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Model of Geometric Neutrino Mixing and Leptogenesis

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Abstract I report results obtained recently in collaboration with on neutrino mixing (hep-ph/0507217). Current neutrino oscillation data are consistent with the neutrino mixing angles taking values $\sin^2\theta_{12}=1/3$, $\sin^2\theta_{23}=1/2$, and $\sin^2\theta_{13}=0$. We present a class of renormalizable gauge models which realize such a geometric mixing pattern naturally. These models, which are based on the non–Abelian discrete symmetry A_4, place significant restrictions on the neutrino mass spectrum. It is shown that baryogenesis via leptogenesis occurs quite naturally, with a single phase (determined from neutrino oscillation data) appearing in leptonic asymmetry and in neutrinoless double beta decay.

Key words neutrino, mass, mixing, lepton, baryon

Our understanding of the fundamental properties of neutrinos has improved dramatically over the last few years. Atmospheric and solar neutrino experiments have by now firmly established occurrences of neutrino flavor oscillations[1]. Combining all positive results[2, 3], one obtains the following neutrino mass and mixing pattern (with 2σ error bars)[3]:

$$\Delta m^2_{\odot} = m_2^2-m_1^2=7.92\times10^{-5}(1\pm0.09)\mathrm{eV}^2, \quad (1)$$

$$\Delta m^2_{\rm atm} = m_3^2-m_2^2=\pm2.4\times10^{-3}(1^{+0.21}_{-0.61})\mathrm{eV}^2, \quad (2)$$

$$\sin^2\theta_{12} = 0.314(1^{+0.18}_{-0.15}), \quad \sin^2\theta_{23}=0.44(1^{+0.41}_{-0.22}),$$
$$\sin^2\theta_{13} = 0.9^{+0.23}_{-0.9}\times10^{-2} \ . \quad (3)$$

Here m_i are the (positive) neutrino mass eigenvalues, and θ_{ij} are the neutrino mixing angles. $m_2^2-m_1^2>0$ in Eq. (1) is necessary for MSW resonance to occur inside the Sun. The sign of $\Delta m^2_{\rm atm}$, which is physical, is currently unknown.

A remarkable feature of the oscillation data is that they are all consistent with a "geometric" neutrino mixing pattern with the neutrino mixing matrix (the MNS matrix) given by[4—7]

$$U_{\rm MNS}=\begin{pmatrix}\sqrt{\dfrac{2}{3}} & \dfrac{1}{\sqrt{3}} & 0\\ -\dfrac{1}{\sqrt{6}} & \dfrac{1}{\sqrt{3}} & -\dfrac{1}{\sqrt{2}}\\ -\dfrac{1}{\sqrt{6}} & \dfrac{1}{\sqrt{3}} & \dfrac{1}{\sqrt{2}}\end{pmatrix}P\ . \quad (4)$$

Here P is a diagonal phase matrix.

In fact, these geometric mixing angles are very close to the central values of Eq. (3). We observe that unlike the quark mixing angles, which are related to the quark mass ratios in many models (eg: $\theta_{\rm c}\approx\sqrt{m_{\rm d}/m_{\rm s}}$), the neutrino mixing angles seem to be unrelated to the neutrino mass ratios. The purpose of this work is toprovide a deriation of such a geometric neutrino mixing based on renormalizable gauge theories.

Our derivation of Eq. (4) will be based on the non–Abelian discrete symmetry A_4, the symmetry group of a regular tetrahedron. This symmetry group has found application in obtaining maximal atmospheric neutrino mixing[8] and in realizing quasi–degenerate

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neutrino mass spectrum[9]. No successful derivation of Eq. (4) has been achieved to our knowledge (based on A_4 or other symmetries) in a renormalizable gauge theory context. For attempts along this line see Refs. [4, 10, 11]. In Refs. [4, 6], Eq. (4) was suggested as a phenomenological ansatz. In Ref. [10], a higher dimensional set up is used to motivate Eq. (4). Ref. [11] analyzes special cases of an A_4 derived neutrino mass matrix towards obtaining the structure of Eq. (4). A large number of models in the literature have derived maximal atmospheric mixing based on non-Abelian symmetries[12], but in most models the solar mixing angle is either maximal (now excluded by data) or is a free parameter.

The Model. We work in the context of low energy supersymmmetry, which is motivated by a solution to the gauge hierarchy problem as well as by the observed unification of gauge couplings. The gauge group of our model is that of the Standard Model, $SU(3)_{\rm C}\times SU(2)_{\rm L}\times U(1)_{\rm Y}$. We augment this symmetry with a non-Abelian discrete symmetry A_4. This order 12 group is the symmetry group of a regular tetrahedron. A_4 has a unique feature in describing the lepton sector: It has one triplet and three inequivalent singlet representations, thus allowing for assigning the left-handed lepton fields to the triplet and the right-handed charged lepton fields to the three inequivalent singlets. In addition to the A_4 symmetry, we assume a $Z_4\times Z_3$ discrete symmetry. The Z_4 is an R-symmetry under which the superpotential carries 2 units of charge. The Z_4 and Z_3 symmetries are broken softly in the superpotential via the lowest dimensional operators.

The lepton and Higgs fields transform under $A_4\times Z_4\times Z_3$ as follows.

$$\begin{aligned}&L:(3,1,0),\ e^{\rm c}:(1+1'+1'',3,0),\ \nu^{\rm c}:(3,0,1),\\&E:(3,1,0),\ E^{\rm c}:(3,1,0),\\&H_{\rm u}:(1,1,2),\ H_{\rm d}:(1,0,0),\\&\chi:(3,2,0),\ \chi':(3,2,1),\ S_{1,2}:(1,2,1).\end{aligned}\tag{5}$$

Here in the fermion sector we have introduced new vector-like iso-singlet fields E and $E^{\rm c}$ transforming under the SM gauge group as (1,1,−1) and (1,1,1), respectively, which will acquire large masses and decouple. $H_{\rm u}$ and $H_{\rm d}$ are the usual Higgs fields of MSSM, while $\chi,\chi',S_{1,2}$ are all SM singlet fields needed for achieving symmetry breaking. The quark fields $(Q,u^{\rm c},d^{\rm c})$ are all singlets of A_4 with $Z_4\times Z_3$ charges of $Q(1,1); u^{\rm c}(0,0)$ and $d^{\rm c}(1,2)$, so that the usual quark Yukawa couplings $Qd^{\rm c}H_{\rm d}+Qu^{\rm c}H_{\rm u}$ are allowed in the superpotential.

The superpotential terms relevant for lepton masses and Higgs superpotential are given by

$$\begin{aligned}W_{\rm Yuk}=&M_E E_i E_i^{\rm c}+f_{\rm e}L_i E_i^{\rm c}H_{\rm d}+h^{\rm e}_{ijk}E_i e^{\rm c}_j\chi_{\rm k}+\\&\frac{1}{2}f_{\rm S_1}\nu^{\rm c}_i\nu^{\rm c}_i S_1+\frac{1}{2}f_{ijk}\nu^{\rm c}_i\nu^{\rm c}_j\chi'_{\rm k}+f_\nu L_i\nu^{\rm c}_i H_{\rm u},\\W_{\rm Higgs}=&\lambda_\chi\chi_1\chi_2\chi_3+\lambda_{\chi'{\rm s}}(\chi_1'^2+\chi_2'^2+\chi_3'^2)S_1+\\&\lambda'_{\chi'}\chi'_1\chi'_2\chi'_3+\lambda_{\rm s_{11}}S_1^3+\lambda_{\rm s_{12}}S_1^2S_2+\lambda_{\rm s_{21}}S_1S_2^2+\\&\lambda_{\rm s_{22}}S_2^3+\mu_1^2S_1+\mu_2^2S_2+\mu_\chi(\chi_1^2+\chi_2^2+\chi_3^2).\end{aligned}\tag{6}$$

Here $\chi=(\chi_1,\chi_2,\chi_3)$, and $\chi'=(\chi'_1,\chi'_2,\chi'_3)$. The $\mu^2_{1,2}$ terms are the lowest dimensional terms that break the Z_3 symmetry softly, while leaving Z_4 unbroken. The μ_χ term is the lowest dimensional term that breaks the Z_4 symmetry softly. Such soft breaking can be understood as spontaneous breaking occurring at a higher scale. We have chosen without loss of generality the combination of S_1 and S_2 that couples to χ' as simply S_1 in Eq. (6).

Minimizing the potential derived from Eq. (6) in the supersymmetric limit, we obtain the following vacuum structure:

$$\begin{aligned}&\langle S_2\rangle=v_{\rm s},\ \langle S_1\rangle=0;\ \langle\chi_1\rangle=\langle\chi_2\rangle=\langle\chi_3\rangle=v_\chi;\\&\langle\chi'_2\rangle=v_{\chi'},\ \langle\chi'_1\rangle=0;\ \langle\chi'_3\rangle=0.\end{aligned}\tag{7}$$

with $v_\chi=-2\mu_\chi/\lambda_\chi$, $v_{\rm s}^2=-\mu_2^2/(3\lambda_{\rm s_{22}})$, and $v^2_{\chi'}=(\lambda_{\rm s_{21}}\mu_2^2-3\lambda_{\rm s_{22}}\mu_1^2)/(3\lambda_{\rm s_{22}}\lambda_{\chi's})$. Electroweak symmetry breaking is achieved in the usual way by $\langle H_{\rm u}\rangle=v_{\rm u}$, $\langle H_{\rm d}\rangle=v_{\rm d}$. We emphasize that vanishing of certain VEVs is a stable result, owing to the discrete symmetries present in the model. This is important for deriving the MNS matrix of Eq. (4). We observe that there are no pseudo-Goldstone modes, as can be seen by directly computing the masses of the Higsinos from Eq. (6).

The mass matrices M_{eE} for the charged leptons and $M_{\nu\nu^{\rm c}}$ for the neutral leptons resulting from Eqs. (6) and (7) are given by (in the notation $\mathscr{L}=$

$(e,E)M_{eE}\ (e^{\rm c},E^{\rm c})^{\rm T})$

$$M_{eE}=\begin{pmatrix}0&0&0&f_e v_{\rm d}&0&0\\0&0&0&0&f_e v_{\rm d}&0\\0&0&0&0&0&f_e v_{\rm d}\\h_1^{\rm e}v_\chi&h_2^{\rm e}v_\chi&h_3^{\rm e}v_\chi&M_E&0&0\\h_1^{\rm e}v_\chi&h_2^{\rm e}\omega v_\chi&h_3^{\rm e}\omega^2 v_\chi&0&M_E&0\\h_1^{\rm e}v_\chi&h_2^{\rm e}\omega^2 v_\chi&h_3^{\rm e}\omega v_\chi&0&0&M_E\end{pmatrix}, \tag{8}$$

$$M_{\nu\nu^{\rm c}}=\begin{pmatrix}0&0&0&f_\nu v_{\rm u}&0&0\\0&0&0&0&f_\nu v_{\rm u}&0\\0&0&0&0&0&f_\nu v_{\rm u}\\f_\nu v_{\rm u}&0&0&f_{{\rm s}_2}v_{\rm s}&0&f_{\chi'}v_{\chi'}\\0&f_\nu v_{\rm u}&0&0&f_{{\rm s}_2}v_{\rm s}&0\\0&0&f_\nu v_{\rm u}&f_{\chi'}v_{\chi'}&0&f_{{\rm s}_2}v_{\rm s}\end{pmatrix}.$$

Since the E and the $E^{\rm c}$ fields acquire large masses, of order the GUT scale, they can be readily integrated out. The reduced 3×3 mass matrices for the light charged leptons is given by

$$M_e=U_{\rm L}\begin{pmatrix}m_e&0&0\\0&m_\mu&0\\0&0&m_\tau\end{pmatrix},\ U_{\rm L}=\frac{1}{\sqrt{3}}\begin{pmatrix}1&1&1\\1&\omega&\omega^2\\1&\omega^2&\omega\end{pmatrix}, \tag{9}$$

where $m_i=\sqrt{3}(f_e v_{\rm d}v_\chi/M_E)h_i^e(1+(h_i v_\chi)^2)/M_E^2)^{-1/2}$. The light neutrino mass matrix is found to be

$$M_\nu^{\rm light}=m_0\begin{pmatrix}1&0&x\\0&1-x^2&0\\x&0&1\end{pmatrix}, \tag{10}$$

where $m_0=f_\nu^2v_{\rm u}^2f_{{\rm s}_2}v_{\rm s}/(f_{{\rm s}_2}^2v_{\rm s}^2-f_{\chi'}^2v_{\chi'}^2)$, and $x=-f_{\chi'}v_{\chi'}/(f_{{\rm s}_2}v_{\rm s})$. We define $x=|x|{\rm e}^{{\rm i}\psi}$.

$M_\nu^{\rm light}$ can be diagonalized by the transformation $M_\nu^{\rm light}=U_\nu^*D_\nu U_\nu^\dagger$ with

$$U_\nu=\frac{1}{\sqrt{2}}\begin{pmatrix}1&0&-1\\0&\sqrt{2}&0\\1&0&1\end{pmatrix}P^*;$$
$$D_\nu=m_0\begin{pmatrix}|1+x|&&\\&|1-x^2|&\\&&|1-x|\end{pmatrix}. \tag{11}$$

P^* is a diagonal phase matrix given by

$$P^*={\rm diag}\{{\rm e}^{-{\rm i}\phi_1/2},\ {\rm e}^{-{\rm i}(\phi_1+\phi_2)/2},\ {\rm e}^{-{\rm i}(\phi_2+\pi)/2}\},$$
$$\phi_1={\rm arg}(1+x),\ \phi_2={\rm arg}(1-x)\ . \tag{12}$$

These Majorana phases will not be relevant for neutrino oscillations, but they will appear in neutrinoless double beta deacy and in leptogenesis. The MNS matricx is given by $U_{\rm MNS}=U_{\rm L}^{\rm T}U_\nu^*$ which has the form given in Eq. (4).

These conclusions can also be arrived at by analyzing the neutrino mass matrix in the flavor basis, i.e., in a basis where the charged lepton mass matrix is diagonal:

$$M_\nu^{\rm flavor}=\frac{m_0}{3}\begin{pmatrix}3+2x-x^2&-x-x^2&-x-x^2\\-x-x^2&2x-x^2&3-x-x^2\\-x-x^2&3-x-x^2&2x-x^2\end{pmatrix}. \tag{13}$$

The expressions for the light neutrino masses can be inverted to obtain

$$\cos\psi=\frac{-(m_3^2-m_1^2)m_2^2}{2\sqrt{2}m_1m_2[m_1^2m_2^2+m_2^2m_3^2-2m_1^2m_3^2]^{1/2}},$$
$$|x|=\frac{1}{\sqrt{2}m_1m_2}[m_1^2m_2^2+m_2^2m_3^2-2m_1^2m_3^2]^{1/2}. \tag{14}$$

Constraints on neutrino masses. There are restrictions arising from the conditions that $|x|$ be real and $|\cos\psi|\leqslant1$ for a given value of $r=\Delta m_\odot^2/\Delta_{\rm atm}^2$ with

$$\cos\psi=\frac{1+1\pm\sqrt{(1+r)^2+4(|x|^2+|x|^4)(1-r)^2}}{4|x|(r-1)}. \tag{15}$$

Both normal (the "+" soluand inverted, the "−" solution) neutrino mass hierarchies are allowed.

The normal hierarchy case occurs for x around-1. The condition $|\cos\psi\leqslant1$ can be satisfied only if $m_1/m_2\approx1/2$ whichleads to $m_1\approx1/2$ which leads to $m_1\approx(\Delta m_\odot^2/3)^{1/2}$. One also has $m_3\approx|\Delta m_{\rm atm}^2|^{1/2}$.

In the inverted hierarchy case, the situation is different. Here m_1 and m_2 are nearly equal, and m_3 is smaller than m_1. In order to satisfy $\Delta m_\odot^2\ll\Delta m_{\rm atm}^2$ it is necessary that $\cos\psi\approx|x|/2$. The value of m_3 is not determined by oscillation data. $m_3^2\gg|\Delta m_{\rm atm}^2|$ (or equivalently, $|x|\ll1$), we have three-fold degeneracy of masses and $m_{\beta\beta}\approx m_3$. This case also coincides with the leading results of Ref. [9].

The results for various effective masses, $m_{\beta\beta}$ for neutrinoless double β decay, $m_{\nu_{\rm e}}=(\sum_i|U_{ei}|^2m_i^2)^{1/2}$ for tritium β decay, and $\sum_i m_i$ for the sum of the three light neutrino masses, and the mass ratios are plotted in Figs. 1(a)—(c) and 2(a)—(c) as functions of $|x|$.

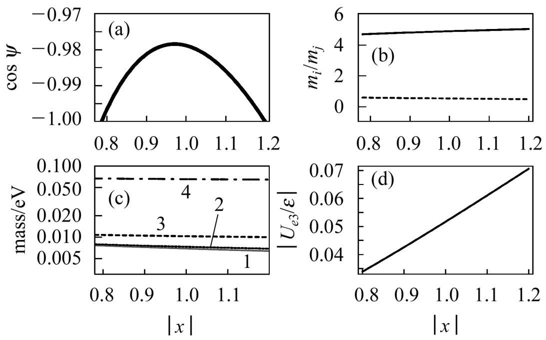

Fig. 1. Various quantities as functions of $|x|$ for normal mass hierarchy case. (a) $\cos\psi$ vs. $|x|$; (b) m_1/m_2 (dashed) and m_3/m_2 (solid) vs. $|x|$; (c) m_{ee} (solid (1)), m_{ν_e} (dotted (2)), m_2 (dashed (3)) and $\sum m_i$ (dot-dashed (4)) (in eV unit) vs. $|x|$; (d) RG running correction to $|U_{e3}/\epsilon|$ vs. $|x|$.

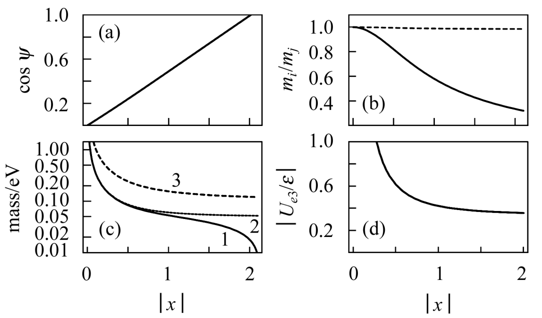

Fig. 2. Various quantities as functions of $|x|$ for inverted mass hierarchy case. (a) $\cos\psi$ vs. $|x|$; (b) m_1/m_2 (dashed) and m_3/m_2 (solid) vs. $|x|$; (c) m_{ee} (solid (1)), $m_{\nu_e} \simeq m_2$ (dashed (2)) and $\sum m_i$ (dot-dashed (3)) (in eV unit) vs. $|x|$; (d) RG running correction to $|U_{e3}/\epsilon|$ vs. $|x|$.

Stability of U_{e3}. A distinctive feature of the geometric mixing pattern is that $U_{e3}=0$ at the scale of A_4 symmetry breaking, which we have taken to be of order the GUT scale. When running from this high scale to low energy scale (M_{EW}), the mixing matrix may change, in particular U_{e3} may not be zero any more. One should ensure that the pattern of Eq. (4) is not destabilized, which can happen if the induced U_{e3} is too large. We demonstrate this stability now.

The leading flavor-dependent effect of the running from high scale to low scale is given by the one-loop RGE[13]

$$\frac{\mathrm{d}M_{\nu}^{\mathrm{e}}}{\mathrm{d}\ln t}=\frac{1}{32\pi^2}[M_{\nu}^{\mathrm{e}}Y_{\mathrm{e}}^{\dagger}Y_{\mathrm{e}}+(Y_{\mathrm{e}}^{\dagger}Y_{\mathrm{e}})^{\mathrm{T}}M_{\nu}^{\mathrm{e}}]+\cdots \tag{16}$$

This leads to correction, to the leading order, to the entries $M_{13,23}(1-\epsilon)$ and $M_{33}(1-2\epsilon)$ with $\epsilon \simeq Y_{\tau}^2\ln(M_{\mathrm{GUT}}/M_{\mathrm{EW}})/32\pi^2$. One obtains to order ϵ,

$$|U_{e3}|\approx\frac{|\epsilon x|\,||x|+\cos\psi+\mathrm{i}\sin\psi|}{3\sqrt{2}|\cos\psi(|x|+2\cos\psi)|}\ . \tag{17}$$

One obtains the induced $|U_{e3}|$ for the normal and inverted hierarchies by inserting the corresponding expressions for $\cos\psi$ and $|x|$ given earlier.

The results are shown in Figs. 1(d) (normal mass ordering) and in 2(d) (inverted ordering) where we plot $|U_{e3}/\epsilon|$ as a function of $|x|$. We see that the induced $|U_{e3}|$ is small, too small to be measured by near future experiments for the normal mass hierarchy case in the whole allowed $|x|$ range. For the inverted mass hierarchy case for $|x|$ larger than about 0.2, $|U_{e3}|$ remains small. For smaller values of $|x|$, with ϵ of order one (corresponding to $Y_{\tau}\approx 1$), $|U_{e3}|$ can be as large as 0.1 which may be measured in the future. In this case, all three neutrinos are nearly degenerate and the cosmological mass limit on neutrinos will be nearly saturated. We conclude that the structure of the mixing matrix derived is not upset by radiative corrections.

Leptogenesis. Leptogenesis occurs in a simple way in this model via the decay of the right-handed neutrinos[14]. The heavy Majorana mass matrix of ν^{c} is given in the model as (see Eq. (8))

$$M_{\nu^{\mathrm{c}}}=M_{\mathrm{R}}\begin{bmatrix}1 & 0 & -x\\ 0 & 1 & 0\\ -x & 0 & 1\end{bmatrix}. \tag{18}$$

The Dirac neutrino Yukawa coupling matrix is proportional to an identity matrix at the scale of A_4 symmetry breaking which we take to be near the GUT scale. The ν^{c} fields will remain light below that scale, down to the scale M_{R}. Renormalization group effects in the momentum range $M_{\mathrm{R}}<\mu<M_{\mathrm{GUT}}$ will induce non-universal corrections to the Dirac neutrino Yukawa coupling matrix. Without such non-universality no lepton asymmetry will be induced in the decay of right-handed neutrinos. The effective theory in this momentum range is the MSSM with the ν^{c} fields.

From the renormalization group equation

$$\frac{\mathrm{d}Y_{\nu}}{\mathrm{d}t}=\frac{1}{16\pi^2}Y_{\nu}(Y_l^{\dagger}Y_l)+\cdots \tag{19}$$

where $W = eY_l LH_\mathrm{d} + \nu^c Y_\nu LH_\mathrm{u} + \cdots$, we obtain at the scale M_R, $Y_\nu = Y_\nu^0 \times \mathrm{diag}(1,1,1-\delta)$ with $\delta \simeq (Y_\tau^2/16\pi^2)\ln(M_\mathrm{GUT}/M_\mathrm{R})$. Y_ν^0 is the value of the universal Dirac Yukawa coupling at the GUT scale.

We diagonalize M_{ν^c} by the rotation $\nu^c = OQN$, where

$$O=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0 & 1\\ 0 & \sqrt{2} & 0\\ -1 & 0 & 1\end{pmatrix}, Q=\mathrm{diag}\{\mathrm{e}^{-\mathrm{i}\phi_1/2}, 1, \mathrm{e}^{-\mathrm{i}\phi_2/2}\} \tag{20}$$

so that the N fields are the mass eigenstates with real and positive mass eigenvalues: $M_\mathrm{N} = M_\mathrm{R}\times\mathrm{diag}(|1+x|,1,|1-x|)$.

In the basis where the heavy ν^c fields have been diagonalized, the Dirac neutrino Yukawa coupling matrix takes the form $\hat{Y}_\nu = QO^\mathrm{T}Y_\nu$, so that

$$\hat{Y}_\nu\hat{Y}_\nu^\dagger = \frac{|Y_\nu^0|^2}{2}\begin{bmatrix}1+(1-\delta)^2 & 0 & -\mathrm{e}^{\frac{\mathrm{i}(\phi_2-\phi_1)}{2}}\{(1-\delta)^2-1\}\\ 0 & 2 & 0\\ -\mathrm{e}^{\frac{\mathrm{i}(\phi_1-\phi_2)}{2}}\{(1-\delta)^2-1\} & 0 & 1+(1-\delta)^2\end{bmatrix}. \tag{21}$$

The CP asymmetry arising from the decay of the field N_i is given by

$$\epsilon_i = \frac{-1}{8\pi}\frac{1}{[\hat{Y}_\nu\hat{Y}_\nu^\dagger]_{ii}}\sum_j \mathrm{Im}\{[\hat{Y}_\nu\hat{Y}_\nu^\dagger]_{ij}^2\} f\left(\frac{M_j^2}{M_i^2}\right), \tag{22}$$

where

$$f(y) = \sqrt{y}\left(\frac{2}{y-1} + \log\frac{1+y}{y}\right) . \tag{23}$$

As $y \gg 1$, $f(y) \to 3/\sqrt{y}$.

In the normal hierarchy case, $m_1/m_3 = M_1/M_3$, so the lightest N field is N_1. In this case we have

$$\epsilon_1 = \frac{-3|Y_\nu^0|^2}{8\pi}\delta^2\left(\frac{m_1}{m_3}\right)\sin(\phi_2-\phi_1) \simeq \frac{\pm 3|Y_\nu^0|^2}{8\pi}\delta^2\left(\frac{m_1}{m_3}\right)\left[1-\frac{(2m_1/m_2-1)^2}{(m_1/m_3)^2}\right]^{1/2} . \tag{24}$$

To see the numerical value of ϵ_1, we note that $|Y_\nu^0|$ can be of order one, $\delta \simeq (0.1Y_\tau^2)$, and $m_1/m_3 \simeq [\Delta m^2_\mathrm{solar}/3\Delta m^2_\mathrm{atm}]^{1/2} \simeq 0.1$. For very large value of $\tan\beta$, $Y_\tau \simeq 1$, and we find $\epsilon_1 \simeq 10^{-4}$. Even for moderate values of $\tan\beta \sim 20$, we find that $\epsilon_1 \simeq 10^{-6}$ is possible. The negative sign will also ensure the correct sign of baryon asymmetry. The induced lepton asymmetry is converted to baryon asymmetry through electroweak sphaleron processes. The baryon asymmetry is given by $Y_\mathrm{B} \simeq -Y_\mathrm{L}/2$, where $Y_\mathrm{L} = \kappa\epsilon_1/g^*$, where $g^* \sim 200$ is the effective number of degrees of freedom in equilibrium during leptogenesis, and κ is the efficiency factor obtained by solving the Boltzman's equations. A simple approximate formula for κ is[15]

$$\kappa \simeq 10^{-2}\left[\frac{0.01}{\tilde{m}_1\mathrm{eV}}\right]^{1.1} , \tag{25}$$

where

$$\tilde{m}_1 = \frac{v_\mathrm{u}^2}{M_1}[\hat{Y}_\nu\hat{Y}_\nu^\dagger]_{11} . \tag{26}$$

For $\delta \sim 0.1$ and $M_1 \sim 10^{14}$GeV, we obtain $Y_\mathrm{B} \sim 7\times10^{-11}$, in good agreement with observations.

For the case of inverted mass hierarchy, N_3 is lighter than N_1, so we focus on ϵ_3. It is given by

$$\epsilon_3 \simeq \frac{\mp 3|Y_\nu^0|^2}{4\pi}\delta^2\left(\frac{m_3}{m_1}\right)\frac{|x|\sqrt{1-|x|^2/4}}{\sqrt{1+2|x|^2}} . \tag{27}$$

Again we see that reasonable lepton asymmetry is generated.

In summary, we have presented a class of renormalizable gauge models based on the non-Abelian discrete symmetry A_4 which realize the geometric neutrino mixing pattern of Eq. (4) naturally. The resulting constraints on the neutrino masses have been outlined. We have also highlighted an intriguing connection between high scale leptogenesis and low energy neutrino experiments.

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中微子几何混合模型与轻子和重子数产生

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摘要　在这一报告中将报告我和BABU教授合作的在hep-ph/0507217一文中有关中微子混合研究结果. 目前中微子实验数据所决定的混合角可归结为几何混合状况: $\sin^2\theta_{12}=1/3$, $\sin^2\theta_{23}=1/2$, 和$\sin^2\theta_{13}=0$. 我们在这一工作中建立了能实现这一几何混合的可重整化模型. 模型以非阿贝尔非连续群 A_4 为描述中微子不同代混合的对称性. 这类模型对中微子质量有很强的限制. 而且能很自然地由轻子数破坏产生重子不对称的实验观测值. 很有趣的是这类模型中出现在轻子不守恒和无中微子双beta衰变中的相位是一样的.

关键词　中微子　质量　混合　轻子　重子

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