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Quark-Gluon Mixed Vacuum Condensates in Dyson-Schwinger Equations *

ZHOU Li-Juan[1;1)] MA Wei-Xing[2;2)]

1 (Department of Information and Computing Sciences, Guangxi University of Technology, Liuzhou 545006, China)

2 (Institute of High Energy Physics, CAS, Beijing 100049, China)

Abstract Based on Dyson-Schwinger Equations of quark propagator, we calculate quark-gluon mixed vacuum condensates $\langle 0|:\bar{q}g_s\sigma_{\mu\nu}G_{\mu\nu}q:|0\rangle$ and quark vacuum condensates $\langle 0|:\bar{q}q:|0\rangle$ which are not only related to virtuality of quark in vacuum state but also characterize the space width of quark distribution in the vacuum. The existence of these vacuum condensates reflects in a direct way the non-perturbative structure of QCD vacuum. Our calculated results on the mixed condensates lead to quark virtualities of $\lambda^2_{u,d} = 0.7\text{GeV}^2$ for u, d quarks, and $\lambda^2_s = 1.6\text{GeV}^2$ for s quark which are consistent with other's calculations using completely different methods.

Key words quark virtuality, vacuum condensate, DSEs, non-perturbative QCD

In order to understand the non-perturbative structure of the QCD vacuum, it is important to study various vacuum condensates such as $\langle 0|:\bar{q}q:|0\rangle$, $\langle 0|:G_{\mu\nu}G^{\mu\nu}:|0\rangle$ and $\langle 0|:\bar{q}g_s\sigma_{\mu\nu}G_{\mu\nu}q:|0\rangle$. Among the various condensates, we emphasize the importance of the quark-gluon mixed condensate $\langle 0|:\bar{q}g_s\sigma_{\mu\nu}G_{\mu\nu}q:|0\rangle \equiv \langle 0|:\bar{q}g_s\sigma_{\mu\nu}G^a_{\mu\nu}\frac{\lambda^a}{2}q:|0\rangle$. First, the mixed condensate represents a direct correlation between quarks and gluons in the QCD vacuum state. In this point, the mixed condensate differs from $\langle 0|:\bar{q}q:|0\rangle$ and $\langle 0|:G_{\mu\nu}G^{\mu\nu}:|0\rangle$ even at the qualitative level. Second, this mixed condensate is another chiral order parameter of the second lowest dimension, because the chirality of the quark flips as $\langle 0|:\bar{q}g_s\sigma_{\mu\nu}G_{\mu\nu}q:|0\rangle = \langle 0|:\bar{q}_R g_s(\sigma_{\mu\nu}G_{\mu\nu})q_L:|0\rangle + \langle 0|:\bar{q}_L g_s(\sigma_{\mu\nu}G_{\mu\nu})q_R:|0\rangle$. Third, the mixed condensate plays an important role in various QCD sum rules, especially in the baryons[1,2], the light heavy mesons[3] and the exotic mesons[4]. In particular, the ratio of $\langle 0|:\bar{q}ig_s\sigma_{\mu\nu}G^a_{\mu\nu}\frac{\lambda^a}{2}q:|0\rangle/\langle 0|:\bar{q}q:|0\rangle$ is closely related to the virtuality of quark in vacuum state which reflects in a direct way the non-perturbative structure of QCD vacuum. Fourth, the mixed condensate is one least known quantity in QCD sum rules but it gives large contributions to QCD sum rule calculations for mesons composed of one light and one heavy quark, and also it is particularly important for the calculation of the πN sigma term. Therefore, a calculation of $\langle 0|:\bar{q}g_s\sigma_{\mu\nu}G_{\mu\nu}:|0\rangle$ on the basis of DSEs is urgently demanded.

We study the quark-gluon mixed vacuum condensates from quark propagator, $S_f(p)$, which can be expressed in Euclidean space[5] as

$$S_f^{-1}(p) = i\not{p}\cdot A_f(p^2) + B_f(p^2) \qquad (1)$$

in a covariant gauge. The propagator is renormalized at space-like point $\bar{\mu}^2$ according to $A_f(\bar{\mu}^2) = 1$ and $B_f(\bar{\mu}^2) = m_f(\bar{\mu}^2)$ with $m_f(\bar{\mu}^2)$ being the current quark mass, which produces a free quark propagator.

Except for the current quark mass and perturbative corrections, the functions $A_f(p^2)$ and $B_f(p^2)$ are non-perturbative quantities, and must satisfy Dyson-Schwinger Equations (DSEs) in the Feynman gauge[6]

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1) E-mail: zljcl2001@tom.com

2) E-mail: mawx@mail.ihep.ac.cn

$$[A_f(p^2) - 1]p^2 =$$
$$\frac{8}{3}g_s^2\int\frac{d^4q}{(4\pi)^4}D(p-q)\frac{A_f(q^2)}{q^2A_f^2(q^2)+B_f^2(q^2)}p\cdot q, \quad (2)$$
$$B_f(p^2) =$$
$$\frac{16}{3}g_s^2\int\frac{d^4q}{(4\pi)^4}D(p-q)\frac{B_f(q^2)}{q^2A_f^2(q^2)+B_f^2(q^2)}. \quad (3)$$

Given a gluon propagator $D(p-q)$, the coupled integral equations of Eqs. (2,3) can be easily solved.

Since the fully dressed gluon propagator $D_{\mu\nu}^{ab}(q)$ is completely unknown at present, we use a phenomenological gluon propagator. The Feynman-like gauge used in derivation of Eqs. (2,3) leads us to the choice of the following empirical form of the $D_{\mu\nu}^{ab}(q)$[5—7]

$$D_{\mu\nu}^{ab}(q) = \delta^{ab}\delta_{\mu\nu}D(q) \quad (4)$$

for the model gluon propagator. The function $D(q)$ is defined usually by the following identity

$$g_s^2D(q^2) = \frac{4\pi\alpha_1(q^2)}{q^2} \quad (5)$$

where $\alpha_1(q^2)$ is formulated phenomenally in terms of parameters χ, Δ, Λ and ϵ as the following

$$\alpha_1(s) = 3\pi s\frac{\chi^2}{4\Delta^2}e^{-s/\Delta} + \frac{\pi d}{\ln(s/\Lambda^2+\epsilon)} \quad (6)$$

which determines the quark-quark interaction through a strength χ and a range parameter Δ. The first term of Eq. (6) simulates the infrared enhancement and confinement, and the second term matches the results of the leading log renormalization group theory predictions[8]. The parameter ϵ can be varied in the range of 1.0 to 2.50. We take $\epsilon = 2.0$ from Ref. [9] in this calculations. In Eqs. (2) and (3), the integration over q is cut off at $q^2 = 4\text{GeV}^2$.

Given A_f and B_f, the vacuum condensates that we want to calculate can be easily obtained because the self-energy functions A_f and B_f are closely related to QCD local vacuum condensates which are vacuum matrix elements of various singlet combinations of quark and gluon fields, i.e

$$\langle 0 \mid :\bar{q}(x)q(0): \mid 0\rangle, \quad \langle 0\mid :G_{\mu\nu}^aG_{\mu\nu}^a: \mid 0\rangle,$$
$$\langle 0\mid :\bar{q}ig_s[\sigma_{\mu\nu}G_{\mu\nu}^a\frac{\lambda^a}{2}]q: \mid 0\rangle, \cdots. \quad (7)$$

The $q(x)$ in Eq. (7) is the quark field, $G_{\mu\nu}^a$ represents the gluon field strength tensor with a being color index ($a = 1, \cdots, 8$).

The nonzero local quark vacuum condensate $\langle 0\mid :\bar{q}(0)q(0): \mid 0\rangle$ is responsible surely for the spontaneous breakdown of chiral symmetry. The nonzero local gluon condensate $\langle 0\mid :G_{\mu\nu}^aG_{\mu\nu}^a: \mid 0\rangle$ defines the mass scale of hadrons through trace anomaly[10].

The non-local vacuum condensates $\langle 0\mid :\bar{q}(x)q(0): \mid 0\rangle$ and $\langle 0\mid :G_{\mu\nu}^a(x)G_{\mu\nu}^a(0): \mid 0\rangle$ describe the distributions of quarks and gluons in the non-perturbative QCD vacuum states. Physically, this means that vacuum quarks and gluons have a nonzero mean-square momentum called virtuality. Indeed, the average virtualities of quarks and gluons are connected with the vacuum expectation values[11]. For instance, the quark average virtuality, λ_q, is related, by the equation of motion in the chiral limit, to the mixed quark-gluon local vacuum condensate[12] and the local quark vacuum condensate as shown in the following equation

$$2\lambda_q^2 = \frac{\langle 0\mid :\bar{q}ig_s\sigma_{\mu\nu}G_{\mu\nu}^a\frac{\lambda^a}{2}]q: \mid 0\rangle}{\langle 0\mid :\bar{q}q: \mid 0\rangle}. \quad (8)$$

Therefore, to obtain quark virtuality, λ_q^2, we have to calculate the quark-gluon mixed local vacuum condensate $\langle 0\mid :\bar{q}ig_s\sigma_{\mu\nu}G_{\mu\nu}^a\frac{\lambda^a}{2}q: \mid 0\rangle$ and the quark local vacuum condensate $\langle 0\mid :\bar{q}q: \mid 0\rangle$.

To this end, we start again, in the way of the operator product expansion, from quark propagator $S_f(x)$ which is determined by various QCD vacuum condensates. We set up first the relationship between the vacuum condensates and the solutions of DSEs, A_f and B_f, and then we use A_f and B_f to calculate the quark and quark-gluon mixed local vacuum condensates of QCD vacuum.

In the quantum field theory, the quark propagator, $S_f(x)$, is given by 2-point Green function $G_2(x_1, x_2) = \langle 0\mid T[q(x_2)\bar{q}(x_1)]\mid 0\rangle$ because it represents a propagation of the quark from x_1 to x_2 (or vice versa). Accordingly, the quark propagator in configuration space is defined by the 2-point Green function

$$S_f(x) = \langle 0 \mid T[q(x)\bar{q}(0)] \mid 0\rangle, \quad (9)$$

where T stands for the time-ordering operator. $T[q(x)\bar{q}(0)]$ can be easily calculated by Wick theorem.

The quark propagator $S_f(x)$ can be divided into two parts: a perturbative part denoted by S_f^{PT} and a non-perturbative part represented by S_f^{NPT}. Namely,

$$S_f(x) = S_f^{PT}(x) + S_f^{NPT}(x). \quad (10)$$

The perturbative part, $S_f^{PT}(x)$, is given in the configuration space by the Fourier transformation of the free quark propagator of $S_f^0(p) = i/(\not{p} - m_f)$ in momentum space with $\not{p} = \gamma^\mu p_\mu$,

$$S_f^{PT}(x) = \frac{i}{2\pi^2x^4}\gamma\cdot x\delta^{ab} - \frac{m_f}{2^2\pi^2x^2}\delta^{ab} + \cdots, \quad (11)$$

where γ is Dirac matrix. The non-perturbative part of Eq. (10), $S_f^{NPT}(x)$, can be written as[5—7,13]

$$S_f^{NPT}(x) = -\frac{1}{12}\{\langle 0 |: \bar{q}(x)q(0) :| 0\rangle + x_\mu \langle 0 |: \bar{q}(x)\gamma^\mu q(0) :| 0\rangle\}. \qquad (12)$$

Evidently, if one wants to study the full non-perturbative part of the quark propagator one must investigate both the scalar part $\langle 0|:\bar{q}(x)q(0):|0\rangle$ and the vector part $\langle 0|:\bar{q}(x)\gamma^\mu q(0):|0\rangle$ of Eq.(12). However, for our present purpose, only considering the scalar part $\langle 0|:\bar{q}(x)q(0):|0\rangle$ of Eq.(12) is sufficiently enough. The scalar part is given[14] by

$$\langle 0 |: \bar{q}(x)q(0) :| 0\rangle = \langle 0 |: \bar{q}(0)q(0) :| 0\rangle - \frac{x^2}{4}\langle 0 |: \bar{q}(0)\sigma\cdot G(0)q(0) :| 0\rangle + \cdots, \qquad (13)$$

which can be found in many references[5—14].

The local ($x=0$) quark and quark-gluon mixed vacuum condensates in Eq.(13) are related to the solutions of DSEs, $A_f(s)$ and $B_f(s)$, via the following identities[5,14], respectively

$$\langle 0 |: \bar{q}(0)q(0) :| 0\rangle = -\frac{3}{4\pi^2}\int_0^\infty ds\cdot s\frac{B_f(s)}{sA_f^2(s)+B_f^2(s)}, \qquad (14)$$

and

$$\langle 0 |: \bar{q}(0)g\sigma G(0)q(0) :| 0\rangle = \frac{9}{4\pi^2}\int ds\cdot s\left[s\frac{B_f(s)[2-A_f(s)]}{sA_f^2(s)+B_f^2(s)} + \frac{81\cdot B_f(s)\{2sA_f(s)[A_f(s)-1]+B_f^2(s)\}}{16[sA_f^2(s)+B_f^2(s)]}\right]. \qquad (15)$$

Eq.(15) has a global sign (−) difference with Ref.[15] but is consistent with many other's calculations such as Ref.[16]. It should be noticed that Eq.(14) is obtained in the "rainbow" approximation, $\Gamma^\nu = \gamma^\nu$, of fully dressed quark propagator. However, the expression for the quark-gluon mixed vacuum condensate, Eq.(15), is derived in a $1/N_C$ expansion of the gluon two-point function[14].

Using Eqs.(14,15), we make our theoretical predictions for the local quark vacuum condensate at the QCD scale parameter of $\Lambda = 0.2$GeV, the range parameter $\Delta = 0.4\text{GeV}^2$, the strength parameter $\chi = 1.84$GeV, and the integral cut-off parameter $\mu^2 = 1\text{GeV}^2$[15,16]. For three flavors $N_f=3$, $d=12/(33-2N_f)=12/27$. These parameters are used by most authors like in Refs.[7—16]. The results are shown as follows

$$\langle 0|:\bar{q}(0)q(0):|0\rangle_{u,d} = -(218\text{MeV})^3, \qquad (16)$$

for u and d quarks. And for s quark

$$\langle 0|:\bar{q}(0)q(0):|0\rangle_s = -(360\text{MeV})^3. \qquad (17)$$

The predicted results obtained here for local quark-gluon mixed vacuum condensates with the same parameters are also given in the following

$$\langle 0|:\bar{q}(0)[g\sigma G(0)]q(0):|0\rangle_{u,d} = -(1.016\text{GeV})^5, \qquad (18)$$

$$\langle 0|:\bar{q}(0)[g\sigma G(0)]q(0):|0\rangle_s = -(0.684\text{GeV})^5. \qquad (19)$$

Accordingly, substituting these predictions of our local quark and quark-gluon mixed vacuum condensates into Eq.(8) gives rise to the nonzero mean-square momentum of quarks in vacuum state as the following

$$\lambda_{u,d}^2 = \frac{1}{2}\frac{\langle 0|:\bar{q}(0)[ig\sigma_{\mu\nu}G_{\mu\nu}^a\frac{\lambda^a}{2}]q(0):|0\rangle}{\langle 0|:\bar{q}(0)q(0):|0\rangle} = 0.70\text{GeV}^2, \qquad (20)$$

for u, d quarks, which is in the acceptable range[14] of $\lambda_{u,d}^2$ between 0.4—1.0GeV^2. The standard QCD sum rule estimation[17] gives $\lambda_{u,d}^2 = 0.4\pm0.1\text{GeV}^2$, the QCD sum rule analysis of pion form factor[18] produces $\lambda_{u,d}^2 = 0.7\text{GeV}^2$ and lattice QCD calculations[19] $\lambda_{u,d}^2 = 0.55\text{GeV}^2$. And for s quark,

$$\lambda_s^2 = \frac{1}{2}\frac{\langle 0|:\bar{q}(0)[ig_s\sigma_{\mu\nu}G_{\mu\nu}^a\frac{\lambda^a}{2}]q(0):|0\rangle}{\langle 0|:\bar{q}(0)q(0):|0\rangle} = 1.60\text{GeV}^2. \qquad (21)$$

The predicted value of λ_s^2 is somewhat larger than that for u, d quarks but consistent with the predictions of lattice QCD[19], $\lambda_s^2 = 2.5\text{GeV}^2$ and instanton model prediction[20], $\lambda_s^2 = 1.4\text{GeV}^2$. The reason for large λ_s^2 is due to the large current mass of strange quark. This is an evident indication of chiral symmetry breakdown and $SU(3)$ flavor symmetry breaking.

In summary, based on Dyson-Schwinger Equations in rainbow approximation, we study the local quark-gluon mixed vacuum condensate and the vacuum condensate of quark with a phenomenological gluon propagator which simulates the infrared enhancement, confinement, and matches the results predicted by the leading log renormalization group theory. The local quark-gluon mixed vacuum condensate, the local vacuum condensate of light quarks and then the quark virtuality are numerically predicted in this work. Our theoretical results for the various condensates are consistent with those empirical values used commonly in QCD sum rules, and also in agreement with lattice QCD calculations. The mixed quark-gluon local vacuum condensates, which give large contributions to QCD sum rules for mesons composed of one light and one heavy quark, represent a direct correlation between quarks and gluons in the QCD vacuum states, and are particularly important for the calculation of the πN sigma term. Not only do they characterize the space width of quark distribution in QCD vacuum , by the equation of motion in the chiral limit, they are also closely related to the quark virtuality in QCD vacuum state.

The QCD vacuum state is densely populated by long-wave fluctuations of quark and gluon fields. The order parameters of this complicated vacuum state are characterized by a variety of local vacuum condensates which are vacuum matrix elements of various singlet combinations of quark and gluon fields. The existence of QCD vacuum condensates reflects in a direct way the non-perturbative structure of the QCD vacuum which needs to be studied.

References

1 Ioffe B L. Nucl. Phys., 1981, **B188**:317; Erratum-ibid. 1981, **B191**:591
2 Dosch H G. Jamin M, Narison S. Phys. Lett., 1989, **B220**:251
3 Dosch H G, Narison S. Phys. Lett., 1998, **B417**:173
4 Latorre J I, Pascual P, Narison S. Z. Phys., 1987, **C34**:347
5 ZHOU Li-Juan, MA Wei-Xing. Chin. Phys. Lett., 2003, **20**:2137; Kisslinger L S, Meissner T. Phys. Rev., 1998, **C57**:1528
6 Frank M R, Meissner T. Phys. Rev., 1996, **C53**:2410
7 Kisslinger L S. Phys. Rev., 1999, **D60**:065204; Frank M R, Tandy P C, Fai G. Phys. Rev., 1991, **C43**:2808
8 Cahill R C, Roberts C D. Phys. Rev., 1985, **D32**:2419; Cahill R C. Nucl. Phys., 1992, **A543**:63c; Tandy P C. Prog. Part. Nucl. Phys., 1997, **39**:117
9 Praschifka J, Cahill R T, Roberts C D. Intern. J. Mod. Phys., 1989, **A4**:4929
10 Ji X D. Phys. Rev. Lett., 1995, **74**:1071; Thomas A W, Weise W. The Structure of the Nucleon. Singapore: John Wiley and Sons, 2000, 170—172
11 Ovchinnikov A A, Pivovarov A A. Yad. Fiz., 1998, **48**:1135; Bakulev A P, Radyushkin A V. Phys. Lett., 1991, **B27**:223; Kremer M, Schierholz G. Phys. Lett., 1987, **B194**:283
12 Shifman M A, Vainshtein A Z, Zakharov V Z. Nucl. Phys., 1979, **B147**:385
13 Wilson K. On Products of Quantum Field Operators at Short Distances. USA Cornell University Press, 1964
14 Novikov V A, Shifman M A, Vainshtein V I et al. Nucl. Phys., 1984, **B237**:525
15 Meissner T. Phys. Lett., 1997, **B405**:8—13
16 Kisslinger L S, Linsuain O. Light-Cone Representation of the Quark Schwinger-Dyson Equation, hep-hp/0110111 V1; Kisslinger L S, Harly M A, Linsuan O. Quark Propagator, Instatons and Gluon Propagator, hep-hp/9906457
17 Belyaev V M, Ioffe B L. Sov. Phys. JEPT, 1982, **56**:493
18 Roberts C D, Williams G A. Prog. Part. Nucl. Phys., 1994, **33**:477
19 Takumi Doi, Noriyoshi Ishii, Makoto Oka et al. Nucl. Phys., 2003, **A721**:934c
20 Polykov M V, Weiss C. Phys. Lett., 1996, **B387**:841

在 Dyson-Schwinger 方程中夸克-胶子的混合真空凝聚*

周丽娟[1;1)] 马维兴[2;2)]

1(广西工学院信息与计算科学系 柳州 545006)

2(中国科学院高能物理研究所 北京 100049)

摘要 基于夸克传播子的 Dyson-Schwinger 方程,计算了夸克胶子混合真空凝聚和夸克真空凝聚.这些凝聚不仅联系着夸克在真空态中的虚度,而且也表征了真空中夸克分布的空间宽度.真空凝聚的存在直接反映着QCD 真空的非微扰结构.计算表明:上夸克与下夸克的虚度为 $\lambda^2_{u,d}=0.7\mathrm{GeV}^2$,奇异夸克的虚度为 $\lambda^2_s=1.6\mathrm{GeV}^2$.这些结果与许多用完全不同的方法得到的结果一致.

关键词 夸克的虚度 真空凝聚 DSEs 非微扰 QCD

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1) E-mail:zljcl2001@tom.com

2) E-mail:mawx@mail.ihep.ac.cn