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Normalization Constants of the Twist-3 Distribution Amplitudes of the Pion and Kaon from the QCD Sum Rules *

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Abstract In this paper we calculate the normalization constants $m_{0\pi}^{p}$ and m_{0K}^{p} of the twist-3 distribution amplitudes of the pion and kaon from the QCD sum rules, instead of using the equations of motion. We find that $m_{0\pi}^{p}=1.00\pm0.17$GeV and $m_{0K}^{p}=1.46\pm0.23$GeV after including α_S corrections to the perturbative part of the sum rules. They are close to the phenomenological values. For the pion case, this shows that the value obtained in QCD sum rules is only 50% of that determined by the equation of motion.

Key words twist-3 distribution amplitude, normalization constant, QCD sum rules

For the exclusive processes involving large momentum transfer, factorization pictures are applicable. By the factorization theorem, the light cone distribution amplitudes appearing in the convolution representations of physical quantities contain non-perturbative information and are universal to different processes. Although these light cone distribution amplitudes take the asymptotic form when the energy scale $Q^2\to\infty$, they deviate apparently from asymptotic forms in the intermediate scale (a few GeV^2).

Higher twist distribution amplitudes make less contribution by further $1/Q^2$ suppression. But for the access region of present experiments (Q^2 not too large), their contributions can't be omitted if one wants to compare the theoretical calculation with the experimental data. So it is important to learn more about the behaviors of higher twist distribution amplitudes so that we can better understand the exclusive processes.

Distribution amplitudes can be obtained from the hadronic wave functions by integrating out the transverse momenta of the quarks in the hadrons. In this paper, we study the normalization constants for two twist-3 distribution amplitudes of the pion and kaon. The pionic distribution amplitudes of the lowest Fork state can be defined as,

$$\langle 0|\bar{d}_\alpha(z)[z,-z]u_\beta(-z)|\pi(q)\rangle = \frac{i}{8}f_\pi\int_{-1}^{1}d\xi e^{i\xi(z\cdot q)}\Big\{\not{q}\gamma_5\phi_\pi(\xi)-m_{0\pi}^{p}\gamma_5\phi_p^\pi(\xi)-\frac{2}{3}m_{0\pi}^{\sigma}\sigma_{\mu\nu}\gamma_5 q^\mu z^\nu\phi_\sigma^\pi(\xi)\Big\}_{\beta\alpha}+\cdots. \quad (1)$$

In this equation, $\phi_\pi(\xi)$ is the twist-2 distribution amplitude, $\phi_p^\pi(\xi)$ and $\phi_\sigma^\pi(\xi)$ are the two twist-3 (non-leading) distribution amplitudes. We introduce two normalization parameters $m_{0\pi}^{p}$ and $m_{0\pi}^{\sigma}$ which can be calculated by QCD sum rules[1].

A similar definition for the K meson can be given by replacing d quark with s quark, $m_{0\pi}^{p,\sigma}$ with $m_{0K}^{p,\sigma}$ and f_π with f_K.

In this paper, we calculate the $m_{0\pi}^{p}$ and m_{0K}^{p} in QCD sum rules.

From Eq.(1) (and a similar equation for the K meson) and the normalization of the zeroth moments of ϕ_π^p and ϕ_K^p,

$$\frac{1}{2}\int_{-1}^{1}d\xi\phi_p^\pi(\xi)=1, \qquad \frac{1}{2}\int_{-1}^{1}d\zeta\phi_p^K(\zeta)=1. \quad (2)$$

we can find:

$$\langle 0|\bar{d}i\gamma_5 u(x)|\pi^+(q)\rangle = m_{0\pi}^{p}f_\pi e^{-iqx}, \quad (3)$$

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$$\langle 0|\bar{s}\mathrm{i}\gamma_5 u(x)|\mathrm{K}^+(q)\rangle = m_{0\mathrm{K}}^{\mathrm{p}} f_{\mathrm{K}} \mathrm{e}^{-\mathrm{i}qx}. \qquad (4)$$

As it is well known, for the twist-2 wave function of the π meson, the normalzation constant f_π (usually called the π decay constant) can be defined as,

$$\langle 0|\bar{d}\gamma_\mu\gamma_5 u(x)|\pi^+(q)\rangle = \mathrm{i}q_\mu f_\pi \mathrm{e}^{-\mathrm{i}qx}, \qquad (5)$$

and f_π is determined from the process $\pi\to\mu\nu$. However, one has no such normalization condition to determine the constants $m_{0\pi}^{\mathrm{p}}$ and $m_{0\mathrm{K}}^{\mathrm{p}}$ in Eqs. (3) and (4) from experiments. One way is to employ the equations of motion for pion by multiplying q^μ on both sides of Eq. (5). From this, one can get

$$m_{0\pi}^{\mathrm{p}} = \frac{m_\pi^2}{m_{\mathrm{u}}+m_{\mathrm{d}}}, \qquad m_{0\mathrm{K}}^{\mathrm{p}} = \frac{m_{\mathrm{K}}^2}{m_{\mathrm{u}}+m_{\mathrm{s}}}, \qquad (6)$$

and $m_{0\pi}^{\mathrm{p}}, m_{0\mathrm{K}}^{\mathrm{p}} \sim 2\mathrm{GeV}$ (for current mass $m_{\mathrm{u}}+m_{\mathrm{d}}\approx 10\mathrm{MeV}$, $m_{\mathrm{u}}+m_{\mathrm{s}}\approx 125\mathrm{MeV}$ at scale $\mu^2=4\mathrm{GeV}^2$ under minimal subtraction with $\Lambda_{\mathrm{QCD}}\approx 250\mathrm{MeV}$). If its running behavior is taken into account[2], the average value of $m_{0\pi}^{\mathrm{p}}, m_{0\mathrm{K}}^{\mathrm{p}}$ over the intermediate energy is up arround 2.4GeV (at scale $\mu^2\approx 20\mathrm{GeV}^2$). The large values of the constants $m_{0\pi}^{\mathrm{p}}$ and $m_{0\mathrm{K}}^{\mathrm{p}}$ make the twist-3 contribution to the exclusive processes overestimated. For example, the twist-3 contribution to the pion form factor is comparable with, even larger than, that of the leading twist in the wide intermediate energy region, e.g., $Q^2\sim(2—40)\mathrm{GeV}^2$ (for example, see Refs. [2,3]). It is hard to believe that these results are reliable since the power suppressed corrections make such a large contribution up to $40\mathrm{GeV}^2$. One way to soften this difficulty is to set $m_{0\pi}^{\mathrm{p}}$ and $m_{0\mathrm{K}}^{\mathrm{p}}$ as phenomenological parameters which can be deviated from those by equations of motion in exclusive processes. For example, a smaller phenomenological value $m_{0\pi}^{\mathrm{p}}\sim 1.4\mathrm{GeV}$ had been used in Ref. [4].

Our solution is to calculate these normalization constants from the QCD sum rules directly. We think the equations of motion are not appropriate for the quarks which can't be considered on-shell in the light mesons. So $m_{0\pi}^{\mathrm{p}}$ and $m_{0\mathrm{K}}^{\mathrm{p}}$ can't be estimated by Eq. (6) in this point of view.

In order to obtain the sum rules of $m_{0\pi}^{\mathrm{p}}$ and $m_{0\mathrm{K}}^{\mathrm{p}}$, two corresponding correlation functions should be introduced,

$$\begin{aligned} I_\pi(q^2) \equiv & -\mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}q\cdot x}\langle 0|T\{\bar{d}(x)\gamma_5 u(x), \\ & \bar{u}(0)\gamma_5 d(0)\}|0\rangle, \end{aligned} \qquad (7)$$

$$\begin{aligned} I_{\mathrm{K}}(q^2) \equiv & -\mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}q\cdot x}\langle 0|T\{\bar{s}(x)\gamma_5 u(x), \\ & \bar{u}(0)\gamma_5 s(0)\}|0\rangle. \end{aligned} \qquad (8)$$

In the deep Euclidean region, $-q^2\gg 0$, Eqs. (7) and (8) can be calculated perturbatively by background field method[5—9]. To leading order in α_{s} and up to dimension six condensate the results are (the α_{s} corrections to the perturbative part are taken from Ref. [6]),

$$\begin{aligned} I_\pi(q^2)_{\mathrm{QCD}} = & -\frac{3}{8\pi^2}\left(1+\frac{11}{3}\frac{\alpha_{\mathrm{s}}}{\pi}\right)q^2\ln\frac{-q^2}{\mu^2} + \\ & \frac{1}{2}\frac{(m_{\mathrm{u}}+m_{\mathrm{d}})\langle\bar{\mathrm{u}}\mathrm{u}\rangle}{q^2} - \frac{1}{8}\frac{\langle(\alpha_{\mathrm{s}}/\pi)G^2\rangle}{q^2} + \\ & \frac{112\pi}{27}\frac{\langle\sqrt{\alpha_{\mathrm{s}}}\bar{\mathrm{u}}\mathrm{u}\rangle^2}{q^4}, \end{aligned} \qquad (9)$$

$$\begin{aligned} I_{\mathrm{K}}(q^2)_{\mathrm{QCD}} = & -\frac{3}{8\pi^2}\left(1+\frac{11}{3}\frac{\alpha_{\mathrm{s}}}{\pi}\right)q^2\ln\frac{-q^2}{\mu^2} - \\ & \frac{[m_{\mathrm{s}}-2m_{\mathrm{u}}]\langle\bar{\mathrm{s}}\mathrm{s}\rangle + [m_{\mathrm{u}}-2m_{\mathrm{s}}]\langle\bar{\mathrm{u}}\mathrm{u}\rangle}{2q^2} - \\ & \frac{1}{8}\frac{\langle(\alpha_{\mathrm{s}}/\pi)G^2\rangle}{q^2} - \frac{16\pi}{27}\frac{\alpha_{\mathrm{s}}[\langle\bar{\mathrm{s}}\mathrm{s}\rangle^2+\langle\bar{\mathrm{u}}\mathrm{u}\rangle^2]}{q^4} + \\ & \frac{16\pi}{3}\frac{\alpha_{\mathrm{s}}\langle\bar{\mathrm{s}}\mathrm{s}\rangle\langle\bar{\mathrm{u}}\mathrm{u}\rangle}{q^4}. \end{aligned} \qquad (10)$$

In the physical region, $q^2>0$, the hadronic spectrum representations for Eqs. (7) and (8) can be represented respectively as (the continuous states contribution are taken to be a theta function with the coefficients from the perturbative parts of Eqs. (9) and (10) by using hadron-quark duality),

$$\begin{aligned} \frac{1}{\pi}\mathrm{Im}I_\pi(q^2)_{\mathrm{had}} = & \delta(q^2-m_\pi^2)f_\pi^2(m_{0\pi}^{p})^2 + \\ & \frac{3}{8\pi^2}(1+11\alpha_{\mathrm{s}}/3\pi)q^2\theta(q^2-s_\pi), \end{aligned} \qquad (11)$$

$$\begin{aligned} \frac{1}{\pi}\mathrm{Im}I_{\mathrm{K}}(q^2)_{\mathrm{had}} = & \delta(q^2-m_{\mathrm{K}}^2)f_{\mathrm{K}}^2(m_{0\mathrm{K}}^{p})^2 + \\ & \frac{3}{8\pi^2}(1+11\alpha_{\mathrm{s}}/3\pi)q^2\theta(q^2-s_{\mathrm{K}}). \end{aligned} \qquad (12)$$

These expressions in two $Q^2=-q^2$ regions can be related by the dispersion relation,

$$\frac{1}{\pi}\int \mathrm{d}s\,\frac{\mathrm{Im}I(s)_{\mathrm{had}}}{s+Q^2} = I(-Q^2)_{\mathrm{qcd}}. \qquad (13)$$

In order to improve the convergence of this dispersion relation, we employ the Borel transformation,

$$\frac{1}{M^2}\int \mathrm{d}s\,\mathrm{e}^{-s/M^2}\frac{1}{\pi}\mathrm{Im}I(s)_{\mathrm{had}} = \hat{L}_{\mathrm{M}}I(-Q^2)_{\mathrm{QCD}}, \qquad (14)$$

where M is Borel parameter. After substituting Eqs. (9), (11) and (10), (12) into Eq. (14) correspondingly, the sum rules for $m_{0\pi}^{\mathrm{p}}$ and $m_{0\mathrm{K}}^{\mathrm{p}}$ can be written as:

$$\begin{aligned} (m_{0\pi}^{\mathrm{p}})^2 = & \frac{\mathrm{e}^{m_\pi^2/M^2}M^4}{f_\pi^2}\left\{\frac{3}{8\pi^2}\left(1+\frac{11}{3}\frac{\alpha_{\mathrm{s}}}{\pi}\right)\left[1-\left(1+\frac{s_\pi}{M^2}\right)\mathrm{e}^{-s_\pi/M^2}\right] + \right. \\ & \left.\frac{1}{8}\frac{\langle(\alpha_{\mathrm{s}}/\pi)G^2\rangle}{M^4} - \frac{1}{2}\frac{(m_{\mathrm{u}}+m_{\mathrm{d}})\langle\bar{\mathrm{u}}\mathrm{u}\rangle}{M^4} + \frac{112\pi}{27}\frac{\langle\sqrt{\alpha_{\mathrm{s}}}\bar{\mathrm{u}}\mathrm{u}\rangle^2}{M^6}\right\}, \end{aligned} \qquad (15)$$

$$(m_{0\mathrm{K}}^{\mathrm{p}})^2 = \frac{\mathrm{e}^{m_{\mathrm{K}}^2/M^2}M^4}{f_{\mathrm{K}}^2}\left\{\frac{3}{8\pi^2}\left(1+\frac{11}{3}\frac{\alpha_{\mathrm{s}}}{\pi}\right)\left[1-\left(1+\frac{s_{\mathrm{K}}}{M^2}\right)\mathrm{e}^{-s_{\mathrm{K}}/M^2}\right] + \right.$$

$$\frac{1}{8}\frac{\langle(\alpha_s/\pi)G^2\rangle}{M^4}+\frac{[m_s-2m_u]\langle\bar{s}s\rangle+[m_u-2m_s]\langle\bar{u}u\rangle}{2M^4}-\frac{16\pi}{27}\frac{\alpha_s[\langle\bar{s}s\rangle^2+\langle\bar{u}u\rangle^2]}{M^6}+\frac{16\pi}{3}\frac{\alpha_s\langle\bar{s}s\rangle\langle\bar{u}u\rangle}{M^6}\Big\}, \qquad (16)$$

where s_π and s_K are the threshold values to be chosen properly.

To analyse Eqs.(15) and (16) numerically, we take the input parameters as usual: $f_\pi = 0.133\text{GeV}$, $f_K = 0.131\text{GeV}$, $m_s = 0.130\text{GeV}$, $m_u = 0.004\text{GeV}$, $m_d = 0.007\text{GeV}$, $\langle\bar{u}u\rangle = \langle\bar{d}d\rangle = -(0.24\text{GeV})^3$, $\langle\bar{s}s\rangle = 0.8\langle\bar{u}u\rangle$, $\langle\frac{\alpha_s}{\pi}GG\rangle = 0.012\text{GeV}^4$, $\alpha_s(1\text{GeV}) = 0.5$. The normalization scale $\mu = M$ is assumed in the analysis.

As to the threshold values $s_{\pi,K}$ in the sum rule, they can be taken to the mass square of the first excited states in corresponding channel. Although the windows become broader when the $s_{\pi,K}$ are larger, the threshold values can't exceed the first excited states. In order to get the maximal stability of the sum rules, we can take them to be the mass square of the first excited states, i.e.,

$$s_\pi = (1.3\text{GeV})^2, \qquad s_K = (1.46\text{GeV})^2$$

where the first excited state is $\pi'(1300)$ for the pion case, and that for the kaon meson case is K(1460) taken from Ref.[10].

For $m_{0\pi}^p$, we need a parameter (M^2) window to determine its value from Eq.(15). It can be obtained by requiring the contribution from the continuous states and the highest dimensional condensate to the total operator product expansion series not to exceed 30% respectively. This requirement leads to a window for $m_{0\pi}^p$, $M^2 \in (0.35, 0.72)\text{GeV}^2$, and the corresponding value is $m_{0\pi}^p = 1.00 \pm 0.17\text{GeV}$. The contribution of the continuous states and highest condensate to the operator expansion series are plotted in Fig.1 (a), and the values of $m_{0\pi}^p$ in this window are shown in Fig.1 (b).

The same procedure can be imposed to get the value of m_{0K}^p, one finds the window is $M^2 \in (0.30, 0.90)\text{GeV}^2$ (see Fig.2 (a)) and the corresponding value is $m_{0K}^p = 1.46 \pm 0.23\text{GeV}$ (see Fig.2 (b)).

In summary, we don not impose the equations of motion. Instead, we calculate the normalization constants $m_{0\pi}^p$ and m_{0K}^p of the twist-3 distribution amplitudes of the pion and kaon from the QCD sum rules. We find that $m_{0\pi}^p$ and m_{0K}^p are close to the phenomenological values. They will decrease the twist-3 contributions to the exclusive processes, such as the hadronic form factors and the B decays, in the intermediate energy region.

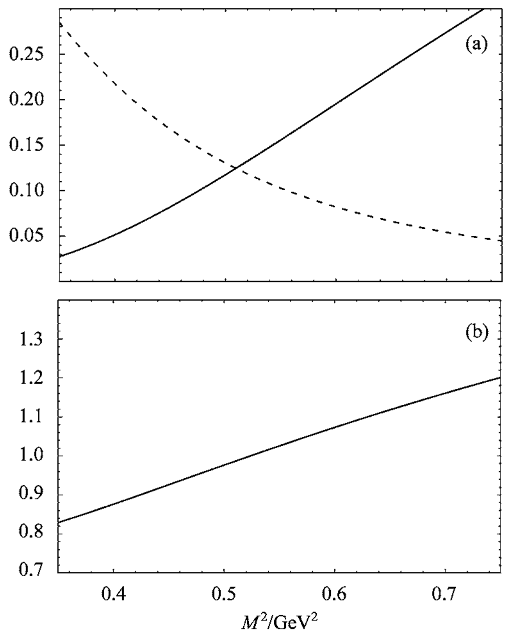

Fig.1. (a) The window for the normalization constant $m_{0\pi}^p$, the dashed and the solid lines indicate the ratio of the contribution from the dimension six condensates and the continuous states in the total sum rule respectively; (b) The corresponding values of $m_{0\pi}^p$ within the window.

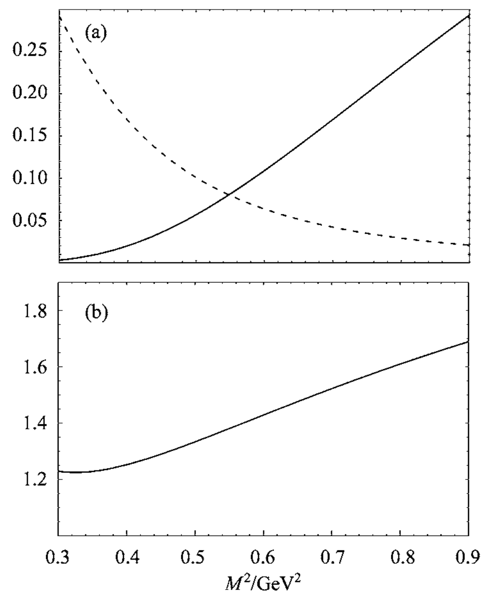

Fig.2. (a) The window for the normalization constant m_{0K}^p, the dashed and the solid lines indicate the ratio of the contribution from the dimension six condensates and the continuous states in the total sum rule respectively; (b) The corresponding values of m_{0K}^p within the window.

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由QCD求和规则计算π介子和K介子twist-3分布振幅的归一化常数*

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摘要　用QCD求和规则计算了π介子和K介子的两个twist-3分布振幅的归一化常数 $m_{0\pi}^{p}$和 m_{0K}^{p}. 与运动方程的要求不同, 计算结果表明(把求和规则微扰部分的 α_s 修正考虑之后), $m_{0\pi}^{p}=1.00\pm0.17$GeV, $m_{0K}^{p}=1.46\pm0.23$GeV. 应该指出的是, 它们与运动方程给出的结果相比要小不少. 比如π介子的情形, QCD求和规则给出的上述结果约是运动方程要求的值的50%左右. 在exclusive的一些过程中, 人们发现, 一直到 Q^2 较大(2—40GeV^2)的区域, 本应受到抑止的非首要的(比如, non-leading twist的贡献)贡献还可以跟首要的贡献(比如, leading twist的贡献)相比, 甚至可以超过. 这是难以相信的. 而较小的归一化常数将有助于弱化这个矛盾. 计算结果支持这一点.

关键词　twist-3分布振幅　归一化常数　QCD求和规则

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