# Scalar-Field Model for Dark Energy with a Fixed Background Vector Field*

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**Abstract** We propose a dark energy model with single dynamical scalar field and fixed background vector field, in which the parameter $w$ can cross $-1$ during the evolution of the universe. It is found that in certain cases $w$ can cross $-1$ and transition from decelerating to accelerating occur at $z\approx 0.2$ and $z\approx 1.7$ respectively, which is consistent with the observations.

**Key words** scalar field, dark energy, vector field, equation of state

## 1 Introduction

The recent observations[1—3] suggest that the Universe consists of dark energy (73%), dark matter (23%) and baryon matter (4%). What the dark energy becomes one of the essential questions in theoretical physics and cosmology. Although the simplest form of dark energy — the cosmological constant — fits the observation[4] quite well, which has the effective equation of state $p=-\rho$, or $w=-1$, there is evidence[5] to show that dark energy might evolve from $\omega>-1$ in the past to $\omega<-1$ today. It seems that the dynamics model with a varying $w$ is preferable rather than the simple constant.

Various models have been proposed in order to describe the evolving dark energy. Most of them are characterized by a scalar field, which grew in part out of the inflation scenario in part from ideas from particle physics. Unfortunately, most of these dynamical dark energy models cannot explain the fact $w$ crossing $-1$. For example, $w$ in the quintessence models[6] is always evolving in the range of $-1\leqslant\omega\leqslant 1$. In the phantom models[7] $w$ only varies below $-1$. The parameter $w$ for the general k-essence models[8] also fails to cross $w=-1$.

## 2 The evolution equation of universe in new model

Recently some dynamical dark energy models with the property of $w$ crossing $-1$ have been proposed[9—14]. In the present letter, we shall further study the scalar-field model for dark energy with a fixed background vector field[14]. In this model the dark energy is effectively described by a dynamical scalar field coupled to a fixed background vector field with a constant norm and constant zeroth-component in a comoving system, which describes some unknown effects of the universe. Namely, the dark energy is supposed to be governed by the Lagrangian,

$$\mathscr{L}_{\phi}^{\rm eff}=\frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}-g^{\mu\nu}\phi_{,\mu}A_{\nu}-V(\phi). \tag{1}$$

where $A_\mu$ is the fixed background covariant vector. In Ref. [14], it has been shown that due to the existence of a fixed background vector field, the parameter $w$ of dark energy for a flat universe may cross $w=-1$ in

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the evolution of the universe. Since the cosmological observations prefer that the universe is closed[3], we will study the model in a closed universe. The present letter is to focus on the problem.

Consider the Lagrangian

$$\mathscr{L}_{\rm T}=\mathscr{L}_{\rm EH}+\mathscr{L}_{\rm m}+\mathscr{L}_{\phi}^{\rm eff}, \tag{2}$$

where $\mathscr{L}_{\rm EH}$ and $\mathscr{L}_{\rm m}$ are the Lagrangian for gravity and for matter, including bayonic matter and dark matter. Suppose the universe is homogenous and isotropic, namely satisfying the cosmological principle as usual, so that the Robertson-Walker metric

$$\mathrm{d}s^2=\mathrm{d}t^2-a^2(t)\left(\frac{\mathrm{d}r^2}{1-kr^2}+r^2\mathrm{d}\Omega_2^2\right) \tag{3}$$

can be used.

The stress-energy tensor for the homogenous and isotropic scalar field is

$$\begin{aligned}T_{\mu\nu} &= \dot\phi^2\delta_\mu^0\delta_\nu^0-2\dot\phi A_0\delta_\mu^0\delta_\nu^0- \\ &\quad \frac{1}{2}g_{\mu\nu}(\dot\phi^2-2\dot\phi A_0-2V(\phi)).\end{aligned} \tag{4}$$

It is equivalent to perfect fluid with energy density

$$\rho_\phi=\frac{1}{2}\dot\phi^2-\dot\phi A_0+V(\phi) \tag{5}$$

and pressure

$$p_\phi=\frac{1}{2}\dot\phi^2-\dot\phi A_0-V(\phi), \tag{6}$$

respectively. The effective equation of state for the dark energy is $p=w\rho$ with

$$w=\frac{\dot\phi^2-2\dot\phi A_0-2V(\phi)}{\dot\phi^2-2\dot\phi A_0+2V(\phi)}. \tag{7}$$

Remember that the energy density $\rho_\phi\geqslant 0$. It can be seen that for $A_0>0$,

$$w\begin{cases}\geqslant -1, & \dot\phi\geqslant 2A_0 \quad\text{or}\quad \dot\phi\leqslant 0,\\ <-1, & 2A_0>\dot\phi>0,\end{cases} \tag{8}$$

for $A_0<0$,

$$w\begin{cases}\geqslant -1, & \dot\phi\geqslant 0 \quad\text{or}\quad \dot\phi\leqslant 2A_0,\\ <-1, & 0>\dot\phi>2A_0.\end{cases} \tag{9}$$

and other cases are not relevant. The equation of motion for $\phi$ takes the form

$$\ddot\phi+3H\dot\phi+\frac{\mathrm{d}V}{\mathrm{d}\phi}=3HA_0, \tag{10}$$

where we have assumed that $A_\mu$ satisfies the cosmological principle and $A_0$ is a constant. Since we are only interested in the late stage of the evolution of the universe, all matter can be regarded as dust, whose energy density satisfies $\rho=\rho_0a_0^3/a^3$. Then, the Friedmann equation reads

$$H^2+\frac{k}{a^2}=\frac{8\pi G}{3}\left(\frac{1}{2}\dot\phi^2-\dot\phi A_0+v(\phi)+\frac{\rho_0a_0^3}{a^3}\right). \tag{11}$$

Let

$$\begin{aligned}\varphi &= \sqrt{\frac{8\pi G}{3}}\phi, \quad \chi=\sqrt{\frac{8\pi G}{3}}\frac{\dot\phi}{H_0},\\ b_0 &= \sqrt{\frac{8\pi G}{3}}\frac{A_0}{H_0}, \quad v(\varphi)=\frac{8\pi GV(\phi)}{3H_0^2},\\ h &= \frac{H}{H_0}.\end{aligned} \tag{12}$$

Eq. (10) and Eq. (11) can be simplified as dimensionless ordinary differential equations

$$\frac{\mathrm{d}\varphi}{\mathrm{d}z} = -\frac{\chi}{h(1+z)}, \tag{13}$$

$$\frac{\mathrm{d}\chi}{\mathrm{d}z} = \frac{3(\chi-b_0)}{1+z}+\frac{v'(\varphi)}{h(1+z)}, \tag{14}$$

and

$$\begin{aligned}h^2 &= \frac{1}{2}\chi^2-b_0\chi+v(\varphi)+\\ &\quad \Omega_{\rm M0}(1+z)^3-\frac{k(1+z)^2}{a_0^2H_0^2}.\end{aligned} \tag{15}$$

In terms of new variables, Eq. (7) becomes

$$w=\frac{\chi^2-2\chi b_0-2v(\varphi)}{\chi^2-2\chi b_0+2v(\varphi)}. \tag{16}$$

For a closed universe the present values which serve as the initial values of integration are taken as

$$h_0=1, \quad \omega_0=-1.33, \quad \Omega_{\rm M0}=0.275, \tag{17}$$

$$\Omega_{\rm DE0}:=(\frac{1}{2}\chi^2-b_0\chi+v)|_{z=0}=0.745, \tag{18}$$

$$(\chi^2-2b_0\chi)|_{z=0}=-0.246, \quad v|_{z=0}=0.868 \tag{19}$$

concordant with the observation data[3, 5].

During the calculation of Eqs. (13)—(15), we need other important parameter $w_1$ from parametrization in model independent analysis of observation data

$$w_1=\frac{\mathrm{d}}{\mathrm{d}z}\frac{\chi^2-2\chi b_0-2v(\varphi)}{\chi^2-2\chi b_0+2v(\varphi)}\bigg|_{z=0}, \tag{20}$$

so we can get

$$\begin{aligned}v'(\varphi)|_{z=0} &= \frac{1}{\chi_0^2(\chi_0-2b_0)+2v_0(\chi_0-b_0)}\\ &\quad \left(\frac{w_1}{4}(\chi_0^2-2b_0\chi_0+2v_0)^2-\right.\\ &\quad \left.6v_0(\chi_0-b_0)^2\right).\end{aligned} \tag{21}$$

## 3 The results by choosing different potentials

Consider the simplest potential,

$$V(\phi)=\frac{1}{2}m^2\phi^2. \tag{22}$$

then $v(\phi)=\frac{m^2}{2H_0^2}\varphi^2$, $v'(\phi)=\frac{m^2}{H_0^2}\varphi$.

Further we take $b_0=-0.659$, $w_1=1.47$, thus we can get the initial conditions of Eqs. (12)—(14), $\varphi_0=5.09$, $\chi_0=-1.09$, and a necessary parameter $\frac{m^2}{2H_0^2}=3.35\times10^{-2}$. The dependence of $w$ on redshift $z$ is shown in Fig. 1 and Fig. 2.

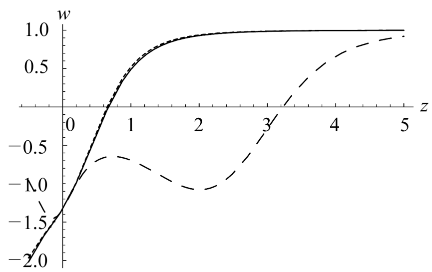


Fig. 1. The evolution of $w$ for a closed universe with a quadratic (dot), quartic (real) and supergravity (dash) potential with the corresponding initial values.

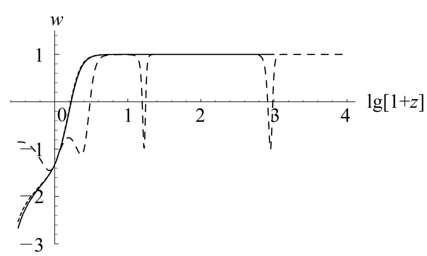


Fig. 2.

In the same way, we can analyze the case with the quartic potential

$$V(\phi)=\frac{1}{4}\lambda\phi^4. \tag{23}$$

$$v(\varphi)=\frac{3\lambda}{32H_0^2\pi G}\varphi^4,\quad \frac{3\lambda}{32H_0^2\pi G}=\left.\frac{v(\varphi)}{v'(\varphi)}\right|_{z=0}. \tag{24}$$

If we take $b_0=-0.662$, $w_1=1.47$, the initial conditions of Eqs. (13)—(15) can be chosen as $\varphi_0=9.16$, $\chi_0=-1.10$, and a necessary parameter $\frac{3\lambda}{32H_0^2\pi G}=1.23\times10^{-4}$. The equation of state can give $w<-1$ at $z<0.2$ which satisfies the requirement given by analysis of supernova data in Refs. [3, 5]. Note that, it is so similar to the case of $\varphi^2$ in Fig. 1 and Fig. 2 that they can not be distinguished clearly.

The next example is the closed universe with the potential inspired from supergravity[15]

$$V(\phi)=V_0\phi^{-\alpha}e^{4\pi G\phi^2}, \tag{25}$$

where $\alpha\geqslant 11$ is a constant. For this potential, if we may take $\alpha=12$, then

$$\chi_0=b_0\pm\sqrt{b_0^2-v_0^2}, \tag{26}$$

$$\varphi_0=\frac{v_0'}{6v_0}\pm\sqrt{\left(\frac{v_0'}{6v_0}\right)^2+4}. \tag{27}$$

If $w_1=1.47$, $b_0=-0.611$, by taking $-$ and $+$ in Eqs. (26)—(27) respectively, then we can get the initial conditions of Eqs. (13)—(15). In Fig. 1 $w=-1$ at $z=0.2$ is satisfied. However, $w\approx-1/3$ at $z\approx2.6$ is not consistent with the transition redshift between the accelerating and decelerating phases occuring at $0.6<z<1.7$ in the SNe Ia observation[1, 2, 16, 17]. The dependence of $w$ on $z$ is not monotonic, for $w$ becomes $-1$ at $z\approx2$ again. The universe experienced another accelerated expansion around $z\approx30$. From Fig. 2 we can see $w$ becomes 1 in the high redshift.

## 4 Concluding remarks

In a closed universe, we studied the new single-dynamical-scalar-field model for dark energy with a fixed background vector field. In the new model the parameter $w$ for the effective equation of state of dark energy can cross $-1$ and $w=-1/3$ during the evolution of the Universe. After proper choice of the potential the crossing $w=-1$ occurs at $z\approx0.2$ and the transition from decelerating phase to accelerating one occurs at $z\approx1.7$, respectively. The dependence of the parameter $w$ on the redshift $z$ is far from the linear relation. Thus, it is needed to re-analyze the observation data based on more general form of $w(z)$. By the way, without the introduction of oscillation potential, the universe may also exhibit some oscillation behaviors.

The present model is in the almost standard framework of physics, e.g. in general relativity in

4-dimension. An a priori vector field, $A_\mu$, is introduced, which is selected in an unspecified way by the cosmological background. It will result in the violation of Lorentz invariance and might be contributed by the collective effects of the vacuum polarization of some fundamental fields. The physical meaning of such a vector field is to be explored in further work.

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# 带有背景矢量场的标量场暗能量模型*


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**摘要**　讨论了带有背景矢量场的动力学标量场作为暗能量在闭宇宙的情形下的演化. 选取适当的标量场势函数后, 在得到的暗能量有效状态方程中, 参数$w$在红移$z\approx0.2$处可以越过$-1$, 而且在$z\approx1.7$处宇宙从减速膨胀的状态转变为加速膨胀状态, 这与最近的宇宙学观测相符.

**关键词**　标量场　暗能量　矢量场　状态方程

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