第 30 卷 第 12 期 高 能 物 理 与 核 物 理 Vol. 30, No. 12
2006 年 12 月 HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS Dec., 2006

Spontaneously Broken Rotational Symmetry in Nuclear Structure: Some New Theoretical Aspects*

CHEN Yong-Shou[1,2;1)] GAO Zao-Chun[1]

1 (China Institute of Atomic Energy, Beijing 102413, China)
2 (Institute of Theoretical Physics, CAS, Beijing 100080, China)

Abstract The Reflection Asymmetric Shell Model has been generated to include the triaxial degree of freedom. The signature inversion and chiral and wobbling band structures can be described by the present model as results of some nuclei having intrinsic mean fields with spontaneously broken rotational symmetry. As a application of the theory, the triaxial rotation, a fundamental question in nuclear structure, has been investigated by examining the signature inversion and chirality phenomena in nuclei. The signature inversion phenomenon is interpreted as the change of the dynamic cranking axis in triaxially deformed nuclei. An other goal is to provide a new understanding of the nature of the doublet bands in ^{134}Pr.

Key words shell model, signature inversion, chiral bands

1 Introduction

The essential issue that underlies the present study is whether some nuclei can be characterized as having intrinsic mean fields with spontaneously broken rotational symmetry, and a full quantum mechanics treatment of which may lead to some new understandings of signature inversion and chirality in rapidly rotating nuclei. Signature is a quantum number related to the invariance of the system with respect to reflection in space and time, and defined as $\alpha_I=\dfrac{1}{2}(-1)^{I-1/2}$ for a nuclear state of spin I. For odd-A nuclei, the yrast band is based upon a single particle high-j orbital, such as $h_{11/2}$ and $i_{13/2}$, and consists of two sequences of $I=j(\mathrm{mod}2)$ and $I=j+1$ (mod2) according to the signature. The former, with signature $\alpha_{\rm f}=\dfrac{1}{2}(-1)^{j-1/2}$, is lower in energy than the latter,with signature $\alpha_{\rm u}=\dfrac{1}{2}(-1)^{j+1/2}$, in almost all the experimental cases, and this energy shift can be well understood in terms of the Coriolis coupling. In some odd nuclei, however, the signature inversion was found, namely, the favored sequence of $\alpha_{\rm f}$ lies higher in energy than the unfavored one of $\alpha_{\rm u}$ at spin $I>I_{\rm rev}$, where $I_{\rm rev}$ is so called inversion spin, e.g., in ^{157}Ho[1]. In some odd-odd nuclei, the signature inversion takes place at low spins and when spin increases up to spin $I>I_{\rm rev}$ the two signature sequences become normal phase in energy, namely no inversion, e.g., in $^{120-130}$Cs[2]. The signature inversions have been widely observed in nuclear rotational spectra in different mass regions, but the phenomenon has no common understanding for more that 20 years since the first discussion of the observed inversion of signature in $\pi h_{11/2}\nu i_{13/2}$ bands in odd-odd nuclei at N=89 in 1984, based on an assumption of significant asymmetry in the γ direction[3]. Since then there have existed different explanations for this phenomenon, for example, by attributing the signature inversion to the

* Supported by National Natural Science Foundation of China (10475115, 10305019, 10435010) and Major State Basic Research Development Program (G20000774)

1) E-mail: ysch@iris.ciae.ac.cn

positive γ rotation[3, 4], to the reversed γ rotation[5, 6], to the specific angular momentum coupling[7] , to the proton-neutron residual interaction[8, 9], to the band crossing[10], and to the quadropole pairing[11]. Very recently, the signature inversion was interpreted as the manifestation of the dynamic drift of the rotational axis in triaxially deformed nuclei[12]. In the present study we will emphasize the possibility that the signature inversion can be used to probe the triaxial rotation, a fundamental question in nuclear structure.

Another interesting phenomenon that is associated with the triaxial rotation is the chirality in nuclei. It was proposed that the spontaneously broken chiral symmetry may take place in triaxially deformed nuclei when a particular angular momentum coupling scheme appears, where three angular momenta of the valence neutrons, the valence protons and the core are mutually perpendicular so that a left- and a right-handed systems can be formed[13—15]. Some of the experimental evidences for the doublet bands of chirality were reported in the mass region A=130, where the proton Fermi level lies in the lower part and the neutron Fermi level in the higher part of the $h_{11/2}$ subshell. The first candidate of the chiral sister bands was reported for ^{134}Pr, where the nearly degenerate doublet bands with the same parity and spins were found experimentally[16, 17]. Very recently, a decisive experiment of measuring electromagnetic transition rates has been done, and a large difference, by a factor of 2 or 3, of the experimental B(E2) values for the doublet bands of ^{134}Pr was reported[18]. Then arises a serious question whether the observed doublet bands in ^{134}Pr can be interpreted as having chirality. At the present, there have been two different explanations for the nature of the measured E2 transition probabilities in ^{134}Pr. First, the fact of the different B(E2) values although does not support the static chirality, but the concept of the dynamic chirality may be applicable, this conclusion was drawn based on the IBFFM calculation[18]. Second, the large difference of the B(E2) values may be attributed to very different shapes, based on an analysis of band interaction between the doublet bands at the nearly degenerate spin region, and thus ^{134}Pr can not be interpreted as chiral bands[19]. As the third explanation, we will show that the large difference of the measured electromagnetic transition probabilities of the two bands can be attributed to very different intrinsic structures of the doublet bands at the nearly degenerate spin region, therefore the chirality picture is not applicable for ^{134}Pr.

The theoretical model is briefly described in Sec. 2. A calculation is performed for the so called double signature inversion in odd-proton nucleus ^{157}Ho, and a comparison between theory and experiment is made in Sec. 3. The energies and the electromagnetic transition probabilities of the doublet bands in ^{134}Pr are calculated and discussed in Sec. 4. The general conclusions are summarized in Sec. 5.

2 Brief description of the model

Reflection Asymmetric Shell Model has been generated to include the triaxial degree of freedom. The shell model Hamiltonian considered involves a large number of nucleons moving in a spherical Nilsson potential and an interaction of separable multipole Q·Q plus monopole pairing plus quardrupole pairing,

$$\begin{aligned} H = & H_0-\frac{1}{2}\sum_{\lambda=2}^{4}\chi_\lambda\sum_{\mu=-\lambda}^{\lambda}Q^\dagger_{\lambda\mu}Q_{\lambda\mu}- \\ & G_0P^\dagger_{00}P_{00}-G_2\sum_{\mu=-2}^{2}P^\dagger_{2\mu}P_{2\mu}\ , \end{aligned} \tag{1}$$

where H_0 is the spherical modified harmonic-oscillator single particle Hamiltonian, and the operators Q and P are expressed as

$$Q_{\lambda\mu} = \sum_{\alpha,\beta}\langle\alpha|\rho^2Y_{\lambda\mu}|\beta\rangle c^\dagger_\alpha c_\beta, \tag{2}$$

$$P^\dagger_{00} = \frac{1}{2}\sum_{\alpha}c^\dagger_\alpha c^\dagger_{\bar\alpha}, \tag{3}$$

$$P^\dagger_{2\mu} = \frac{1}{2}\sum_{\alpha,\beta}\langle\alpha|\rho^2Y_{2\mu}|\beta\rangle c^\dagger_\alpha c^\dagger_{\bar\beta}\ , \tag{4}$$

where, the eigenstates of a single particle moving in the spherical Nilsson potential are described by the quantum numbers $\alpha=nljm$, and $\bar\alpha$ denotes the time-reversed state of α. The QQ interaction strength χ is determined in such a way that it has a self-consistent relation with the quadrupole deformation[20, 21].

The monopole pairing strength $G_{\rm M}$ is of the stand- ard form G/A (MeV) in the A=130 region and $G=(g_1 \mp g_2(N-Z)/A)/A$ (MeV), where $\mp$ for neutrons and protons respectively, in the A=160 region. The quadrupole pairing strength $G_{\rm Q}$ is proportional to $G_{\rm M}$. For the present calculations, $G_{\rm M}$=19.6 for neutrons and 17.2 for protons and $G_{\rm Q}$=0.16$G_{\rm M}$ in the A=130 region, and g_1=20.12, g_2=13.12 and $G_{\rm Q}$=0.22$G_{\rm M}$ in the A=160 region. These pairing parameters approximately reproduce the observed odd-even mass differences in the considered mass regions.

The trial wave function may be constructed by means of the projection method. For the present purpose, we do not need the parity projection, the wave-function is written as,

$$|\Psi^{\sigma}_{IM}\rangle = \sum_{K\kappa} f^{\sigma}_{IK_\kappa}\, \hat{P}^{I}_{MK}\,|\Phi_\kappa\rangle\,, \tag{5}$$

where $\hat{P}^{I}_{MK}$ is the three-dimensional angular-momentum-projection operator

$$\hat{P}^{I}_{MK} = \frac{2I+1}{8\pi^2}\int {\rm d}\Omega\, D^{I}_{MK}(\Omega)\,\hat{R}(\Omega), \tag{6}$$

and σ in Eq. (5) specifies the states with the same angular momentum I. The dimension of the summation in Eq. (5) is $K\times\kappa$, where $|K|\leqslant I$ and κ is usually in the order of 10^2.

Where $|\Phi_\kappa\rangle$ represents a set of multi-qp states associated with the triaxially deformed qp vacuum $|0\rangle$. For odd-odd nuclei one has

$$\begin{aligned}&\{\alpha^{\dagger}_{\nu_1}\alpha^{\dagger}_{\pi_1}|0\rangle,\ \ \alpha^{\dagger}_{\nu_1}\alpha^{\dagger}_{\nu_2}\alpha^{\dagger}_{\nu_3}\alpha^{\dagger}_{\pi_1}|0\rangle,\ \ \alpha^{\dagger}_{\nu_1}\alpha^{\dagger}_{\pi_1}\alpha^{\dagger}_{\pi_2}\alpha^{\dagger}_{\pi_3}|0\rangle,\\ &\alpha^{\dagger}_{\nu_1}\alpha^{\dagger}_{\nu_2}\alpha^{\dagger}_{\nu_3}\alpha^{\dagger}_{\pi_1}\alpha^{\dagger}_{\pi_2}\alpha^{\dagger}_{\pi_3}|0\rangle\,\},\end{aligned} \tag{7}$$

and for odd proton nuclei,

$$\{\alpha^{\dagger}_{\pi_1}|0\rangle,\ \alpha^{\dagger}_{\nu_1}\alpha^{\dagger}_{\nu_2}\alpha^{\dagger}_{\pi_1}|0\rangle,\ \alpha^{\dagger}_{\nu_1}\alpha^{\dagger}_{\nu_2}\alpha^{\dagger}_{\pi_1}\alpha^{\dagger}_{\pi_2}\alpha^{\dagger}_{\pi_3}|0\rangle\,\}. \tag{8}$$

The triaxially deformed single particle states are generated by the Nilsson Hamiltonian

$$H_{\rm N} = H_0 - \frac{2}{3}\hbar\omega\epsilon_2\left(\cos\gamma Q_0 + \sin\gamma\frac{Q_{+2}+Q_{-2}}{\sqrt{2}}\right), \tag{9}$$

Where H_0 is the spherical single-particle Hamiltonian, which contains a proper spin-orbit force[22]. The parameters ϵ_2 and γ describe quadrupole deformation and triaxial deformation, respectively.

By solving the Hamiltonian equation

$$\hat{H}\,|\Psi^{\sigma}_{IM}\rangle = E^{\sigma}(I)\,|\Psi^{\sigma}_{IM}\rangle\,, \tag{10}$$

we obtain energy eigenvalue $E^{\sigma}(I)$ of state σ as a function of spin I and the corresponding wave function $f^{\sigma}_{IK\kappa}$. The reduced rate of electromagnatic transitions between the eigenstates $|\Psi^{\sigma}_{IM}\rangle$, induced by a spherical tensor operator $\hat{M}_{\lambda\mu}$, is

$$B(M\lambda; i\to f) = \frac{2I_f+1}{2I_i+1}\left|\left\langle \Psi^{\sigma_f\pi_f}_{I_f}\|\hat{M}_\lambda\|\Psi^{\sigma_i\pi_i}_{I_i}\right\rangle\right|^2, \tag{11}$$

and the reduced matrix elements become

$$\begin{aligned}\left\langle \Psi^{\sigma_f\pi_f}_{I_f}\|\hat{M}_\lambda\|\Psi^{\sigma_i\pi_i}_{I_i}\right\rangle &= \frac{1}{2}(1+\pi_f\pi_i\pi_\lambda)\\ \sum_{K_i\kappa_i K_f\kappa_f} f^{\sigma_f\pi_f}_{I_fK_f\kappa_f} f^{\sigma_i\pi_i}_{I_iK_i\kappa_i}&\sum_{\mu}\langle I_iK_f-\mu\lambda\mu|I_fK_f\rangle\\ &\left\langle \Phi_{\kappa_f}\left|\hat{M}_{\lambda\mu}P^{I_i\pi_i}_{K_f-\mu K_i}\right|\Phi_{\kappa_i}\right\rangle.\end{aligned} \tag{12}$$

3 Signature inversion as a prob for the triaxial rotation

The signature inversion phenomenon that has been widely observed in nuclear rotational spectra, but no common understanding for more that 20 years. Recently, the signature inversion was interpreted as the manifestation of drift of the dynamic cranking axis in triaxially deformed nuclei[12]. Here we point out that the signature inversion is strongly related to the triaxial rotation, a fundamental question in nuclear structure. The present study attempts to achieve a more thorough understanding for the signature inversion. The signature is associated with the invariance of a system with intrinsic quadrupole deformation under a rotation of 180° around a principal axis, and is first defined in the cranking model. We define the dynamic cranking axis as the axis along which the total angular momentum has a largest component. Note that the dynamic cranking axis can be any one of three principal axises of a rotating triaxial nucleus, while the cranking axis defined in the cranking model is a fixed one. We interpret the signature inversion as the change of the dynamic cranking axis in the rotational triaxial nuclear system. Take the yrast band of old proton nucleus ^{157}Ho as an example, where the twice signature inversions were observed experimentally around spin I=37/2 and 53/2 respectively[1]. This band is based on an intrinsic configuration of the Fermi aligned proton orbital in the proton $h_{11/2}$ shell, namely the proton Fermi level lies in the middle of the shell, which has a large alignment along the y-axis,the intermediate axis, I_y, therefore, the occurrence of the signature inversion in the band is mainly determined by the characteristic of the y-axis, namely being as the dynamic cranking axis or not. In order to investigate the double signature inversions and explore the triaxial rotation, the quantity $S(I)=E(I)-E(I-1)$ and the expectation values of I_i^2, where $i=x,y,z$, for the yrast band of ^{157}Ho are calculated. The deformation parameters used in the basis construction are $\varepsilon_2=0.26$, $\varepsilon_4=-0.02$ and $\gamma=26°$, which reproduce approximately the γ-band energy in the neighbor even-even nuclei. The calculated result of $S(I)$ is compared with data in Fig. 1.

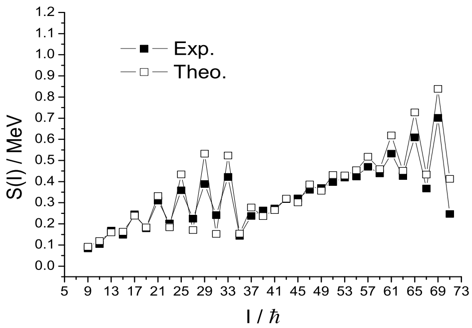

Fig. 1. Comparison of calculated $S(I)=E(I)-E(I-1)$ with experimental data for ^{157}Ho[1].

It is seen that the twice signature inversions are well reproduced and the large signature splitting before the first and after the second inversions together with the quenched signature splitting between the two inversion points are also reproduced. The calculated expectation values of the squares of the components of total angular momentum along the three intrinsic principal axis, I_x^2, I_y^2 and I_z^2, as functions of spin I are shown in Fig. 2.

From Fig. 2 it is seen that the I_y is largest one at lower spins and becomes smaller than the I_x after the first inversion point and then becomes largest one again after the second inversion point. According to the present definition of the dynamic cranking axis, these calculations show that the first signature inversion is caused by the change of the dynamic cranking axis from the y-axis to the x-axis, while the second inversion is due to the change back of the dynamic cranking axis to the y-axis. The cause of change of the dynamic axis may be attributed to the alignments of the pair of neutrons in the $i_{13/2}$ shell.

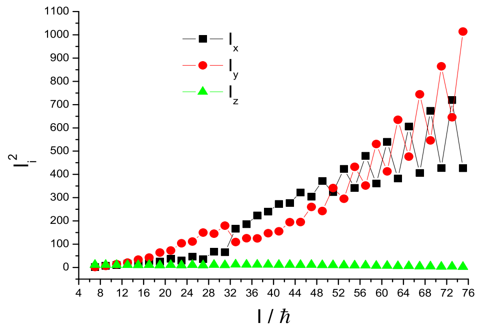

Fig. 2. Calculated expectation values of I_x^2, I_y^2 and I_z^2 as functions of spin I for ^{157}Ho.

4 Nature of the candidate chiral bands in ^{134}Pr

The energy degeneracy of doublet band states with the same spin and parity is an important indication for the occurrence of chirality and seems to be observed in ^{134}Pr. Among other things, however, the most crucial criteria for the chirality is the equivalence of the reduced E2 transition probabilities in the doublet bands. Very recently, it has been reported that the measured electromagnetic transition rates show a large difference of the B(E2) values for the doublet bands of ^{134}Pr. It is important to understand the nature of the observed energy degeneracy and the difference of the E2 transition rates in the spin region of I=14 to 17. To explore the nature of these observations we carried out calculations of electromagnetic transition probabilities and energy levels of the doublet bands with the present model. The deformation parameters used in the basis construction are ε_2=0.19, ε_4=0.0 and $\gamma=34°$, which reproduce approximately the γ-band energy in the neighbor even-even nuclei. The calculated band energies are compared with experimental data and shown in Fig. 3, and it is seen that a good agreement between theory and experiment has been achieved.

Particularly, the crossing behavior of the band 1 (yrast) and the band 2 (nonyrast) was reproduced by the present calculation. The crossing is realized by the cross over interband E2 transition rate which is calculated to be larger for the transition from the nonyrast state of I=18 to the yrast state of I=16 than for the transition from the yrast state of I=18 to the nonyrast state of I=16. At the nearly degenerate region, from I=14 to 17, calculated B(E2) values are larger for band 1 than for band 2 by about a factor of 3, in average, and this is in a quite nice agreement with the experimental B(E2) data[18]. The present calculations reproduce simultaneously both the energy degeneracy and the large difference of the reduced E2 transition probabilities in the considered spin region for the doublet bands. A detailed analysis for the intrinsic structures of calculated doublet bands indicates that in the spin region from I=14 to 18, the doublet bands have completely different nature, namely the band 1 is a 2 q.p. state , 1n1p, e.g., of $\mu h_{11/2}\pi h_{11/2}$ configuration, while the band 2 has mainly a 4 q.p. configuration ,1n3p, e.g., of $\mu h_{11/2}\pi h_{11/2}d_{5/2}g_{7/2}$ configuration. The shell model configuration mixing of 4 q.p. component gives rise to a strong reduction of B(E2) values of the band 2 in the band interaction region, I=14—17. The chiral doublet bands should have a similar intrinsic structure, therefore our theoretical results lead to a conclusion that the observed doublet bands in ^{134}Pr can not be interpreted as chiral bands although the energy nearly degeneracy is observable.

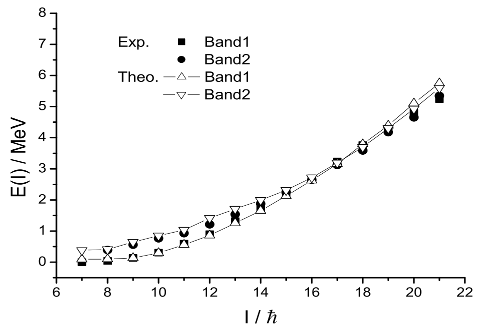

Fig. 3. Comparison of calculated the doublet bands with experimental data for ^{134}Pr [16].

5 Summary

The signature inversion and chiral band structures are described by the recently developed triaxial projected shell model as results of some nuclei having intrinsic mean fields with spontaneously broken rotational symmetry. The triaxial rotation, a fundamental question in nuclear structure, has been investigated by examining the signature inversion and chirality phenomena in nuclei. The signature inversion phenomenon is interpreted as the change of the dynamic cranking axis in triaxially deformed nuclei. The double signature inversions observed experimentally in ^{157}Ho are reproduced quite well by the present calculation and the phenomenon is then interpreted as the twice changes of the dynamic cranking axis, caused by the alignments of a pair of $i_{13/2}$ neutrons. The energies of the candidate chiral band states in ^{134}Pr are calculated and the results are in a good agreement with experimental data, particularly the nearly degeneracy and crossing feature of the two bands in the range of spin 14 to 18 are reproduced. The same wave functions are then used to calculate the E2 transition probabilities of the doublet bands, and the results again reproduce the striking feature of the measured B(E2) probabilities, namely a large difference of B(E2) values between band 1 and band 2 in the nearly degenerate spin region. We pointed out that this large difference of the E2 transition probabilities between the doublet bands can be attributed to very different intrinsic structures of the doublet bands at the nearly degenerate spin region, therefore, the observed doublet bands in ^{134}Pr can not be interpreted as chiral bands.

References

1 Hagemann G B et al. Nucl. Phys., 1984, **A424**: 365
2 LIU Y Z et al. Phys. Rev., 1996, **C54**: 719
3 Bengtsson R, Frisk H, May F R et al. Nucl. Phys., 1984, **A415**: 189
4 Ikeda A, Aberg S. Nucl. Phys., 1988, **A480**: 85
5 Akitsu Ikeda, Takafumi Shimano. Phys. Rev. Lett., 1989, **63**: 139
6 Hamamoto I, Mottelson B R. Phys. Lett., 1983, **B132**: 7
7 Hamamoto I. Phys. Lett., 1990, **B232**: 221
8 Cederwall B et al. Nucl. Phys., 1992, **A542**: 454
9 Tajima N. Nucl. Phys., 1994, **A572**: 365
10 Hara K. Nucl. Phys., 1993, **A557**: 449c
11 XU F R, Satula W, Wyss R. Nucl. Phys., 2000, **A669**: 119
12 GAO Z C, CHEN Y S, SUN Yang. Phys. Lett., 2006, **B 634**: 195
13 Frauendorf S, Meng J. Nucl. Phys., 1997, **A617**: 131
14 Dimitrov V I et al. Phys. Rev. Lett., 2000, **84**: 5732
15 Starosta K et al. Nucl. Phys., 2001, **A682**: 375c
16 Petrache C M et al. Nucl. Phys., 1996, **A597**: 106
17 Starosta K et al. Phys. Rev. Lett., 2001, **86**: 971
18 Tonev D et al. Phys. Rev. Lett., 2006, **96**: 052501
19 Petrache C M et al. Phys. Rev. Lett., 2006, **96**: 112502
20 CHEN Y S, GAO Z C. Phys. Rev., 2001, **C63**: 014314
21 Hara K, Sun Y. Int. J. Mod. Phys., 1995, **E4**: 637
22 Bengtsson T, Ragnasson I. Nucl. Phys., 1985, **A436**: 14

核结构中转动对称性自发破缺：某些理论问题*

陈永寿[1,2;1)]　高早春[1]

1 (中国原子能科学研究院　北京　102413)
2 (中国科学院理论物理研究所　北京　100080)

摘要　反射不对称壳模型被推广为包含三轴不对称自由度. 旋称反转, 摇摆带和手征带结构, 可由本理论模型描述为某些原子核具有自发破缺的内禀平均场的结果. 作为该理论的应用, 通考察原子核的旋称反转和手征性现象, 研究了三轴转动这个核结构中的基本问题. 旋称反转现象被解释为三轴变形核中动力学推转轴的改变. 对 ^{134}Pr 中的候选手征二重带的性质给出了新的认识.

关键词　壳模型　旋称反转　手征带

* 国家自然科学基金(10475115, 10305019, 10435010)和国家重点基础研究发展规划(G20000774)资助

1) E-mail: yschen@iris.ciae.ac.cn