Magnetic Moment in Relativistic Mean Field Theory*

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Abstract The magnetic moments of nuclei near double closed-shell with A=133, 207, 209 are calculated within spherical and triaxial deformed relativistic mean field theory. The relativistic effect and core polarization effect on nuclear magnetic moment have been examined. The observed large discrepancy between Schmidt values and experimental data for ^{133}Sb and ^{209}Bi can be reduced significantly with the consideration of time-odd magnetic potential in triaxial deformed RMF theory.

Key words magnetic moment, heavy nuclei, triaxial deformation, relativistic mean field theory

Nuclear magnetic dipole moments of nuclei have drawn the attentions of nuclear physicist since the early days. Besides the measurement of magnetic dipole moments of ordinary nuclei[1], new techniques have made possible the determination of nuclear ground state moments in regions far from stability[2]. Since that the electromagnetic matrix elements are sensitive to the wave functions of the many-body systems, the measurement of magnetic moments can probe microscopic aspects such as the configurations of nucleons and it thus provides us a stringent test of nuclear structure models.

Relativistic mean field (RMF) theory has been successfully applied in analysis of nuclear structure over the whole nuclear chart, from stable nuclei to exotic nuclei[3—6]. To apply RMF theory to describe the nuclear magnetic moments, the spatial-component of vector fields, i.e. time-odd nuclear magnetic potential, which turns out to be very important for the description of the magnetic moments[7, 8] haven been considered self-consistently in the recently developed time-odd triaxial RMF theory[9]. In view of these facts, it is interesting to apply this approach to describe the magnetic moments of nuclei near double closed-shell in heavy-mass region.

The starting point of RMF theory is a local Lagrangian density. Besides nucleon field, it contains the isoscalar scalar σ- and isoscalar vector ω-mesons, providing medium-range attractive and short-range repulsive interactions respectively, the isovector vector ρ-meson giving the necessary isospin asymmetry and electromagnetic field providing coulomb repulsive interaction[3—6]. The equation of motion for the nucleons can be obtained with Euler-Lagrangian equation. The magnetic potential in the Dirac equation $\boldsymbol{V}(\boldsymbol{r}) = g_\omega \boldsymbol{\omega}(\boldsymbol{r}) + g_\rho \tau_3 \boldsymbol{\rho}^3(\boldsymbol{r}) + e\dfrac{1-\tau_3}{2}\boldsymbol{A}(\boldsymbol{r})$ will cause the time reversal invariance breaking in the nuclear states and remove the Krammer's degeneracy in odd nuclei. The details can be found in Ref. [9].

The nuclear magnetic moments can be obtained from the effective electromagnetic current operator[9]. The Dirac magnetic moment $\mu_{\rm D}^{\rm Rel.}$ of single-particle ψ_{jm}^{ω} can be divided into two terms: renormalized mass $M^*(r) = M + g_\sigma \sigma(r)$ independent term $\mu_{\rm D}^0$ and renormalized mass dependent term $\Delta\mu_{\rm D}$[10],

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$$\mu_{\mathrm{D}}^{0}=\begin{cases}\left(j+\dfrac{1}{2}\right)\left[1-\dfrac{2j+1}{j+1}\displaystyle\int \mathrm{d}rG^{2}(r)\right], & j=l+1/2\\ \dfrac{j(j+\frac{1}{2})}{(j+1)}\left[1-\dfrac{2j+1}{j}\displaystyle\int \mathrm{d}rG^{2}(r)\right], & j=l-1/2\end{cases} \tag{1}$$

and

$$\Delta\mu_{\mathrm{D}} = \frac{\kappa j}{(\kappa-\frac{1}{2})(\kappa+\frac{1}{2})}\int \mathrm{d}r\left(\frac{M}{M^{*}}-1\right)\{rG'(r)G(r)+ rF'(r)F(r)+\omega\left(j+\frac{1}{2}\right)[F^{2}(r)-G^{2}(r)]\} \tag{2}$$

with $\kappa=\omega(j+1/2)$, and prime means the derivative with respective to radial coordinate. The anomalous moment of single-particle can be obtained as

$$\mu_{A}^{\mathrm{Rel.}}=\begin{cases}\lambda_{\mathrm{a}}\left[1-\dfrac{1}{j+1}\displaystyle\int \mathrm{d}rG^{2}(r)\right], & j=l+1/2\\ \lambda_{\mathrm{a}}\dfrac{-j}{(j+1)}\left[1+\dfrac{1}{j}\displaystyle\int \mathrm{d}rG^{2}(r)\right], & j=l-1/2\end{cases} \tag{3}$$

where the free nucleon anomalous magnetic moment λ_{a} is 1.793 for proton and −1.913 for neutron. Compared with the magnetic moments in the extreme single-particle model, i.e. the Schmidt values, it is noticed that the relativistic correction term to the magnetic moment is proportional to the amplitude of small component. Due to the small renormalized mass $M^{*}(\simeq 0.6\mathrm{M})$, the spherical relativistic description of Dirac magnetic moment without magnetic potential will give enhanced Dirac moment $\mu_{\mathrm{D}}^{\mathrm{Rel.}}(\sim 5\mu_{\mathrm{D}}^{0}/3)$[11, 12]. Moreover, the relativistic effect makes the magnitude of anomalous magnetic moment slightly smaller for $j_{>}$ orbit and slightly larger for $j_{<}$ orbit.

In deformed odd nuclei, the unpaired valence nucleon can generate current (nuclear magnetism), which forms the source of magnetic potentials in the Dirac equation. The time-odd magnetic potentials will give rise to the degeneracy breaking of the level with opposite spin projections, playing the role of anti-pairing. This kind of effect, so called the core polarization effect, turns out to be very important for magnetic moment. After consideration of the full four-vector potential in deformed RMF theory, $\mu_{\mathrm{D}}^{\mathrm{Rel.}}$ becomes close to the Schmidt value $\mu_{\mathrm{D}}^{\mathrm{Sch.}}$ for light nuclei near double closed-shells[7—9].

The Dirac magnetic moments $\mu_{\mathrm{D}}^{\mathrm{Rel.}}$, anomalous magnetic moments $\mu_{\mathrm{A}}^{\mathrm{Rel.}}$ and total magnetic moments $\mu^{\mathrm{Rel.}}$ of nuclei near double closed-shell with A=133, 207,209 are calculated within spherical RMF theory with PK1 parameter set, as shown in Table 1. The Schmidt values are also given for comparison. It is found that spherical RMF theory gives the similar anomalous magnetic moments as the Schmidt values. For Dirac magnetic moments, however, spherical RMF gives about 1.5 times than the Schmidt values. Moreover, the total nuclear magnetic moments given by spherical RMF theory are systematic larger than the experimental data due to the small renormalized mass.

Table 1. The Dirac magnetic moments, anomalous magnetic moments and total magnetic moments in units of μ_{N}. The superscript "Sch." and "Rel." denote the values given by Schmidt formula and spherical RMF calculation with PK1 parameter set respectively. The experimental data are from Ref. [1].

nuclei	j^{π}	μ_{D}^{0}	$\mu_{\mathrm{D}}^{\mathrm{Sch.}}$	$\mu_{\mathrm{D}}^{\mathrm{Rel.}}$	$\mu_{\mathrm{D}}^{\mathrm{Rel.}}/\mu_{\mathrm{D}}^{0}$	$\mu_{\mathrm{D}}^{\mathrm{Rel.}}/\mu_{\mathrm{D}}^{\mathrm{Sch.}}$	$\mu_{\mathrm{A}}^{\mathrm{Sch.}}$	$\mu_{\mathrm{A}}^{\mathrm{Rel.}}$	$\mu^{\mathrm{Sch.}}$	$\mu^{\mathrm{Rel.}}$	Exp.
^{207}Pb	$\frac{1}{2}^{-}$	0	0	0		0	0.638	0.649	0.638	0.649	0.593
^{209}Pb	$\frac{9}{2}^{+}$	0	0	0		0	-1.913	-1.907	-1.913	-1.907	-1.474
^{133}Sb	$\frac{7}{2}^{+}$	2.899	3.111	4.626	1.60	1.49	-1.394	-1.407	1.717	3.219	3.00
^{207}Tl	$\frac{1}{2}^{+}$	0.995	1.000	1.533	1.54	1.53	1.793	1.789	2.793	3.322	1.876
^{209}Bi	$\frac{9}{2}^{-}$	3.820	4.091	5.992	1.57	1.46	-1.467	-1.477	2.624	4.516	4.110

The enhancement of Dirac magnetic moment in spherical RMF calculation will be reduced if the spatial-component of vector meson fields are considered in deformed RMF theory. The magnetic moments of nuclei with A=133, 207, 209, calculated within time-odd triaxial RMF theory with PK1 parameter set are plotted in Fig. 1, where the early calculations of light nuclei near double closed-shell with $A \leqslant 41$ from Ref. [9] are also given for comparison. The numerical details can be found in Ref. [9].

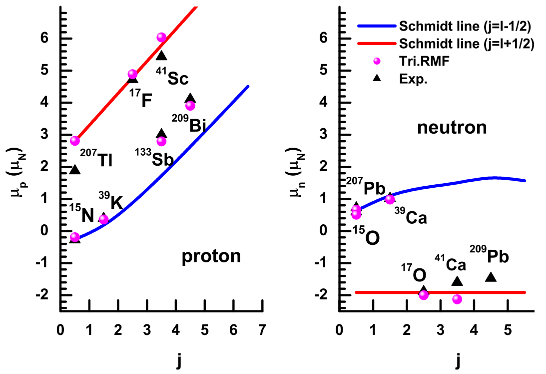

Fig. 1. The magnetic moments of nuclei near double closed-shell with A=15, 17, 39, 41, 133, 207, 209.

Fig. 1 shows that the magnetic moments of nuclei with A=15, 17, 39, 41, 133, 207, 209 (except ^{209}Pb) have been reproduced well with the consideration of time-odd magnetic potential in triaxial deformed RMF theory. Especially for ^{133}Sb, ^{209}Bi, the large discrepancies between the measured moments and the Schmidt values can be reduced greatly after considering the relativistic effect and core polarization effect.

In summary, the magnetic moments in relativistic description have been compared with the Schmidt formula. The relativistic enhancement of Dirac magnetic moment ($\mu_D^{\rm Rel.}/\mu_D^{\rm Sch.} \sim 1.5$) has been observed in the spherical RMF calculation of the magnetic moments of nuclei near double closed-shell with A=133, 207, 209. It is found that after considering the spatial-component of vector meson fields in triaxial RMF theory, the enhancement of Dirac magnetic moments have been reduced and the magnetic moments of ^{133}Sb and ^{209}Bi are quite close to the experimental data, which shows the success of relativistic description of nuclear magnetic moments.

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相对论平均场理论研究原子核的磁矩*

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摘要 采用球形以及三轴形变的相对论平均场理论研究了满壳附近质量数 A=133, 207, 209 的原子核的磁矩, 分析了相对论效应以及核芯极化对磁矩的影响. 计算结果表明用包含矢量介子空间分量(奇时间分量)的三轴的相对论平均场理论可以很好的描述满壳附近重质量区原子核 ^{133}Sb 以及 ^{209}Bi 的磁矩.

关键词 磁矩 重核 三轴形变 相对论平均场理论

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