fig. 1(c).

Here, we discuss only the influences of Σ potential on the symmetry to analyze the correlations between the pseudospin symmetry and spin symmetry and the parameters of square well potential. By solving Dirac equation with the scalar and vector square well potential, the variations of the pseudospin energy splitting ($\Delta E=E_{\tilde{l}j=\tilde{l}-1/2}-E_{\tilde{l}j=\tilde{l}+1/2}$) and the spin energy splitting ($\Delta E=E_{lj=l-1/2}-E_{lj=l+1/2}$) with the parameters are shown in Fig. 2. Because the splitting of wavefunction is proportional to the area between the lower components of the doublets, the variations of the wavefunction splitting with the parameters are also drawn in Fig. 2. In the figures below, all the pseudospin-orbital partners appear in the order of the energy

$$1(1\tilde{p}_{3/2},1\tilde{p}_{1/2}),\quad 2(1\tilde{d}_{5/2},1\tilde{d}_{3/2}),\quad 3(1\tilde{f}_{7/2},1\tilde{f}_{5/2}),$$
$$4(2\tilde{p}_{3/2},2\tilde{p}_{1/2}),\quad 5(1\tilde{g}_{9/2},1\tilde{g}_{7/2}),\quad 6(2\tilde{d}_{5/2},2\tilde{d}_{3/2}),$$
$$7(1\tilde{h}_{11/2},1\tilde{h}_{9/2}),\quad 8(2\tilde{f}_{7/2},2\tilde{h}_{5/2}),\ \text{and}\ 9(3\tilde{p}_{1/2},3\tilde{p}_{3/2}).$$

All the spin-orbital partners appear in the order of the energy $1(1p_{1/2},1p_{3/2})$, $2(1d_{3/2},1d_{5/2})$, $3(1f_{5/2},1f_{7/2})$, $4(2p_{1/2},2p_{3/2})$, $5(1g_{7/2},1g_{9/2})$, $6(2d_{3/2},2d_{5/2})$, $7(1h_{9/2},1h_{11/2})$, $8(2f_{5/2},2f_{7/2})$, $9(3p_{1/2},3p_{3/2})$, $10(1i_{11/2},1i_{13/2})$, $11(2g_{7/2},2g_{9/2})$, $12(3d_{3/2},3d_{5/2})$.

Fixing Σ_0 and Δ_0, we vary r_0 to see how the pseudospin splitting and spin splitting are sensitive to the radius. These dependences are shown in Fig. 2. we observe that both the energy splitting and the wavefunction splitting between the doublets are sensitive to the parameter r_0. In Fig. 2(a), It is seen that all the splittings of the pseudospin energy decrease with r_0 increasing. In Fig. 2(b), as r_0 increases, all the spin energy splittings also decrease. Similarly, in Fig. 2(c) and Fig. 2(d), both the spin wavefunction splittings and pseudospin wavefunction splittings decrease with r_0 increasing, which is in agreement with the variation of the energy splittings with the parameter r_0. It means that when r_0 increases, the symmetry of the energy levels and the symmetry of the wavefunctions both become better.

If we keep a,R and Δ_0 fixed, but vary Σ_0 to study the sensitiveness of the pseudospin doublets and spin doublets with the depth of the central Σ mean field potential, we can conclude that all the pseudospin doublets and spin doublets are almost invariant with $|\Sigma_0|$ increasing, because Σ_0 is just a constant added to the potential. Therefore, the pseudospin splittings and spin splittings are all insensitive to the depth of well, which is in accordance with Ginocchio predictions for pseudospin symmetry breaking due to the finiteness of the mean field.

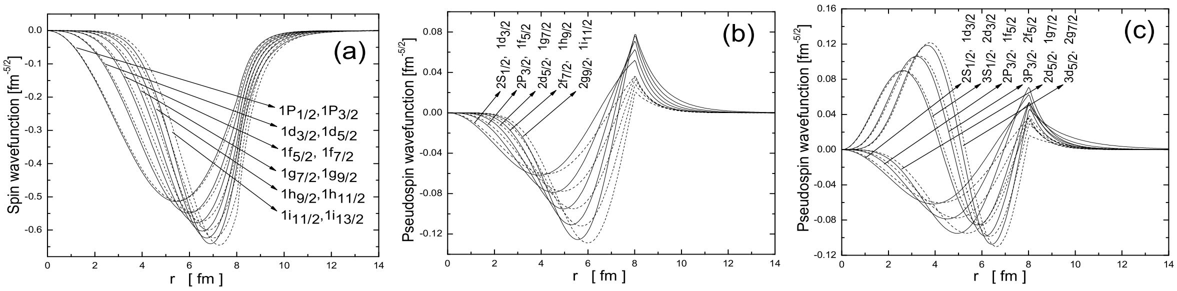

Fig. 1. (a) The radial upper components($F(r)$) of spin partners for the relativistic SW with the parameters determined by fitting the results from the RMF calculations with the interactions NL3 for ^{208}Pb; (b) The same as (a), but for the radial lower components($G(r)$) of pseudospin partners; (c) The same as (b).

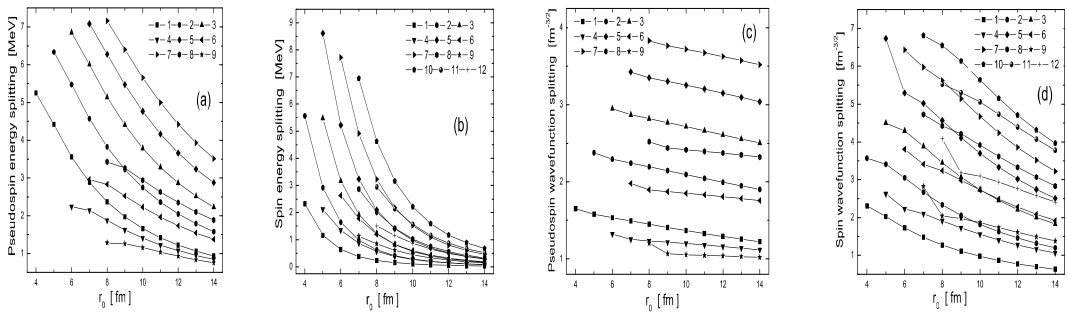

Fig. 2. (a) Pseudospin energy splitting as a function of r_0 for the different pseudospin doublets; (b) spin energy splitting as a function of r_0 for the different spin doublets; (c) pseudospin wavefunction splitting as a function of r_0 for the radial lower components($G(r)$) of the different pseudospin doublets; (d) spin wavefunction splitting as a function of r_0 for the radial upper components($F(r)$) of the different spin doublets.

Furthermore, as for a certain nucleus, different partners are in the same potential. So it is necessary to analyze the dependence of splittings with quantum numbers. In Fig. 2, when $\tilde{n}$ is fixed, all the energy splittings and the wavefunction splittings increase with $\tilde{l}$ increasing. However, when $\tilde{l}$ is fixed, both the pseudospin energy splittings in Fig. 2(a) and the pseudospin wavefunction splittings in Fig. 2(c) decrease with $\tilde{n}$ increasing, while both the spin energy splittings in Fig. 2(b) and the spin wavefunction splittings in Fig. 2(d) increase with $\tilde{n}$ increasing. All these show that both the pseudospin breaking and spin breaking are different for different pseudospin partners and spin partners, respectively. Although some of these results have been derived in specific relativistic field theories[6,7], these results probably are the general feature of any relativistic model which fits nuclear binding energies, and hence very likely the general feature independent of any one model.

4 Conclusion

At the present work, the pseudospin symmetry and the spin symmetry in the relativistic square well are investigated systemically by solving the Dirac equation with scalar and vector radial potentials. The symmetry is found to be a good approximation in realistic nuclei such as ^{208}Pb. The pseudospin breaking and spin breaking are shown in correlation with the nuclear mean field which is shaped by the depth and the radius of the potential. The radius r_0 is found to play an important role in the energy splittings and wavefunction splittings for all the partners. Furthermore, the dependence of pseudospin splittings and

spin splittings with quantum numbers are also analyzed. when l or $\tilde{l}$ increases, the maximum splittings of all the wavefunction move towards the outside of the nucleus for the spin doublets n=1 and psudospin doublets $\tilde{n}$=1. when $\tilde{n}$ is fixed, all the energy splittings and the wavefunction splittings increase with $\tilde{l}$ increasing. However, when $\tilde{l}$ is fixed, both the pseudospin energy splittings and the pseudospin wavefunction splittings decrease while both the spin energy splittings and the spin wavefunction splittings increase with $\tilde{n}$ increasing.

References

1 Arima A, Harvey M, Shimizu K. Phys. Lett., 1969, **B30**: 517
2 Hecht K T, Adler A. Nucl. Phys., 1969, **A137**: 129
3 Ginocchio J N. Phys. Rev. Lett., 1997, **78**: 436
4 Ginocchio J N, Leviatan A. Phys. Rev. Lett., 2001, **87**: 072502
5 Ginocchio J N. Phys. Rev., 2002, **C66**: 064312
6 MENG J, Sugawara-Tanabe K, Yamaji S et al. Phys. Rev., 1998, **C58**: R628
7 MENG J, Sugawara-Tanabe K, Yamaji S et al. Phys. Rev., 1999, **C59**: 154

方阱势下的赝自旋劈裂和自旋劈裂*

徐强[1)] 朱胜江

(清华大学物理系 北京 100084)

摘要 通过求解具有方阱势的径向标量势与矢量势的Dirac方程, 分别分析了原子核中赝自旋和自旋双重态的能级劈裂和波函数劈裂随着方阱势的势阱深度参数和势阱半径参数的变化关系, 此外, 能级劈裂和波函数劈裂随着量子数的变化关系也被进行了研究. 由于在特定的同位素链中, 势阱深度参数和势阱半径参数与核子数有关系, 所以以这些参数为变量对于赝自旋劈裂和自旋劈裂的研究是有意义的, 研究的结果至少可以定性的应用到大部分原子核中.

关键词 赝自旋对称性 自旋对称性 方阱势 Dirac方程

* 国家自然科学基金(10375032)和国家教育部博士点专项基金(20030003090)资助

1) E-mail: xuqiang05@mails.tsinghua.edu.cn