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Relativistic Effects in Two-Photon Decay of Quarkonium *

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Abstract Relativistic effects in two-photon decay of quarkonium are investigated with a relativistic phenomenological approach. Compared with the NR approximation usually used, the relativistic phenomenological approach gives corrections coming from three sources: $q\bar{q}$ relative momentum distribution, $q\bar{q}$ relative energy distribution and description of quark spinors in the meson. These relativistic effects are studied in detail for $c\bar{c}$ and $s\bar{s}$ systems.

Key words two-photon decay, quarkonium, relativistic effects

1 Introduction

The study of two-photon decay of heavy quarkonium has a long history, from the original non-relativistic (NR) approximation in 1975[1,2] to the QCD Next-Leading-Order (NLO) correction in 1980[3,4], systematic construction of NRQCD in 1994[5] and correction of order v^4[6] in 2002, together with many other discussions of relativistic corrections[7—16]. Because this process is the most simple and pure process related to QCD bound state, it is an ideal place to test various descriptions of QCD bound state from theoretical point of view.

Up to now, there are two main methods to deal with the two-photon decay of the heavy quarkonium: NRQCD[5] and Mandelstam formalism with $q\bar{q}$ wave functions from Salpeter equation[12,17—19].

The former factorizes the decay width into two factors: nonperturbative matrix elements and corresponding perturbatively calculated coefficients. Using this effective field theory, one can systemically expand the width in α_S and v order by order. But the problem is: as the order increases, the number of needed nonperturbative parameters also increases greatly.

The latter, by using the instantaneous approximation with a phenomenological 3-D kernel, can give satisfactory meson spectrum and decay width at the same time and is more commonly used. In practical calculations, two approximations are usually assumed. One is assuming equal energy for quark and anti-quark in the meson and another is assuming a free spinor for a bound quark. Then the relativistic effect is mainly coming from the relative momentum distribution. In this paper, with a relativistic phenomenological approach, we study in detail possible relativistic corrections from all three sources: $q\bar{q}$ relative momentum distribution, $q\bar{q}$ relative energy distribution and description of quark spinors in the meson.

The organization of this paper is as follows: In Sec. 2 we summarize the basic formalism for calculating the two-photon decay width of quarkonium and discuss the possible sources of relativistic corrections. Then we investigate these effects for $\eta_c \to \gamma\gamma$ and $\chi_{c0} \to \gamma\gamma$ in Sec. 3 and for light quarkonium in Sec. 4. In Sec. 5, we give our conclusion.

2 Basic formalism

For two-photon decay of a $q\bar{q}$ meson of mass M in its rest frame, the general formulae for calculating the decay width in the Mandelstam formalism is[15,16]

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$$d\Gamma = \frac{1}{2}\frac{d\Omega_{cm}}{64\pi^2 M}\sum_{\lambda_1\lambda_2}|M_{fi}|^2 \tag{1}$$

with

$$M_{fi} = \sqrt{3}\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}[\chi(q)\Gamma^{\mu\nu}(q)]\varepsilon_\mu(k_1,\lambda_1)\varepsilon_\nu(k_2,\lambda_2), \tag{2}$$

where $\sqrt{3}$ is the color factor, $\chi(q)$ is meson's Bethe-Salpeter wave function, ε is photon polarization vector, $\Gamma^{\mu\nu}(q)$ is the $q\bar{q}\to\gamma\gamma$ scattering kernel. In pQCD leading order, $\Gamma^{\mu\nu}$ is

$$\Gamma^{\mu\nu} = -e^2e_q^2\Big[\gamma^u\frac{i}{\not p_1-\not k_1-m_c}\gamma^v + \gamma^v\frac{i}{\not p_1-\not k_2-m_c}\gamma^u\Big] \tag{3}$$

with $p_1 = \frac{P}{2}+q$, $p_2 = \frac{P}{2}-q$ and $P=(M,0,0,0)$. In practice, since we cannot get the BS wave function exactly, we have to make some approximations to deal with the integration of the product of BS wave function $\chi(q)$ and $\Gamma^{\mu\nu}(q)$. In literature, there are many approximate methods. Most of them are in fact equivalent to assuming the following BS wave function for a meson of spin J with internal $q\bar{q}$ spin S and orbital angular momentum L:

$$\chi(q) = N\sum_{s_1,s_2,S_z,M}\langle\frac{1}{2},s_1;\frac{1}{2},s_2\mid S,S_z\rangle\langle S,S_z;L,M\mid J,J_z\rangle u_{s_1}(p_1)\bar{v}_{s_2}(p_2)Y_{LM}(\Omega_q)R(|q|)\delta(q_0) \tag{4}$$

with normalization $N=\frac{i2\pi\sqrt{2M}}{2E_p}$ [8,9,11,13] or $N=\frac{i2\pi\sqrt{2M}}{2E_p}\times\frac{2m}{M}$ [16],

$$\int_0^\infty R^2(q)q^2dq = (2\pi)^3,$$

$$u_s(p) = \sqrt{E_p+m}\begin{bmatrix}\chi_s\\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E_p+m}\chi_s\end{bmatrix}. \tag{5}$$

Obviously, the $\delta(q_0)$ in Eq.(4) corresponds to the non-relativistic heavy quark limit. There are also some methods going beyond the $\delta(q_0)$ assumption in dealing with relativistic corrections. For example, in Ref.[14], by defining the equal time amplitude $\Phi(\boldsymbol{p}) := \int dq_0/2\pi\chi(p_0,\boldsymbol{p})$, which can be obtained by solving the corresponding Salpeter equation with an instantaneous interaction $V(\boldsymbol{p},\boldsymbol{p})$, they get the BS amplitude $\chi(q)$ as

$$\chi(q) = S(p_1)\int\frac{d^3q'}{(2\pi)^3}[-iV(\boldsymbol{q},\boldsymbol{q})\Phi(\boldsymbol{q}')]S(-p_2)$$

where $S(p)$ is the free quark propagators which reduces to the leading correction $\delta(q_0+M/2-\sqrt{\boldsymbol{q}^2+m^2})$. In relativistic case when considering the nonperturbative property of quark propagator, q_0 is not necessary to be limited to be zero or even function of $\boldsymbol{q}$. In order to investigate possible relativistic effects due to the extension of q_0 from zero, in this paper, we phenomenologically replace the $\delta(q_0)$ in Eq.(4) by a simple Gaussian function

$$f(q_0) = \frac{a}{\pi^{1/2}}e^{-a^2q_0^2} \tag{6}$$

which goes to $\delta(q_0)$ at the limit $a\to\infty$.

Usually, the quark spinor $u_s(p)$ is assumed to be the same as for a free quark, i.e., taking $E_p=\sqrt{\boldsymbol{p}^2+m^2}$. But we know that the quark is bound in a meson and its spinor should be different from a free one. For example, if we choose

$$u_s(p) = \frac{\frac{M}{2}+m}{\sqrt{E_p+m}}\begin{bmatrix}\chi_s\\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{\frac{M}{2}+m}\chi_s\end{bmatrix}, \tag{7}$$

we will get the form usually used in covariant projection method[20].

$$\chi(q) = N\sum_{S_z,M}\langle S,S_z;L,M\mid J,J_z\rangle\times Y_{LM}(\Omega_q)R(|\boldsymbol{q}|)\delta(q_0)\frac{1}{2\sqrt{2}(E_q+m)}\times(m+\not p_1)(1+\not P/M)\Pi_{s,s_z}(m-\not p_2) \tag{8}$$

with $\Pi_{S,s_z}=-\gamma_5$ for $S=0$, $-\not\varepsilon(s_z)$ for $S=1$.

Here we also consider a new kind of spinor from the Dirac equation for a relativistic spin-$\frac{1}{2}$ particle in a certain scalar potential $V(r)$. We have

$$\chi = \frac{1}{E_c+m-V}\boldsymbol{\sigma}\cdot\boldsymbol{p}\varphi$$

with

$$\Psi = \begin{bmatrix}\varphi\\ \chi\end{bmatrix},$$

instead of

$$\chi = \frac{1}{\sqrt{\boldsymbol{p}^2+m^2}+m}\boldsymbol{\sigma}\cdot\boldsymbol{p}\varphi$$

in the case for a free quark. To investigate possible effect of this deviation, we assume

$$u_s(p) = \sqrt{c+2m}\begin{bmatrix}\chi_s\\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{c+2m}\chi_s\end{bmatrix} \tag{9}$$

with $c\approx\langle E_c-V-m\rangle\approx(\frac{M}{2}-m)$.

For the spatial part, we take $R(q)$ as a simple harmonic-oscillator wave function

$$R(q) = \frac{1}{(\pi\beta)^{3/4}}\exp(-\frac{q^2}{2\beta})\sqrt{32}\pi^2 \tag{10}$$

for η_c and

$$R(q)=\sqrt{\frac{2}{3}}\frac{q}{\pi^{3/4}\beta^{5/4}}\exp(-\frac{q^2}{2\beta})\sqrt{32}\pi^2 \qquad (11)$$

for χ_{c0}. Where the parameter β has a relation to the mean-square-radius $\overline{r^2}$ of the meson η_c as $\beta\overline{r^2}=3/2$, χ_{c0} as $\beta\overline{r^2}=5/2$. So in our relativistic phenomenological approach, relativistic corrections from three sources are determined by two parameters, $b\equiv\sqrt{\overline{r^2}}$, a and with an uncertainty from the different descriptions of three spinor forms: BS1 for free spinor; BS2 for covariant form Eq.(7) and BS3 for Eq.(9).

3 The relativistic effects for two-photon decay of η_c and χ_{c0}

In this section, we present our numerical analysis of the relativistic effects from three sources for a heavy quark system, η_c and χ_{c0}.

1) Effect from $c\bar{c}$ relative 3-D momentum distribution

For the charm quark, its mass is in the range of 1.2—1.6GeV[21]. We choose values at two ends $m=1.2$GeV and $m=1.6$GeV to examine the correction for all the three kinds of spinor. The results are shown in Fig. 1, together with data and the NR result for comparison.

The results show that the relativistic correction from 3-D momentum distribution is about -60% in the physical region around $b=0.3$—0.4fm for η_c and 0.4—0.5fm for χ_{c0}. So in general it cannot be ignored even in $c\bar{c}$ system.

2) Effect from $c\bar{c}$ relative energy distribution

Here we take $b=0.36$fm for $m=1.2$GeV and $b=0.32$fm for $m_c=1.6$GeV to examine the effect with a finite a-value. We expect $a\approx b/v_q\approx b^2M/2$ with a value around 5GeV^{-1}. Fig. 2 shows the dependence of decay width on the $c\bar{c}$ energy distribution parameter a. Compared with results at $a=\infty$, the corrections with $a=5$GeV^{-1} are only about +2% for both η_c and χ_{c0}. The effect is much smaller than that from the relative momentum distribution for the $c\bar{c}$ system.

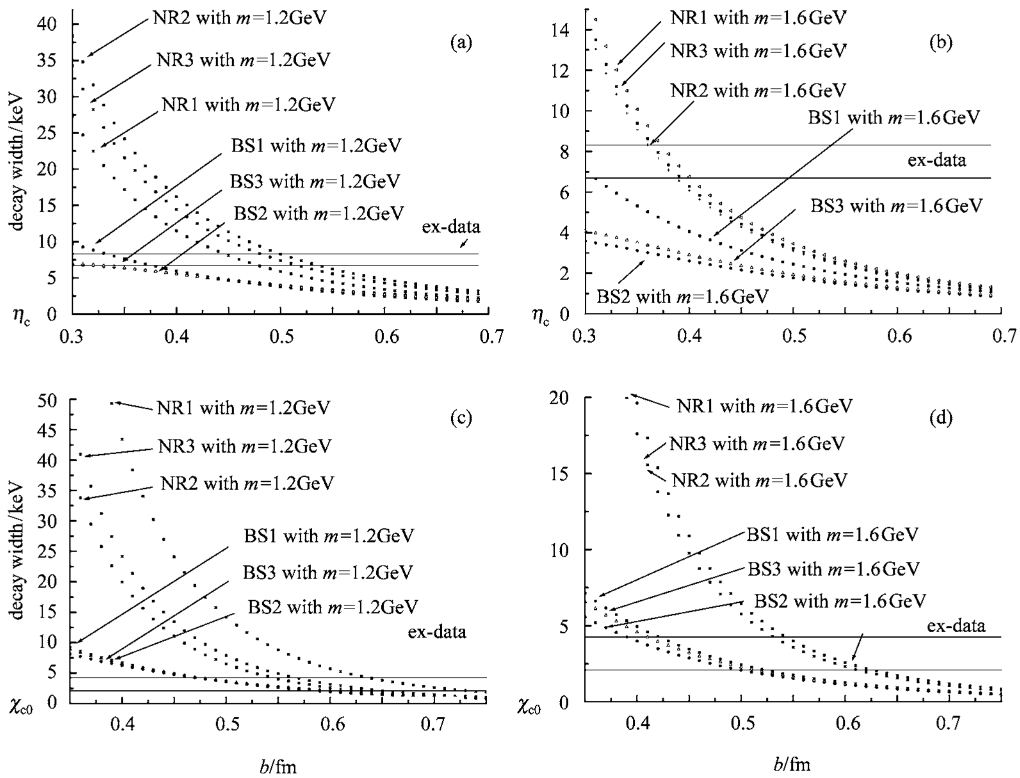

Fig. 1. Dependence of $\Gamma(\eta_c\to2\gamma)$, $\Gamma(\chi_{c0}\to2\gamma)$ on the size parameter b, compared with experimental data. NR for corresponding non-relativistic static limit; BS1 for relativistic case with spinor assuming $E=\sqrt{q^2+m^2}$; BS2 for relativistic case with spinor assuming Eq.(7). BS3 for relativistic case with spinor assuming $E=m+M/2$.

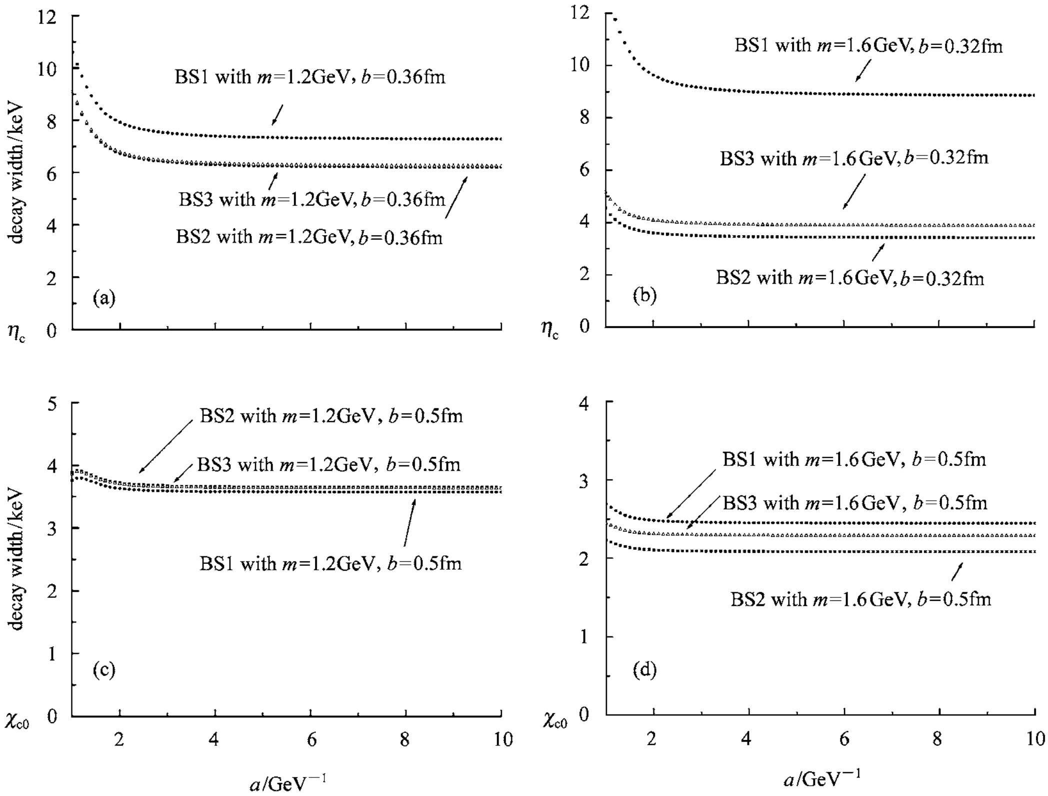

Fig.2. Dependence of $\Gamma(\eta_c\to 2\gamma)$, $\Gamma(\chi_{c0}\to 2\gamma)$ on the parameter a; BS1 for relativistic case with spinor assuming $E=\sqrt{q^2+m^2}$; BS2 for relativistic case with spinor assuming Eq.(7). BS3 for relativistic case with spinor assuming $E=m+M/2$.

3) Effect from assumption for the quark spinor

The results are also shown in Fig. 1. For η_c, when choosing $m=1.2$GeV, the BS1 treatment of the spinor gives about 15% larger value for the 2γ decay width than BS2 and BS3 treatments. For the case with $m=1.6$GeV, BS1 gives about 40% larger value than BS2 and BS3 which even cannot fit the data. For χ_{c0}, the BS1 treatment of the spinor gives about 4% larger value than BS2 and BS3 treatments with $m=1.2$GeV, and 15% with m = 1.6GeV. We see the effect from the different descriptions is sensitive to the mass of quark and meson.

4 Direct extension to light quark mesons

In this section, we examine the relativistic effects from the three sources by a direct extension of the formulae in previous sections to light quark mesons. For simplicity, we assume an $s\bar{s}$ 0^{-+} meson η_s of mass 0.958GeV with s-quark mass of $m=0.45$GeV.

1) Effect from $s\bar{s}$ 3-momentun distribution

The dependence of $\Gamma(\eta_s\to 2\gamma)$ on the size parameter b is shown in the left of Fig.3, compared with non-relativistic (NR) static limit. For the physical region of b around 0.7 fm, the correction is about -70% in average.

2) Effect from $s\bar{s}$ relative energy distribution

Choosing the parameters $b=0.7$ fm, we examine the effect with a finite a-value for the $s\bar{s}$ relative energy distribution. The results are shown in the right of Fig.3. Similar to the case of η_c, we expect $a\approx b^2M_{\eta_s}/2$ with a value around 6GeV^{-1}. Compared with the result at $a=\infty$, the correction with $a=6$GeV^{-1} is about 10%. The effect is much larger than that for the $c\bar{c}$ system as expected.

3) effect from assumption for the bound-quark spinor

We choose parameters $a=\infty$ and $b=0.7$ fm for this investigation. The different choosing of spinor gives about 50% uncertainty. The effect is also much larger than that for the $c\bar{c}$ system as expected.

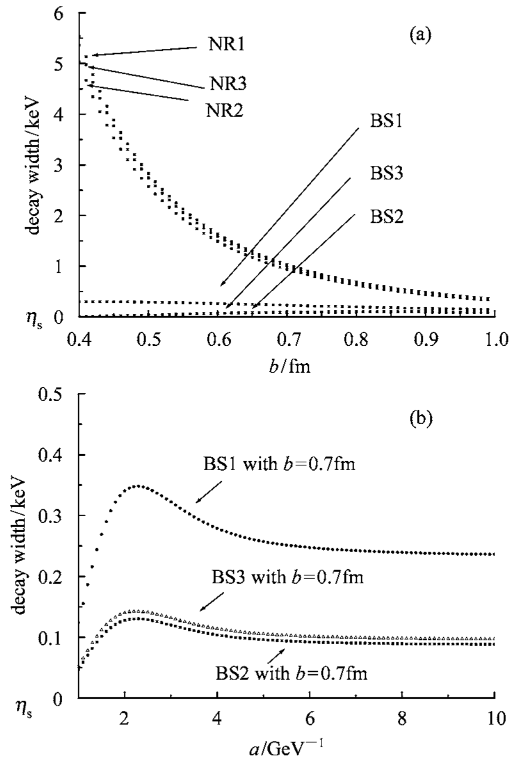

Fig.3. Dependence of $\Gamma(\eta_s\rightarrow 2\gamma)$ on the size parameter b and a. NR for corresponding non-relativistic static limit; BS1 for relativistic case with spinor assuming $E=\sqrt{q^2+m^2}$; BS2 for relativistic case with spinor assuming Eq. (7). BS3 for relativistic case with spinor assuming $E=m+M/2$.

5 Conclusion

In summary, relativistic effects in two-photon decay of 0^{-+} and 0^{++} quarkonium are investigated with a relativistic phenomenological approach. Compared with the non-relativistic approximation usually used in the decay of heavy quarkonium, the relativistic phenomenological approach gives corrections coming from three sources: (1) $q\bar{q}$ relative momentum distribution, (2) $q\bar{q}$ relative energy distribution and (3) description of quark spinors in the meson. These relativistic effects are studied in detail for $c\bar{c}$ and $s\bar{s}$ systems. In most previous works, only the effect from the first source was considered.

From our analysis, we conclude:

1) For the η_c and χ_{c0} charmonium system, the main correction comes from the $c\bar{c}$ relative momentum distribution. Its effect is about 60%, compared with the NR static limit, and hence cannot be neglected. Various treatments for the bound quark spinor could cause about 20% uncertainty in average, while the $c\bar{c}$ relative energy distribution gives little correction of 2% level.

2) For the 0^{-+} $s\bar{s}$ system, relativistic effects from all three sources are important. While the effect from the first source is still the largest one (about 70%), the source (2) gives about 10% correction and the source (3) gives about 50% uncertainty.

Therefore the usual treatment of relativistic effect by considering only the first source is reasonablly good for the charmonium system, but not good enough for direct application to the light quark mesons. The effects from the other two sources should be taken into account.

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夸克偶素双光子衰变的相对论效应*

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摘要　考虑到各种相对论效应,唯象地研究了 0^{-+} 和 0^{++} 夸克偶素的双光子衰变.相比于通常用的非相对论近似,我们的相对论唯象方法给出了3种分别由介子内夸克相对动量,相对能量,以及它的自旋形式的选取带来的效应.在 $c\bar{c}$ 和 $s\bar{s}$ 中我们做了具体的分析.

关键词　双光子衰变　夸克偶素　相对论效应

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